Review of the book

"Introduction to Number Theory"

by Martin Erickson and Anthony Vazzana
Chapman & Hall CRC, 2008

ISBN: 978-1-58488-937-3

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May 2011

1 Summary of the review

Introduction to Number Theory is a well-written book on this important branch of mathematics. The author organizes the work in a very structured way, dividing it into a first part about core topics that starts from the very basics, and a second and a third part regarding advanced topics, such as elliptic curves or Hilbert's tenth problem. The book is hence suitable for a wide range of readers, and the clear, almost story-like structure makes it easy to follow. As a plus, every chapter is correlated with interesting anecdotes about famous mathematicians from the past that gave important contributions to number theory, such as Euler, Gauss, or Euclid. Examples and calculations for two popular softwares like Mathematica and Maple are also provided, as well as an appendix describing how to use them. Strongly recommended to anyone interested in number theory.

2 Summary of the book

Number theory is one of the oldest branches of mathematics, its roots going back to the times of the legendary greek mathematicians such as Euclid or Erathostenes. The importance of this field of study has greatly increased in recent times due to applications, especially in cryptography, that are vital in the modern society.
The book aims to give a detailed introduction to this beautiful subject and to provide the reader a complete and solid understanding of it. It is divided into three main sections.

2.1 Part I

In Part I, the core topics are treated, starting from the very basics (natural numbers, principle of induction) and moving on to fundamental concepts like primes and divisibility (chapter 2), congruences (chapter 3) and quadratic residues (chapter 5). An exception is constituted by chapter 4, which is a brief overview of cryptography and its connection with number theory (ciphers, primes factorisation and the RSA cryptosystem). All of these chapter constitute the foundation and have to be considered as prerequisites for the following chapters.

2.2 Part II

Part II is about further topics in number theory. This includes arithmetic functions (chapter 6), a study of large primes (chapter 7), continued fractions (chapter 8) and diophantine equations (chapter 9).
All of these topics require a greater mathematical maturity and a certain confidence with proofs and notation. Despite the name "further topics", this is still to be considered an essential part of number theory.

2.3 Part III

Part III covers advanced topics and offers a spotlight on important and actual mathematical problems such as the very famous Fermat's Last Theorem (presented in chapter 11) and Hilbert's Tenth Problem (described in chapter 12). The rest of the chapters introduce ideas from analytic number theory (chapter 10) such as the well-known Riemann Zeta Function, an introduction to the theory of elliptic curves, and connections with logic. Knowledge of fundamentals of analysis and algebra is strongly recommended, as well as a cryptographic background (like the one given in chapter 4) for the part about elliptic curves.

2.4 Appendices

Finally, in the end of the book four appendices are provided, respectively about Mathematica basics, Maple basics, Web resources and notation, and the first two are especially useful if the reader has never approached these softwares before.

3 Style of the book

The text is written in a clear and reader-friendly style, and the topics are described carefully and naturally, almost like a story. That makes it easy to follow and very enjoyable.
Throughout the book a wide range of applications to "real-world" problems is presented, relatively to each topic, such as the case of RSA and the ISBN system. Many exercises and worked examples are provided, using both Mathematica and Maple packages. As a plus, almost every chapter is correlated with interesting anecdotes about the great mathematicians of the past that gave a contribution to number theory, from the already cited Euclid and Erathostenes to Euler, Fermat, and Gauss.

4 Would you recommend the book?

The author succeeds in presenting the topics of number theory in a very easy and natural way, and the presence of interesting anecdotes, applications, and recent problems alongside the obvious mathematical rigor makes the book even more appealing. I would certainly recommend it to a vast audience, and it is to be considered a valid and flexible textbook for any undergraduate number theory course.

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