Review of the book

“Problem Solving Through Problems”

Loren C. Larson

Problem Book in Mathematics
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1 What the book is about

What do Don Coppersmith, Berlekamp, Jeff Lagarias, Peter Shor, Paul Vojta, Peter Montgomery, Neal Koblitz and Richard Feynman have in common? Well, they are all Putnam Fellows. These are undergraduate students who have been ranked top 5 participants in the famous annual Putnam math competition in USA/Canada. All of them in this list are great problem solvers, and many of them have made significant contributions to cryptography (eg. Don Coppersmith's work on RSA via lattices-he was Putnam Fellow for 4 consecutive years). Math problem solving skills are obviously important to cryptographers and this book covers many ways of solving math problems. It is highly recommended by many, including Stanford math professor Ravi Vakil, himself a 4-time Putnam fellow, UC-Irvine math professor Karl Rubin, also a Putnam fellow.

The purpose of this book is to draw attention to the most important problem solving techniques encountered in undergraduate math and to illustrate their use by interesting examples and problems. Each section features a single idea and versatility of which is demonstrated in the examples and reinforced in the problems.

This 332-page book is both an anthology of problems and a manual of instruction. It contains overs 700 problems, over 33% of which are worked out in detail. The book's aim throughout is to show how a basic set of simple techniques can be applied in diverse ways to solve an enormous variety of math problems.

2 What the book is like

There are 8 chapters in this book:

Chapter 1: Heuristics (basic problem solving techniques such as searching for patterns, drawing relevant figures, modifying or formulating an equivalent simpler problem, exploiting symmetry, dividing into simpler cases, working backwards, argue by contradiction, pursuing parity, considering extreme cases and sometimes generalising the problem).

Chapter 2: Induction and Pigeonhole, which is a simple but powerful principle to prove existence results)
Chapter 3: Arithmetic (elementary number theory)
Chapter 4: Algebra
Chapter 5: Summation of Series
Chapter 6: Intermediate Real Analysis
Chapter 7: Inequalities
Chapter 8: Geometry
Many examples are given to illustrate these concepts. The writing style is casual and detailed and is thus easy to follow the arguments. Sometimes multiple solutions are given to illustrate the different angles in attacking a problem (e.g. in page 281, 5 solutions are given).

3 What I like about this book

The book considers interesting problems (with full solutions) like

P1. Prove there exists *a,b,c* integers not all zero, with absolute value $<10^6$ such that

$|a + b\sqrt{2} + c\sqrt{3}| < 10^{-11}$ (pp 81).

P2. Given 9 lattice points in R3, show that there is a lattice point on the interior of one of the line segments joining two of these lattice points. (pp 47)

P3. Find positive integers *n* and $a_1, \ldots, a_n$ such that $a_1 + \ldots + a_n = 1000$ and the product of the a_1's is as large as possible. (pp 7)

P4. If 3n +1 is a perfect square, show that n+1 is the sum of 3 perfect squares. (pp 26)

P5. symetryEvaluate the integral $\int 1/[\, 1 + (\tan x)^{\sqrt{2}}]dx$ from 0 to $\pi/2$. (pp 32)

P6. pGiven any set of 10 integers between 1 and 99 inclusive, prove that there are 2 disjoint nonempty subsets of the set with equal sum of their elements. (pp 81)

P7. Are there 1million consecutive integers each of which contains a repeated prime factor? (pp 97).

P8. Does $[x] + [2x] + [4x] + \ldots + [16x] + [32x] = 12345$ have a real number solution? (pp 107)

P9. Prove that there are no prime numbers in the infinite sequence of integers

10001, 100010001, 1000100010001,….(pp 123)

P10. The n- polygon of greatest area that can be inscribed in a circle must be the regular n-polygon (pp 70).

Hints: P1, P2, P6 (pigeonhole principle), P3 (observation), P4,P9 (algebra), P8 (positional notation), P10 (induction & trigonometry), P5 (symmetry), P7 (Chinese Remainder Theorem).

Many of the problems in the book are from math olympiads, Putnam exams and math journals (index to problems given at the end of book). Many important topics are covered and they are done in detail. Plenty of interesting examples and exercises (with solutions to about 33% of them). The authors also covered many formulations of important problems.

The font size is quite large and the arguments are neatly displayed.

4 Possible Improvements

I feel that the author should add 2 very important topics: counting and probability. Counting theory is extremely beautiful and many faceted. Probability is fundamental in math.
Brief answers or hints to every problem in the book will be helpful to those who want to check if they have done the problem correctly.

5 Would you recommend this book?

The book's intended audience: upper undergraduates or even a mature sophomores who has some exposure to elementary number theory, algebra analysis and geometry. However the problems chosen are deeper than typical undergraduat problems, though the tools needed are basic. This book is especially suited to prepare students for math competitons. It can also be used by scientific/technical professionals who are interested to develop their mental agility and creativity.

By now the reader should know that I strongly endorse this book.
The reviewer is a researcher in infocomm security with specialty in math and cryptography. He was formerly a professor in math.