Review of the book

"Chromatic Graph Theory"

by Gary Chartrand and Ping Zhang

CRC Press, Taylor & Francis Group, 2009

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1 Summary of the review

In graph theory, there exists many major topics, graph coloring being one of them. Chromatic Graph Theory is a self-contained book on graph theory with emphasis on graph coloring. By including fundamentals of graph theory in the first few chapters, Chromatic Graph Theory allows any reader to pick up the necessary background to understand the building blocks of graph coloring described in the middle section of the book. Readers will then be able to access a carefully selected list of advanced and recent topics in graph coloring, inscribed in the latter part of the book.

Proofs are carefully explained and easy to follow with a good number of examples given, although some of the facts used are not immediate and instead due to the original assumptions. This requires just a little bit of thinking on the readers' side but makes for a succinct exposition. For each chapter, there is a decent amount of questions with good varying levels of difficulty. No solutions are provided (unlike some books where odd/even numbered questions have solutions), although instructors will be able to order the solution manual. This makes the questions suitable to be used for courses. Chromatic Graph Theory also features 14 suggested study projects aimed to guide readers in future exploration of the advanced topics.

Chromatic Graph Theory also includes a good portion of historical and applied aspect for selected topics, which makes the book more interesting to read.

2 Summary of the book

The book starts off with chapter 0, which is a historical summary on the origin of graph coloring. The following 5 chapters cover the fundamentals of graph theory, presented in an introductory manner, so that any reader without the relevant background will be able to absorb them easily. This builds up nicely to the next 5 chapters that can be seen as the basics in graph colorings. It is worthwhile to note that these 5 chapters are also the ones that are usually covered in general graphy theory texts (Perhaps not so much for chapter 9), although it is presented in more detail in this book. The final 4 chapters of the book can be seen as more advanced or recent topics in graph coloring. At the end of the book, the authors proposed 14 graph coloring study projects to aid readers in independent study. The problems are not open in general and should be solvable given time and effort.

Chapter 0: The Origin of Graph Colorings
As graph coloring originated from the famous four color problem, this chapter is dedicated to the recount of its history. With full 26 pages used to provide a rather detailed recount of its origin up till the point

where its solution was announced, this chapter allows the reader to follow an interesting and exciting development of the problem over the span of more than 100 years. Also, the chapter is written in a way where general reader will be able to understand and follow, as the difficult results are not explained in details. In comparison, most texts only provide a couple of pages of information.

Chapters 1-5: Fundamentals of Graph Theory
These chapters are provided to make the book standalone. Anyone without background in graph theory will be able to pick up the basics required to understand the subsequent chapters. Although they provide sufficient information for the basics, due to the limitation in space only the essential results are presented. Hence, experienced readers are unlikely acquire new knowledge. On the other hand, some of the harder practice questions may still be of interest.

Chapters 6-10: Basic Graph Coloring Topics
For these 5 chapters, they can be considered as basic graph coloring topics and are usually covered in general graph theory courses and texts. However, in this book these chapters tend to contain more contents and more advanced topics as compared to general graph theory texts. Chapter 6 starts off with introduction to vertex coloring and the chromatic number, the least number of colors required to color a graph so that adjacent vertices are colored differently. There is also a section on the applications of colorings. This chapter establishes the basis and definitions of graph coloring on vertices for following chapters, as well as providing some insight to the importance of graph coloring in general. Chapter 7 follows up on the analysis on the chromatic number, establishing bounds under various considerations. More importantly, this chapter describes how the better bounds are obtained using the greedy algorithm approach. On chapter 8, the book considers the vertex coloring problem applied to graph embedded on surfaces. This links back to the original four color problem and the reader should be in a position to better appreciate some of the earlier results described in chapter 0. Chapter 9 considers the vertex coloring problem when restrictions are imposed on the set of vertices. Finally, chapter 10 introduces the concept of graph coloring on edges instead of vertices, with similar considerations and discussions as the case with vertices.

Chapters 11-14: Advanced Graph Coloring Topics
As mentioned, the last 4 chapters cover advanced and recent topics in graph coloring. Chapter 11 provides a discussion on monochromatic and rainbow edge colorings, where adjacent edges are allowed to have the same color. For these colorings, usually there are restrictions imposed, for example the number of colors are limited or certain subgraphs are forbidden. Under this context, the common aim is to characterize the types of graph that satisfies these requirements. Chapter 12 presents complete colorings, which is a study on the ways to color graphs such that for any two colors used, there exists two adjacent vertices of these two colors. This is in effect another way of analyzing the bound of the chromatic number as the conditions for both are essentially same. Chapter 13 is about distinguishing colorings, where vertices or edges are made distinct from each other due to the colorings assigned to them. The final chapter, "Colorings, Distance and Domination", describes distance-related vertex colorings as well as domination and is closely related to the Channel Assignment Problem.

3 What is the book like (style)?

This book is written in a style suitable for usage in introductory and advanced courses to graph theory with emphasis on graph colorings. In this context, each chapter has a good amount of questions with varying difficult of question, making it very suitable for courses. This is made possible as no solutions are provided for the questions and instead instructors may order a copy from the authors. For each topic discussed, most if not all of the central results are provided with carefully written proofs. Examples and diagrams are also well chosen for some of the topics, making it much easier to study.

Some of the facts stated in proofs are actually not immediate and will require readers to think a little. These are usually due to some of the assumptions and propositions involved at the start of the proofs. However, they are easy to derive and makes for very succinct expositions.

As the book has to cover a large range of topics, some of them are only covered in exact sufficiency and not too in depth. The content of the book is in fact carefully chosen and the result is a rather compact book yet having enough material for its original theme. A couple of sub-chapters of topics not related to graph coloring do exist, for example de Bruijn digraphs. These may be removed or skipped without affecting one's ability to understand the remaining chapters. Perhaps the authors may consider replacing them with something else, but since they only account for a small portion of the book it should not matter too much.

Aside from the central results, Chromatic Graph Theory also adds historical discussions and describes applied aspects on some of the topics, which gives more flavor to the book and makes it more interesting to read. Occasionally, the book also lists some of the well known and important conjectures and theorems. These are based on difficult problems and thus not explained in detail, with their presence meant to give insight on the depth of the topics and to highlight some of the beautiful results for the readers to appreciate and savour.

4 Would you recommend this book?

In the preface of the book, it is explicitly stated that the book is intended for the following groups:

- A graph theory course with emphasis on graph colorings, either a beginning course in graph theory or as a follow up course to an elementary graph theory course
- A reading course on graph colorings
- A seminar on graph colorings
- As a reference book for individuals interested in graph colorings

In regards to the first group, I should point out that the same authors had previously wrote an introductory book for graph theory: "Introduction to Graph Theory", published by McGraw Hill in 2005. Incidentally, I had also read that book a while back and I find that it transitions nicely into the current one. This is likely what the authors had in mind when they recommended this book as a follow up course. In light of this experience, I would recommend this book to anyone who is planning for an advanced undergraduate or graduate course on graph coloring.

On the other hand, if the book is used as a beginning course, one should note that the depths on the fundamentals (chapters 1-5) are not explored as much as some other texts. Instead, this book provides more content on basic graph coloring topics (chapters 6-10). This will result in less time spent on fundamentals and more focus on graph coloring. Whether it is suitable as an introductory course for graph theory will depend on the instructors' intention. Also, it will be rather unlikely that the advanced chapters can be covered in time. In general, I would not recommend this book for a beginner graph theory course.

This book is especially good for a reading course on graph coloring or for any individuals who wish to spend the least amount of time to pick up advanced topics on graph coloring from scratch. For these two groups of readers, I would highly recommend the book. In fact, they are likely the groups to benefit the most.

I had previously read 2 introductory and 1 graduate level graph theory texts, each of them having a few chapters on graph colorings. Perhaps not surprisingly, all their contents can also be found in this current book. On the contrary, approximately half of this book's cannot be found in the ones I read. On top of covering the basics of graph coloring fully, this book also contains much emerging areas. This makes for a good indicator of using this book as a reference in graph colorings. I would recommend this book to anyone who has such intention in mind.

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