Composition Does Not Imply Adaptive Security

Krzysztof Pietrzak
ETH Zürich

a 08/15 talk

Pseudorandom Functions

A PRF is a family of functions $\mathcal{F}$ indexed by $\mathbb{N}$. For $n \in \mathbb{N}$ the function $\mathsf{F} \in \mathcal{F}$

$$\mathsf{F} : \mathcal{K}_n \times \mathcal{X}_n \to \mathcal{Y}_n \qquad \text{notation : } \mathsf{F}_k(.) = \mathsf{F}(k, .)$$

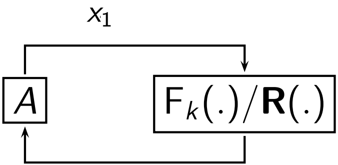
is such that for all efficient distinguishers A.

$$\left| \Pr[A^{\mathsf{F}_k(.)} \to 1] - \Pr[A^{\mathbf{R}} \to 1] \right| = \mathrm{negl}(n)$$

Here $\mathbf{R}$ is a uniform random function and $k \in_R \mathcal{K}_n$ is a random key.

Adaptive vs. Non-Adaptive Distinguisher

Adaptive distinguisher A



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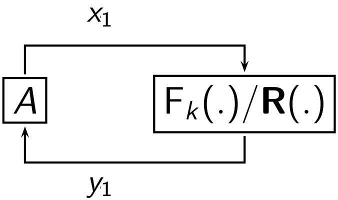

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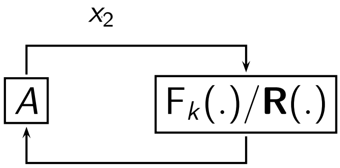

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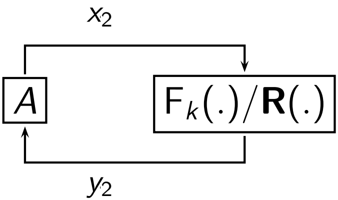

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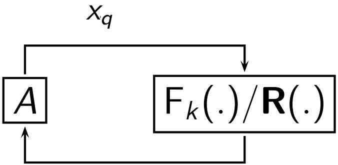

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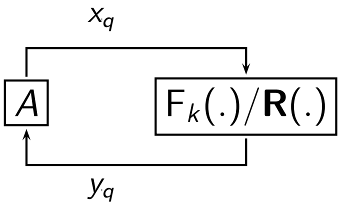

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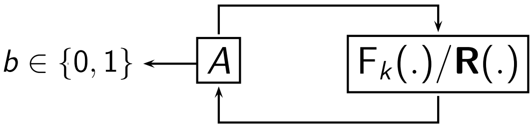

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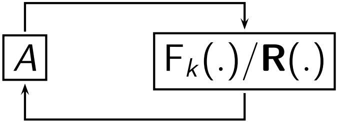

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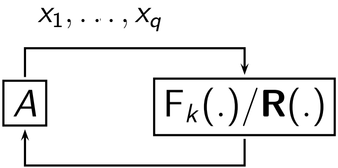

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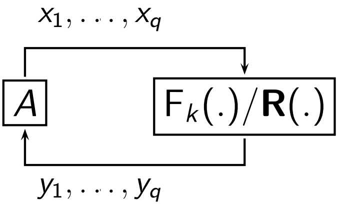

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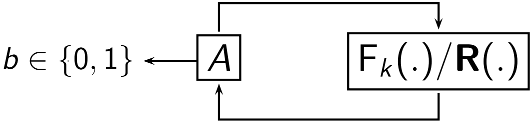
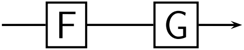


Composition of PRFs

Sequential composition: $(\mathsf{G} \circ \mathsf{F})(x) = \mathsf{G}(\mathsf{F}(x))$



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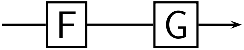
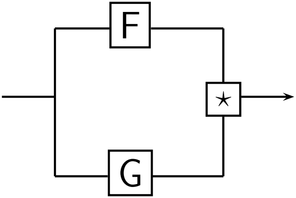


Parallel composition: $(F \star G)(x) = G(x) \star F(x)$.



Adaptive Security by Composition?

Question:

If $\mathcal{F}, \mathcal{G}$ are non-adaptively secure PRFs, is $\mathcal{G} \circ \mathcal{F}$ or $\mathcal{F} \star \mathcal{G}$ adaptively secure?

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- If the answer is yes in the computational setting, then there is *no black-box proof* for it. [Myers, EC04].

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- Yes, in the information theoretic setting [Maurer,P., TCC04].
- If the answer is yes in the computational setting, then there is *no black-box proof* for it. [Myers, EC04].
- No, in the computational setting under DDH.

Distinguishing advantage

Let $\mathcal{A}(t, q)$ denote all distinguishers running in time t and making at most q oracle queries.

$$\mathbf{Adv}_{\mathbf{F}}(t, q) = \max_{A \in \mathcal{A}(t,q)} \left| \Pr[A^{\mathbf{F}(.)} \rightarrow 1] - \Pr[A^{\mathbf{R}(.)} \rightarrow 1] \right|$$

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$\mathbf{Adv}_{\mathbf{F}}^{non-adaptive}(t,q)$ defined like $\mathbf{Adv_F}(t,q)$ but for non-adaptive distinguishers.

The Information Theoretic Setting

Theorem (Maurer,P.04)

If $\mathbf{Adv}_{\mathbf{E}}^{non-adaptive}(\infty, q) \leq \epsilon$ *and* $\mathbf{Adv}_{\mathbf{F}}^{non-adaptive}(\infty, q) \leq \epsilon$
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$$\mathbf{Adv}_{\mathbf{E} \star \mathbf{F}}(\infty, q) \leq 2\epsilon(1 + \ln \epsilon^{-1})$$

The Computational Setting

There exist non-adaptively secure PRFs $\mathcal{F}$ and $\mathcal{G}$ where $\mathcal{G} \circ \mathcal{F}$ $(\mathcal{F} \star \mathcal{G})$ are not adaptively secure.

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Let $\mathcal{H} = \mathcal{H}(n)$ be a prime order cyclic group and g a generator of $\mathcal{H}$. Then for random a, b, c the distributions

$$(g^a, g^b, g^c) \text{ and } (g^a, g^b, g^{ab})$$

are indistinguishable.

El-Gamal Encryption

Public: group $\mathcal{H}$ of prime order P and generator g of $\mathcal{H}$.

Secret key is a random $x \in \mathbb{Z}_P$, public key is g^x. Encryption of $m \in \mathcal{H}$ with randomness $r \in \mathbb{Z}_P$:

$$\mathrm{Enc}_{g^x}(m, r) = (mg^{xr}, g^r)$$

Decryption of $(a, b) \in \mathcal{H}^2$:

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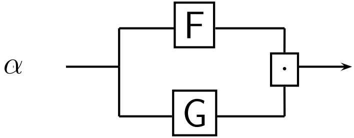
Intuition for the Parallel Counterexample



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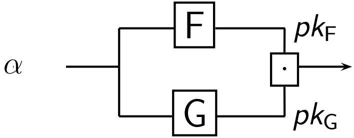

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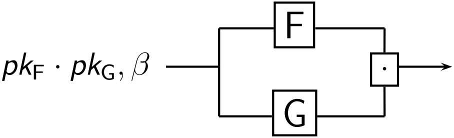

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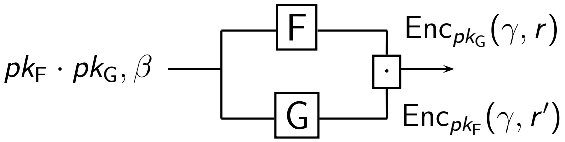

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2. $pk_F \cdot pk_G, \beta \longrightarrow \mathrm{Enc}_{pk_G}(\gamma, r) \cdot \mathrm{Enc}_{pk_F}(\gamma, r')$

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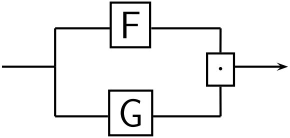

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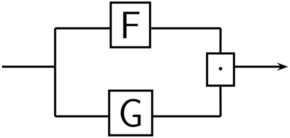

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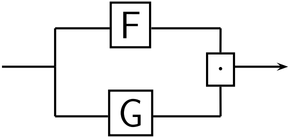

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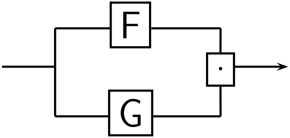

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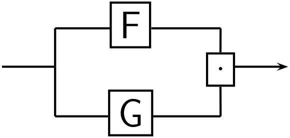

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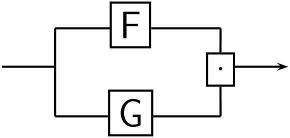

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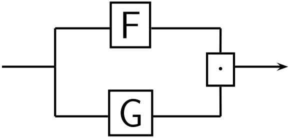

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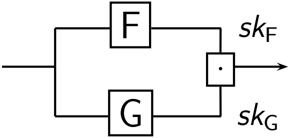

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Definition of the PRFs

Group $\mathcal{H}$ of prime order P, $\langle g \rangle = \mathcal{H}$.
A PRF $\mathsf{R} : \mathcal{K}_R \times \mathcal{H}^3 \to \mathbb{Z}_P^3$.

$\mathsf{F} : \mathcal{K} \times \mathcal{H}^3 \to \mathcal{H}^3$ where $\mathcal{K} = \mathbb{Z}_P \times \mathcal{K}_R$.

$F(\{x, k_F\}, u, v, w)$ is computed as:

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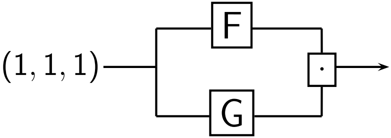
$F(\{x, k_F\}, u, v, w)$ is computed as:

$$(r_1, r_2, r_3) \leftarrow R_{k_F}(u, v, w)$$

$$\begin{array}{rcl}
F(1,1,1) & \rightarrow & (g^x, g^{r_2}, g^{r_3}) \\
F(u \neq 1, 1, 1) & \rightarrow & ((u/g^x)^{r_1}, g^{r_1}, g^{r_3}) \\
F(u \neq 1, v \neq 1, w \neq 1) & \rightarrow & (a, b, c) \quad \text{where} \\
& & (d, e, f) \leftarrow F(u, 1, 1) \\
& & \text{if } (v/d) = (w/e)^x \text{ then} \\
& & \quad (a, b, c) = (x, 1, 1) \\
& & \text{otherwise } (a, b, c) = (g^{r_1}, g^{r_2}, g^{r_3}) \\
F(\text{ all other cases }) & \rightarrow & (g^{r_1}, g^{r_2}, g^{r_3})
\end{array}$$

$\alpha = (1,1,1), \beta = (1,1)$ and $\gamma = 1$

The Parallel Counterexample



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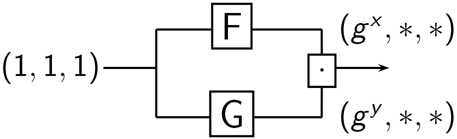

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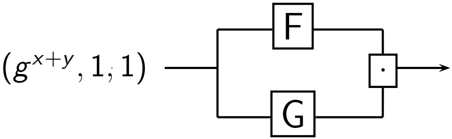

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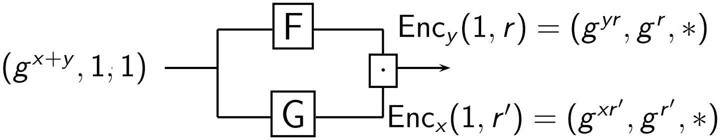

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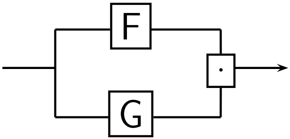
2. $(g^{x+y}, 1, 1) \to (g^{xr'+yr}, g^{r+r'}, *)$

The Parallel Counterexample

$$(g^{x+y}, g^{xr'+yr}, g^{r+r'})$$



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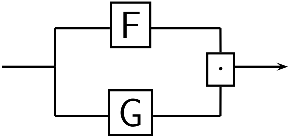
The Parallel Counterexample

$(g^{x+y}, g^{xr'+yr}, g^{r+r'})$



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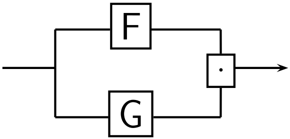

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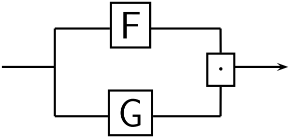

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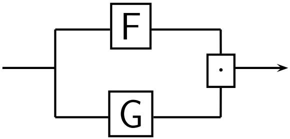

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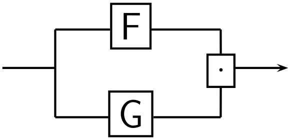

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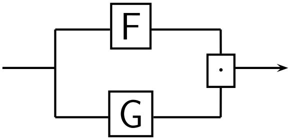

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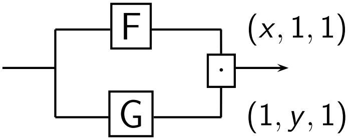

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\mathsf{F}(u \neq 1, v \neq 1, w \neq 1) &\rightarrow (a, b, c) \quad \text{where} \\
& \quad (d, e, f) \leftarrow \mathsf{F}(u, 1, 1) \\
& \quad \boxed{\text{if } (v/d) = (w/e)^x \text{ then}} \\
& \quad\quad (a, b, c) = (x, 1, 1) \\
& \quad \text{otherwise } (a, b, c) = (g^{r_1}, g^{r_2}, g^{r_3}) \\
\mathsf{F}(\text{ all other cases }) &\rightarrow (g^{r_1}, g^{r_2}, g^{r_3})
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Which is equivalent to distinguish

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- Counterexamples under weaker/other assumptions or even unconditionally? Probably no as counterexample for sequential composition implies key-agreement...
- Counterexample for sequential composition with pseudorandom *permutations*. This would show that cascading non-adaptively secure block-ciphers will not give adaptive security in general.