Improved Rebound Attack on the Finalist Grøstl

Jérémy Jean[1] María Naya-Plasencia[2] Thomas Peyrin[3]

[1]École Normale Supérieure, France

[2]University of Versailles, France

[3]Nanyang Technological University, Singapore

FSE'2012 – March 19, 2012

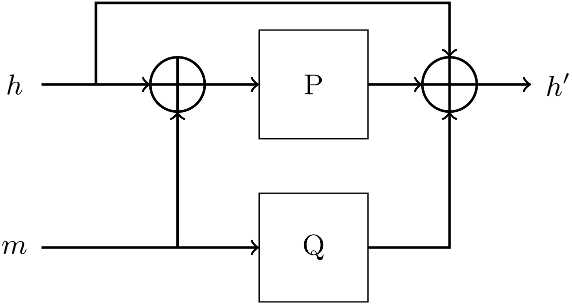
SHA-3 Competition Finalists

- In December 2010, the NIST chose the 5 finalists of the `SHA-3` competition:
  - `BLAKE`
  - `Grøstl`
  - `JH`
  - `Keccak`
  - `Skein`
- This year, the winner will be chosen.

Grøstl: Compression Function (CF)

`Grøstl-v0` [Knudsen et al. 08] has been tweaked for the final:

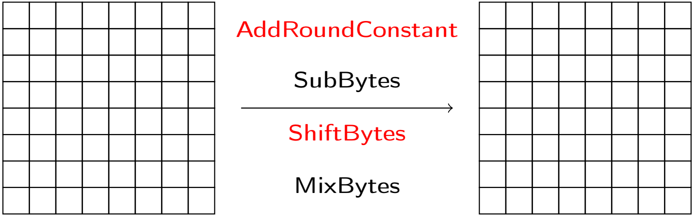
- `Grøstl-256`: $|h| = |m|$=512 bits.
- `Grøstl-512`: $|h| = |m|$=1024 bits.



Grøstl: Internal Permutations

Permutations P and Q apply the wide-trail strategy from the AES.

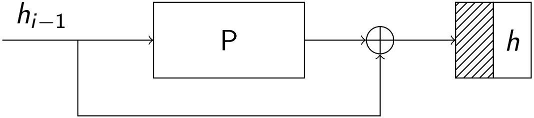
- Grøstl-256: 10 rounds on state a 8 × 8.
- Grøstl-512: 14 rounds on state a 8 × 16.



Tweak: constants in ARK and Sh changed to introduce asymmetry between P and Q

Grøstl: Finalization Round

Once all blocks of message have been treated: truncation.



Grøstl: Best Analysis After the Tweak

- Grøstl-256:
  - [Sasaki et al A10]: 8-round permutation distinguisher.
  - [Gilbert et al. FSE10]: 8-round CF distinguisher.
  - [Boura et al. FSE11]: 10-round zero-sum.
- Grøstl-512
  - [Schläffer 2011]: 6-round collision on the CF.

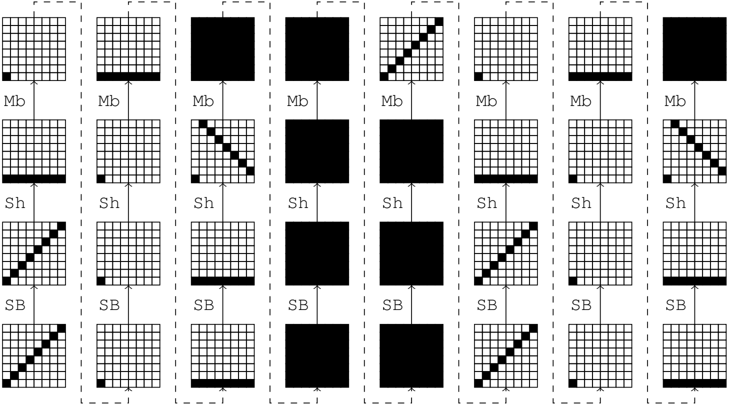
Our New Results 1/2

- Based on the rebound technique [Mendel et al. FSE09].
- Based on a way of finding solutions for three consecutive full active rounds: new.
- They apply both to 256 and 512 versions.

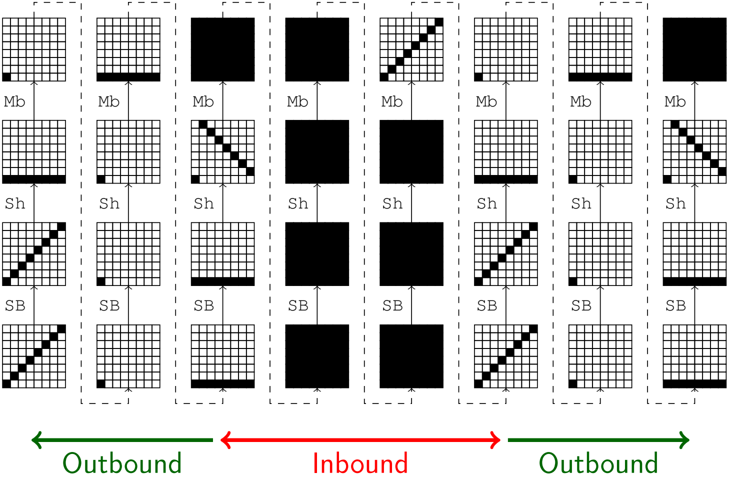
Our New Results 2/2

- On `Grøstl-256`, we provide distinguishers for 9 rounds of the permutation (total: 10).
- On `Grøstl-512`, we provide distinguishers for 8, 9 and 10 rounds of the permutation (total: 14).

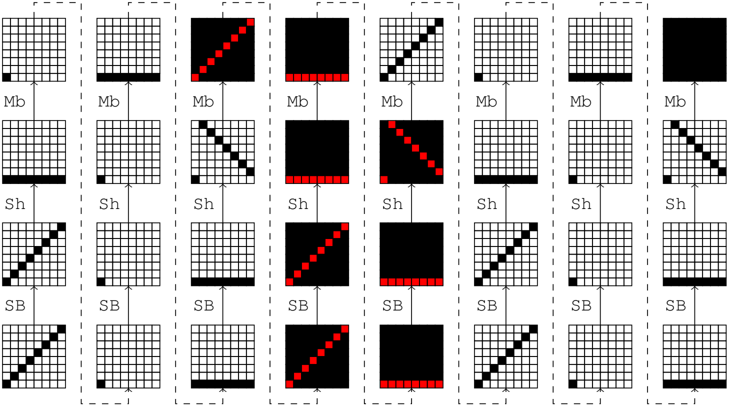
Rebound Attack



Rebound Attack



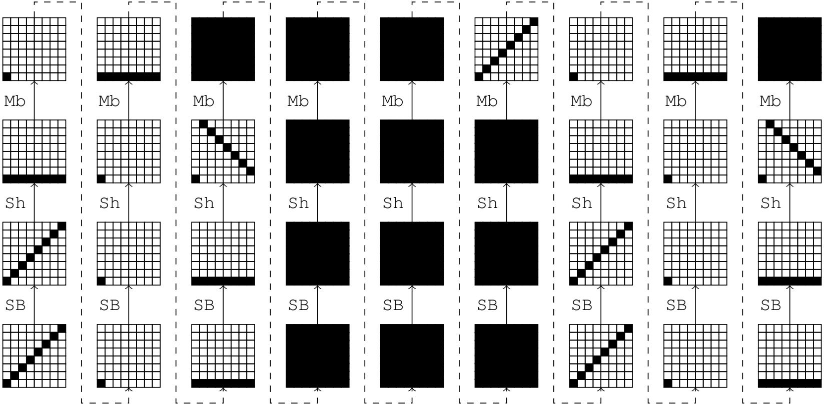
SuperSBox



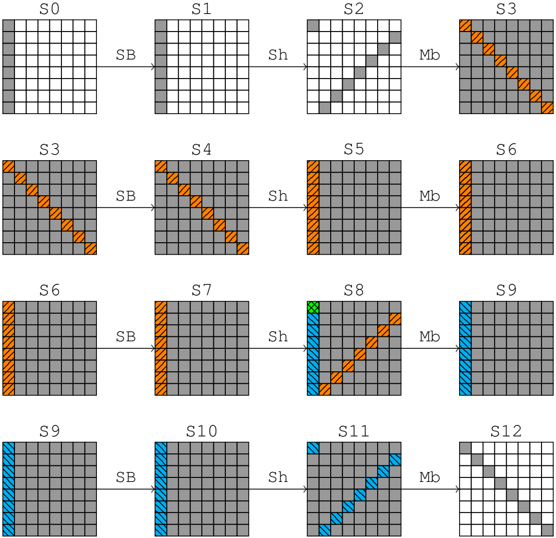
SuperSBox = $\mathtt{SB} \circ \mathtt{MC} \circ \mathtt{SB}$

Grøstl-256 Permutation

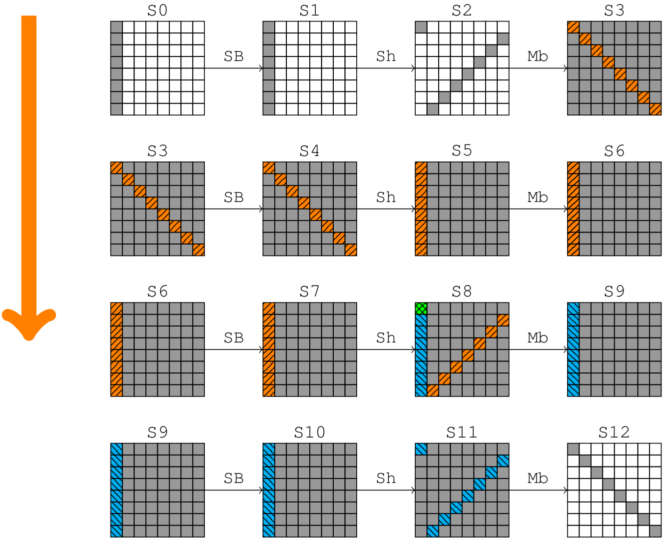
Differential Characteristic for 9 rounds



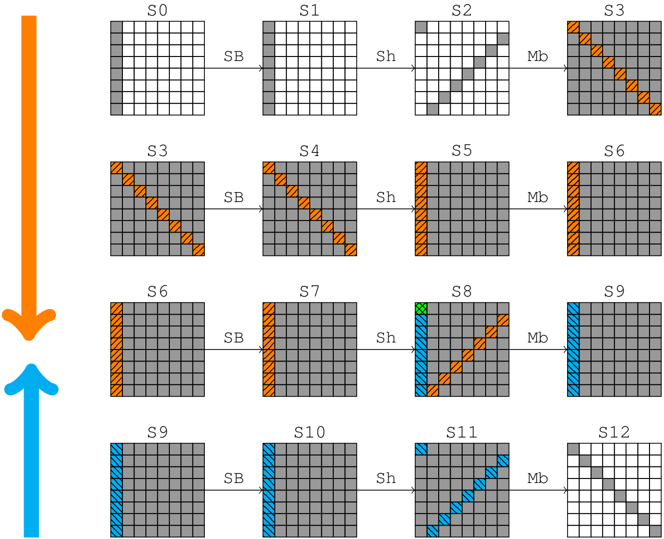
Inbound for 3 Full-Active Rounds



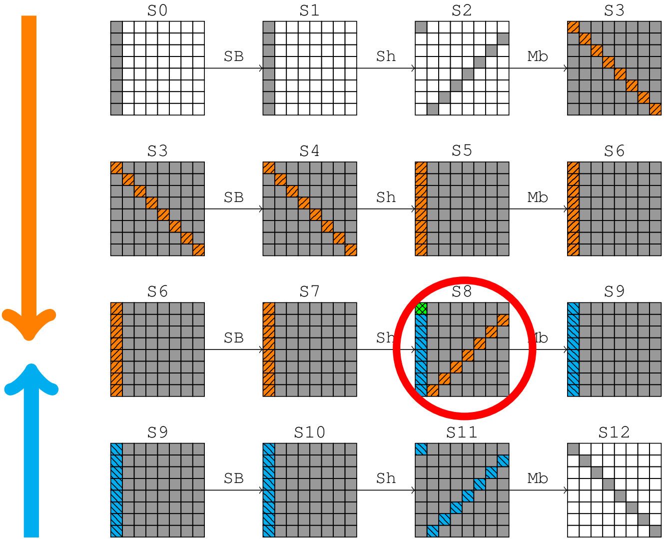
Inbound for 3 Full-Active Rounds



Inbound for 3 Full-Active Rounds



Inbound for 3 Full-Active Rounds



Inbound for 3 Full-Active Rounds: Analysis

Counting

- 8 forward SuperSBox sets of 2^{64} values and differences 🟧
- 8 backward SuperSBox sets of 2^{64} values and differences 🟦
- Overlapping on 512 bits of values + 512 bits of differences

Number of Solutions Expected

$$2^{8\times 64}\, 2^{8\times 64}\, 2^{-512-512} = 2^{512+512-512-512} = 1$$

Limited Birthday

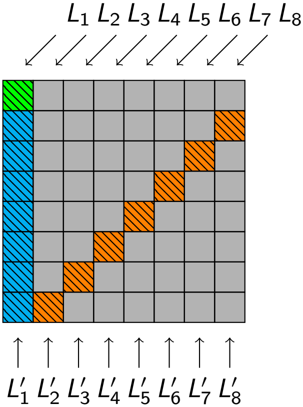
2^{384} operations

Our Algorithm

2^{256} operations, memory 2^{64}

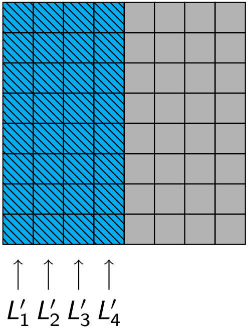
Solving the 3 Active Rounds: Context

The 8 forward L_i overlaps the 8 backwards L'_i like this:



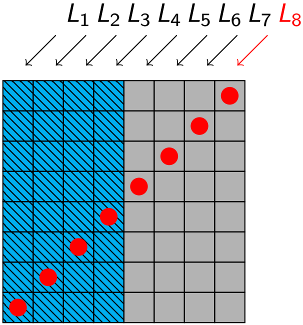
Solving the 3 Active Rounds: Step 1

We start by choosing one element in each of the four first L'_i.



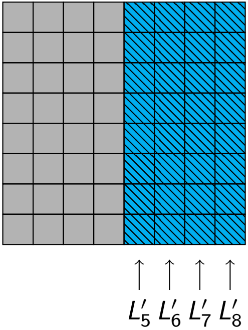
Solving the 3 Active Rounds: Step 2

This determines a single element in each L_i.



Solving the 3 Active Rounds: Step 3

Each determined element in the remaining L'_i exists with $p = 2^{-8\times8}$.



Summing Up

Inbound Phase

In total we try 2^{256} combinations of (L'_1, L'_2, L'_3, L'_4) and each gives a solution with probability: $2^{-4\times8\times8} = 2^{-256}$.

Outbound Phase

Probability $2^{-2\times56}$ to pass two $8 \rightarrow 1$ transitions in the MixBytes.

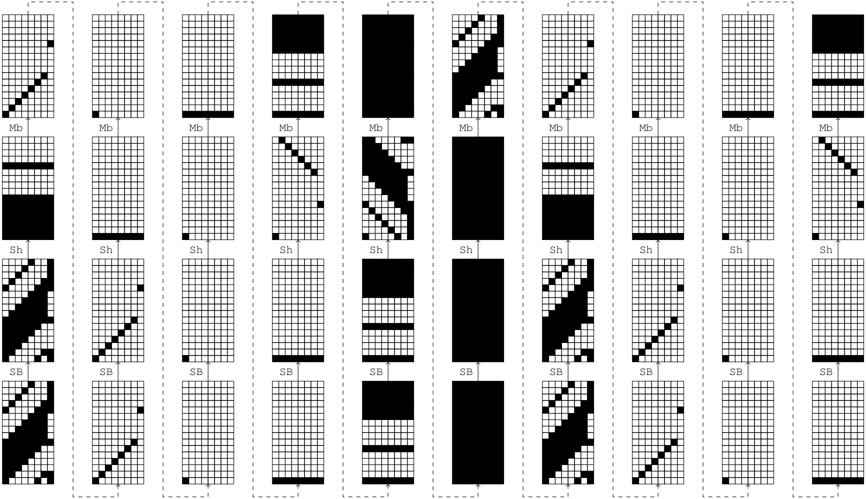
Distinguisher

We distinguish the 9-round permutation in $2^{256+112} = 2^{367}$ operations and 2^{64} in memory.

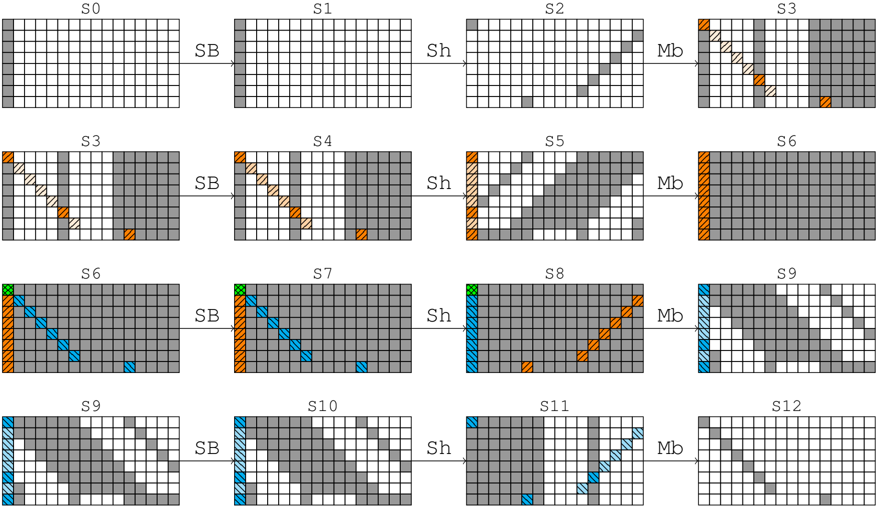
Note: This compares to a generic complexity of 2^{384} operations.

Grøstl-512 Permutation

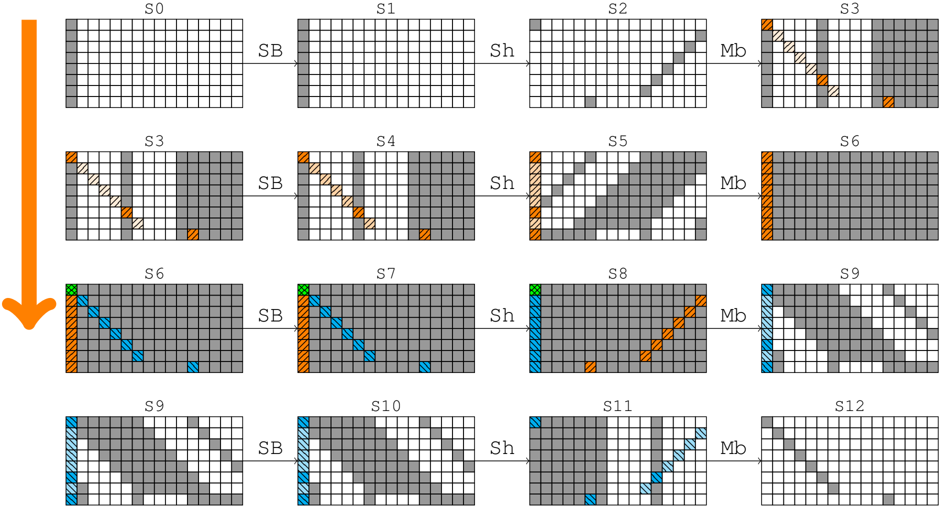
Differential Characteristic for 10 rounds



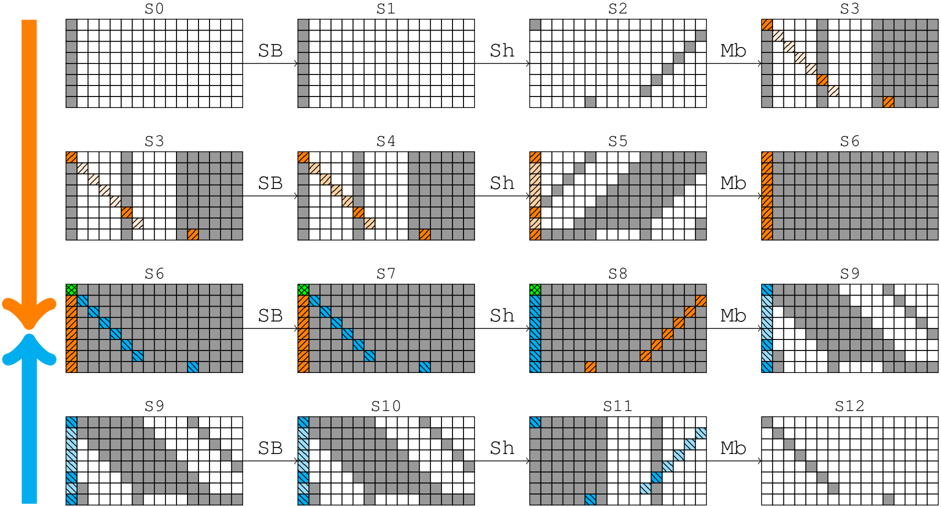
Inbound Phase



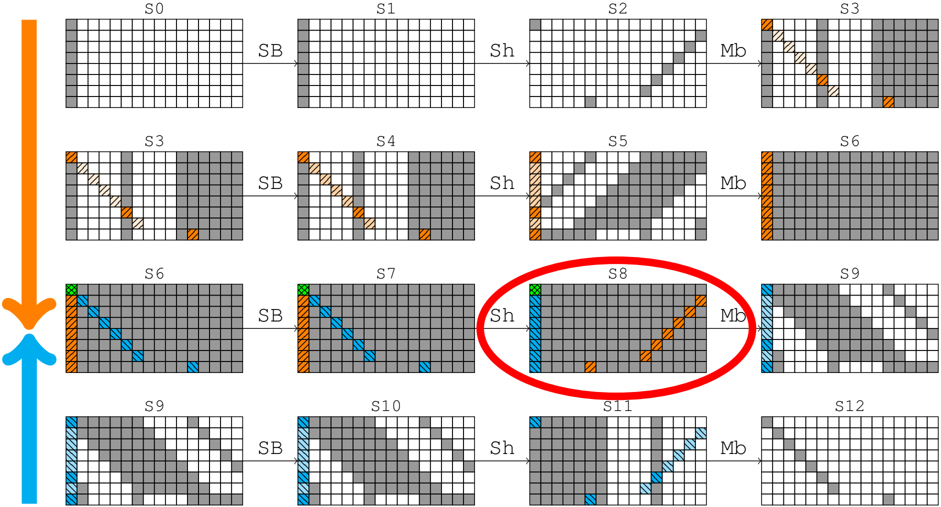
Inbound Phase



Inbound Phase



Inbound Phase



Observations

Counting

- 16 forward SuperSBox sets of 2^{64} values and differences 🟧
- 16 backward SuperSBox sets of 2^{64} values and differences 🟦
- Overlapping on 1024 bits of values + 1024 bits of differences

Number of Solutions Expected

$$2^{16\times64}\,2^{16\times64}\,2^{-1024-1024}=2^{1024+1024-1024-1024}=1$$

Limited Birthday

2^{896} operations

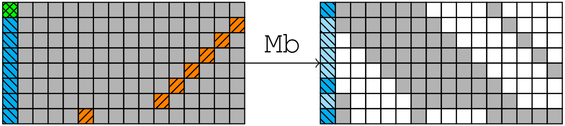
Our Algorithm

2^{280} operations, memory 2^{64}

Algorithm: Guess-and-Determine Approach

Constraints

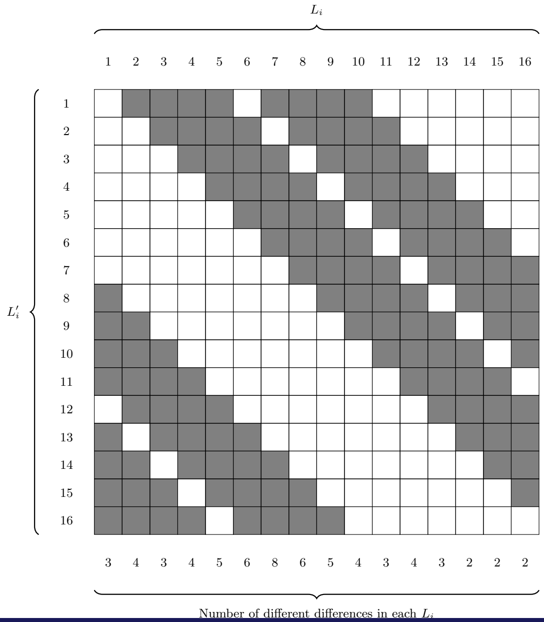
The differences around the MixBytes layer are restricted since the right state is not *fully* active.



Notations

- Forward SuperSBoxes: $L_1, \ldots, L_{16}$. 🟧
- Backward SuperSBoxes: $L'_1, \ldots, L'_{16}$. 🟦

Algorithm: Guess-and-Determine Approach



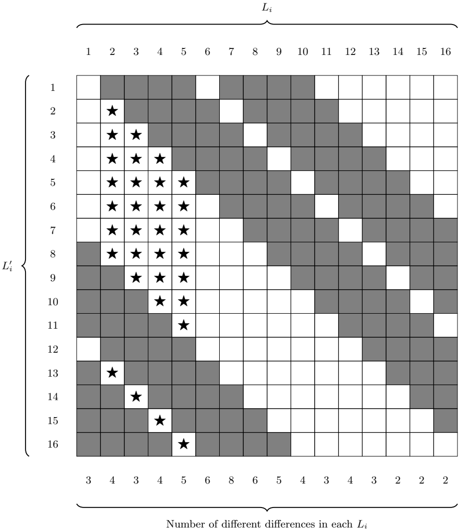
L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		■	■	■	■		■	■	■	■						
2			■	■	■	■		■	■	■	■					
3				■	■	■	■		■	■	■	■				
4					■	■	■	■		■	■	■	■			
5						■	■	■	■		■	■	■	■		
6							■	■	■	■		■	■	■	■	
7								■	■	■	■		■	■	■	■
8	■								■	■	■	■		■	■	■
9	■	■								■	■	■	■		■	■
10	■	■	■								■	■	■	■		■
11	■	■	■	■								■	■	■	■	
12		■	■	■	■								■	■	■	■
13	■		■	■	■	■								■	■	■
14	■	■		■	■	■	■								■	■
15	■	■	■		■	■	■	■								■
16	■	■	■	■		■	■	■	■							
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		▨	▨	▨	▨		▨	▨	▨	▨						
2		★	▨	▨	▨	▨		▨	▨	▨	▨					
3		★	★	▨	▨	▨	▨		▨	▨	▨	▨				
4		★	★	★	▨	▨	▨	▨		▨	▨	▨	▨			
5		★	★	★	★	▨	▨	▨	▨		▨	▨	▨	▨		
6		★	★	★	★		▨	▨	▨	▨		▨	▨	▨	▨	
7		★	★	★	★			▨	▨	▨	▨		▨	▨	▨	▨
8	▨	★	★	★	★				▨	▨	▨	▨		▨	▨	▨
9	▨	▨	★	★	★					▨	▨	▨	▨		▨	▨
10	▨	▨	▨	★	★						▨	▨	▨	▨		▨
11	▨	▨	▨	▨	★							▨	▨	▨	▨	
12		▨	▨	▨	▨								▨	▨	▨	▨
13	▨	★	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	★	▨	▨	▨	▨								▨	▨
15	▨	▨	▨	★	▨	▨	▨	▨								▨
16	▨	▨	▨	▨	★	▨	▨	▨	▨							
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256}

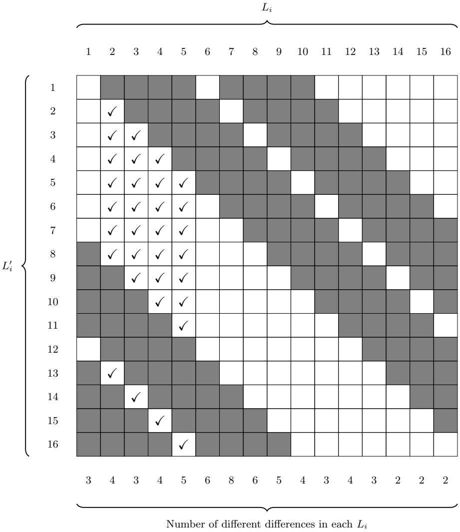
Current Probability

1

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		▨	▨	▨	▨		▨	▨	▨	▨						
2		✓	▨	▨	▨	▨		▨	▨	▨	▨					
3		✓	✓	▨	▨	▨	▨		▨	▨	▨	▨				
4		✓	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			
5		✓	✓	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨		
6		✓	✓	✓	✓		▨	▨	▨	▨		▨	▨	▨	▨	
7		✓	✓	✓	✓			▨	▨	▨	▨		▨	▨	▨	▨
8	▨	✓	✓	✓	✓				▨	▨	▨	▨		▨	▨	▨
9	▨	▨	✓	✓	✓					▨	▨	▨	▨		▨	▨
10	▨	▨	▨	✓	✓						▨	▨	▨	▨		▨
11	▨	▨	▨	▨	✓							▨	▨	▨	▨	
12		▨	▨	▨	▨								▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨								▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨								▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨							
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256}

Current Probability

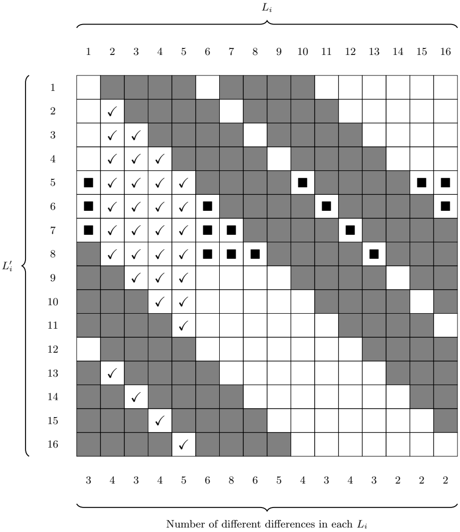
1

Next step: L'_5, L'_6, L'_7, L'_8.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		▨	▨	▨	▨		▨	▨	▨	▨						
2		✓	▨	▨	▨	▨		▨	▨	▨	▨					
3		✓	✓	▨	▨	▨	▨		▨	▨	▨	▨				
4		✓	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			
5	■	✓	✓	✓	✓	▨	▨	▨	▨	■	▨	▨	▨	▨	■	■
6	■	✓	✓	✓	✓	■	▨	▨	▨	▨	■	▨	▨	▨	▨	■
7	■	✓	✓	✓	✓	■	■	▨	▨	▨	▨	■	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	■	■	■	▨	▨	▨	▨	■	▨	▨	▨
9	▨	▨	✓	✓	✓					▨	▨	▨	▨		▨	▨
10	▨	▨	▨	✓	✓						▨	▨	▨	▨		▨
11	▨	▨	▨	▨	✓							▨	▨	▨	▨	
12		▨	▨	▨	▨								▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨								▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨								▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨							
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256}

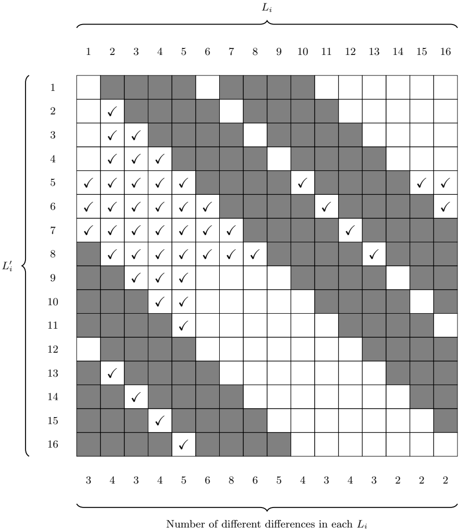
Current Probability

1

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		▨	▨	▨	▨		▨	▨	▨	▨						
2		✓	▨	▨	▨	▨		▨	▨	▨	▨					
3		✓	✓	▨	▨	▨	▨		▨	▨	▨	▨				
4		✓	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓					▨	▨	▨	▨		▨	▨
10	▨	▨	▨	✓	✓						▨	▨	▨	▨		▨
11	▨	▨	▨	▨	✓							▨	▨	▨	▨	
12		▨	▨	▨	▨								▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨								▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨								▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨							
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256}

Current Probability

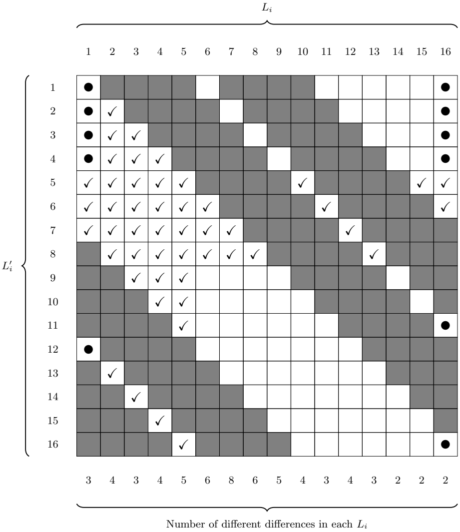
1

Next step: L_1, L_{16}.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	●	▨	▨	▨	▨		▨	▨	▨	▨						●
2	●	✓	▨	▨	▨	▨		▨	▨	▨	▨					●
3	●	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨				●
4	●	✓	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			●
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓					▨	▨	▨	▨		▨	▨
10	▨	▨	▨	✓	✓						▨	▨	▨	▨		▨
11	▨	▨	▨	▨	✓							▨	▨	▨	▨	●
12	●	▨	▨	▨	▨								▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨								▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨								▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨							●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Current Complexity

2^{256}

Current Probability

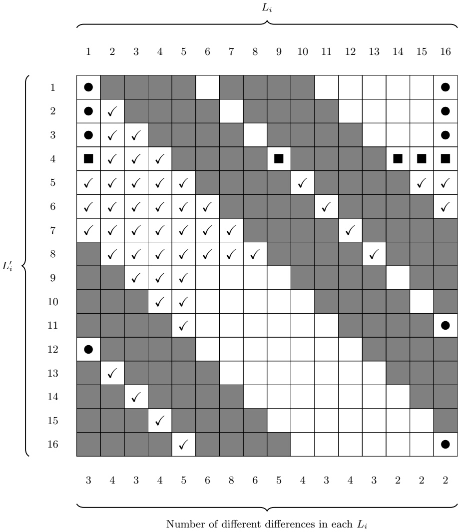
1

Next step: L'_4.

Legend

✓	Known value and difference
●	Known difference
★	Guessed value and difference
🟩	Highlight current step

Guess-and-Determine Algorithm



Current Complexity

2^{256}

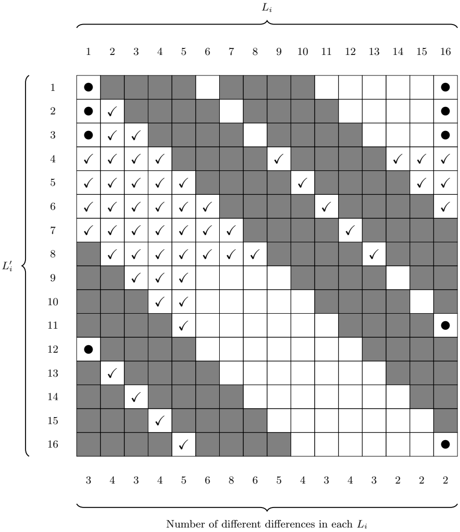
Current Probability

1

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	●	▨	▨	▨	▨		▨	▨	▨	▨						●
2	●	✓	▨	▨	▨	▨		▨	▨	▨	▨					●
3	●	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨				●
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓					▨	▨	▨	▨		▨	▨
10	▨	▨	▨	✓	✓						▨	▨	▨	▨		▨
11	▨	▨	▨	▨	✓							▨	▨	▨	▨	●
12	●	▨	▨	▨	▨								▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨								▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨								▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨							●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Current Complexity

2^{256}

Current Probability

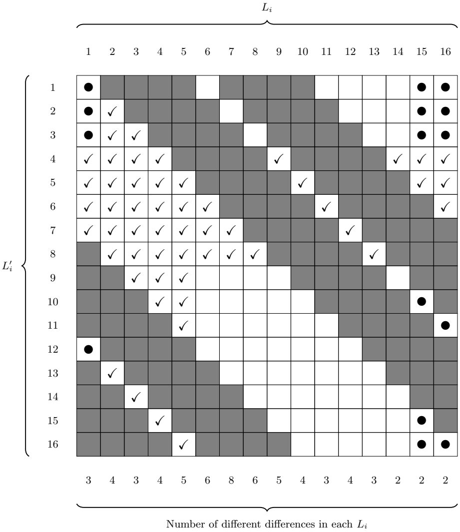
1

Next step: L_{15}.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	●	▨	▨	▨	▨		▨	▨	▨	▨					●	●
2	●	✓	▨	▨	▨	▨		▨	▨	▨	▨				●	●
3	●	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓					▨	▨	▨	▨		▨	▨
10	▨	▨	▨	✓	✓						▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓							▨	▨	▨	▨	●
12	●	▨	▨	▨	▨								▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨								▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨							●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨						●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256}

Current Probability

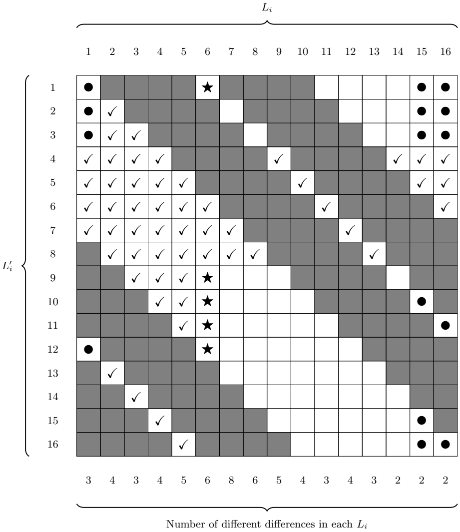
1

Next step: L_6.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	●	▨	▨	▨	▨	★	▨	▨	▨	▨					●	●
2	●	✓	▨	▨	▨	▨		▨	▨	▨	▨				●	●
3	●	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	★				▨	▨	▨	▨		▨	▨
10	▨	▨	▨	✓	✓	★					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	★						▨	▨	▨	▨	●
12	●	▨	▨	▨	▨	★							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨								▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨							●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨						●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256+16}

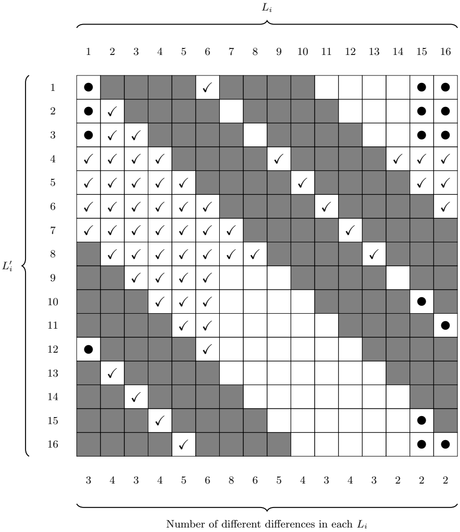
Current Probability

1

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	●	▨	▨	▨	▨	✓	▨	▨	▨	▨					●	●
2	●	✓	▨	▨	▨	▨		▨	▨	▨	▨				●	●
3	●	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓				▨	▨	▨	▨		▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	●	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨								▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨							●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨						●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256+16}

Current Probability

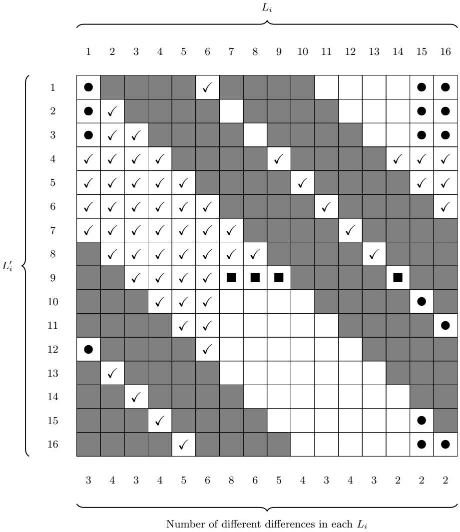
1

Next step: L'_9.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	●	▨	▨	▨	▨	✓	▨	▨	▨	▨					●	●
2	●	✓	▨	▨	▨	▨		▨	▨	▨	▨				●	●
3	●	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	■	■	■	▨	▨	▨	▨	■	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	●	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨								▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨							●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨						●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Current Complexity

2^{256+16}

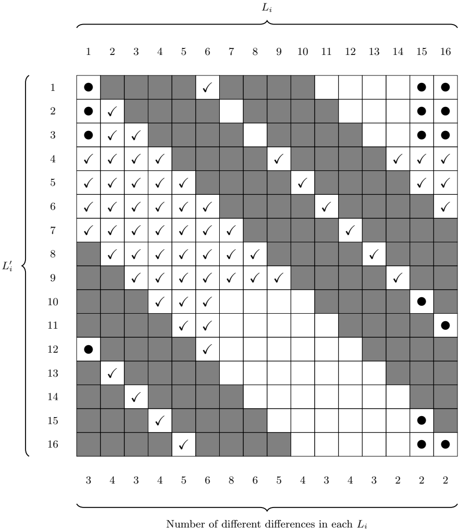
Current Probability

1

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	●	▨	▨	▨	▨	✓	▨	▨	▨	▨					●	●
2	●	✓	▨	▨	▨	▨		▨	▨	▨	▨				●	●
3	●	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	●	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨								▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨							●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨						●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256+16}

Current Probability

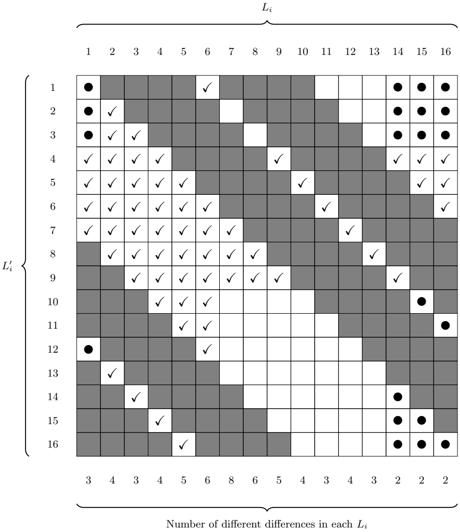
1

Next step: L_{14}.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	●	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●
2	●	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●	●
3	●	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨		●	●	●
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	●	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨							●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨						●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256+16}

Current Probability

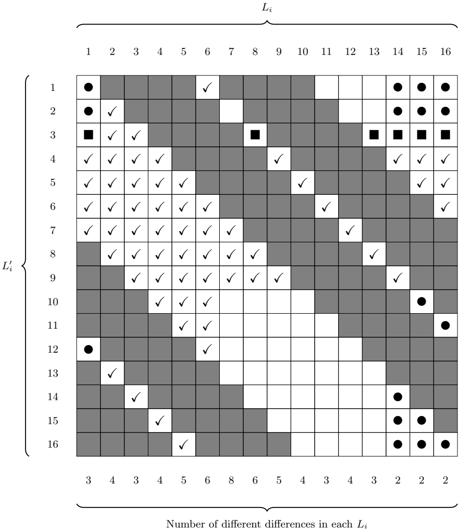
1

Next step: L'_3.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	●	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●
2	●	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●	●
3	■	✓	✓	▨	▨	▨	▨	■	▨	▨	▨	▨	■	■	■	■
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	●	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨							●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨						●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256+16}

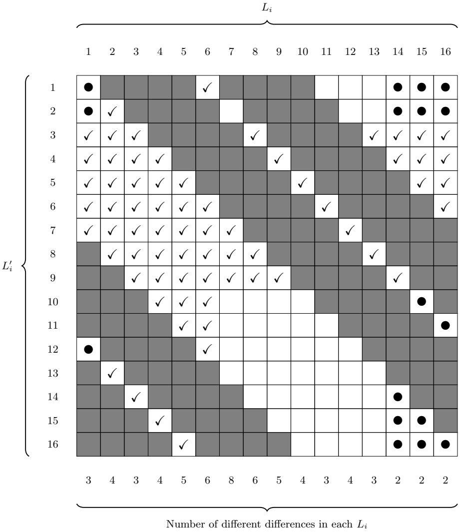
Current Probability

1

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	●	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●
2	●	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●	●
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	●	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨							●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨						●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256+16}

Current Probability

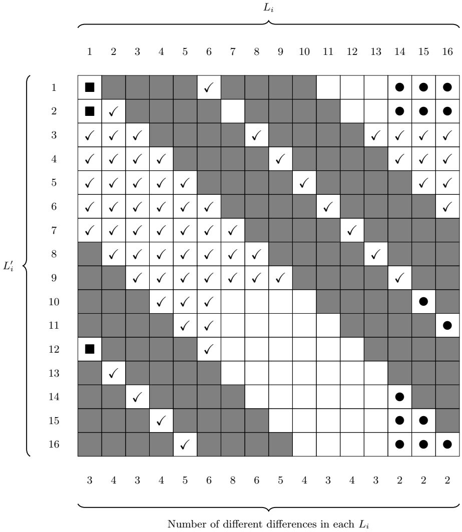
1

Next step: L_1.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	■	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●
2	■	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●	●
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	■	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨							●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨						●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256+16}

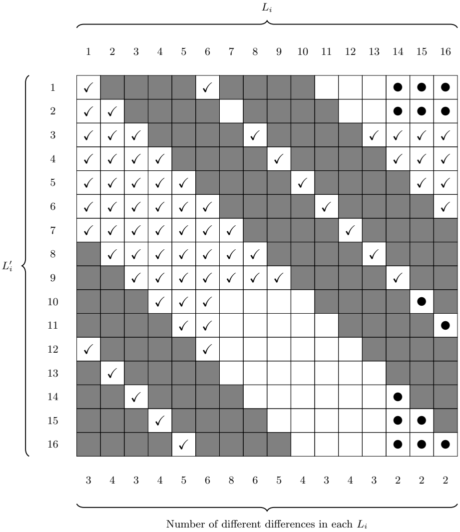
Current Probability

1

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●
2	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●	●
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	✓	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨							●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨						●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

2^{256+16}

Current Probability

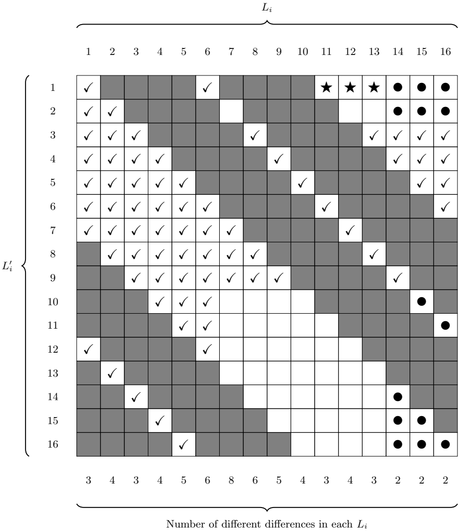
1

Next step: L'_1.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	★	★	★	●	●	●
2	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●	●
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	✓	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨							●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨						●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$2^{256+16+8}$

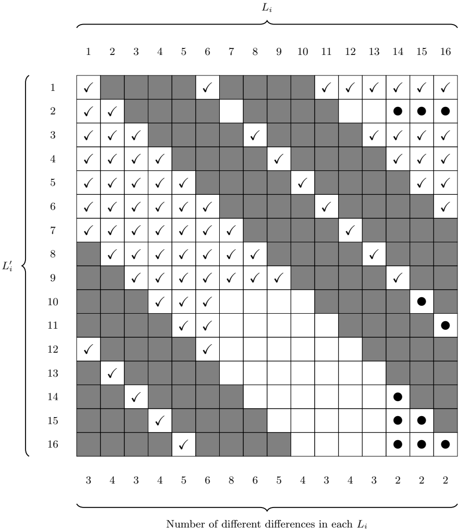
Current Probability

1

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨			●	●	●
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	✓	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨								▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨							●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨						●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$2^{256+16+8}$

Current Probability

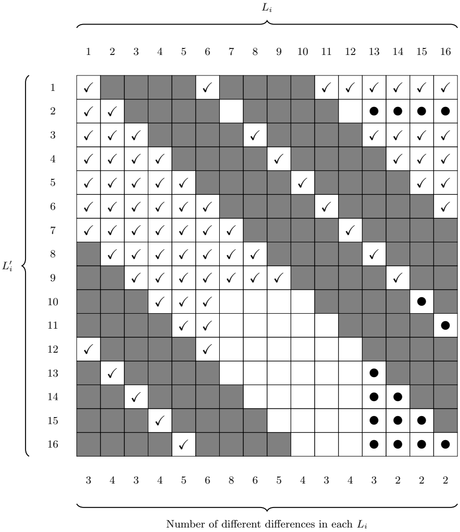
1

Next step: L_{13}.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨		▨	▨	▨	▨		●	●	●	●
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	✓	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨							●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨						●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$2^{256+16+8}$

Current Probability

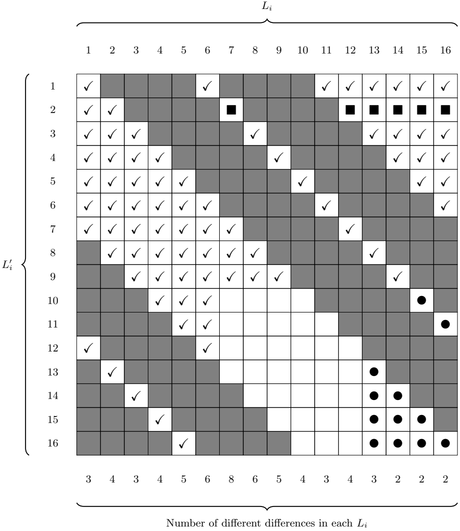
1

Next step: L'_2.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨	■	▨	▨	▨	▨	■	■	■	■	■
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	✓	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨							●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨						●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$2^{256+16+8}$

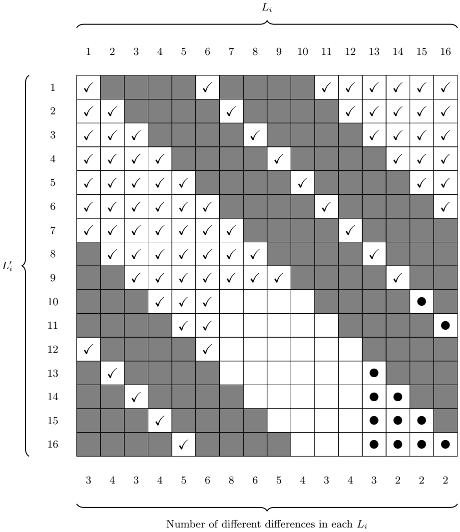
Current Probability

1

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨	●
12	✓	▨	▨	▨	▨	✓							▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨							●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨						●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●	●
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$2^{256+16+8}$

Current Probability

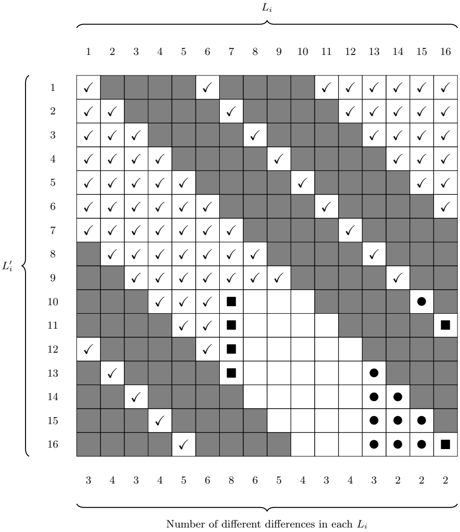
1

Next step: L_7, L_{16}.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L_i'

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓	■				▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓	■					▨	▨	▨	▨	■
12	✓	▨	▨	▨	▨	✓	■						▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨	■						●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨						●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●	■
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$2^{256+16+8}$

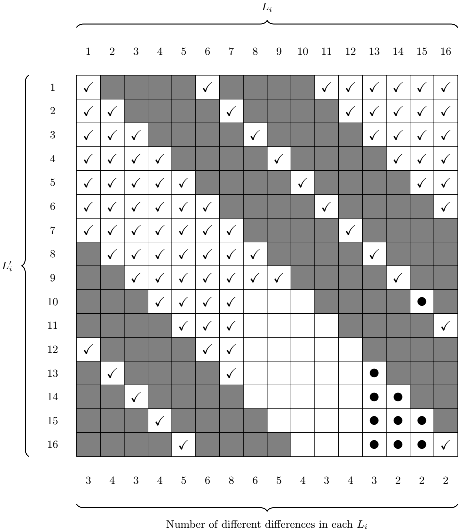
Current Probability

1

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓	✓				▨	▨	▨	▨	●	▨
11	▨	▨	▨	▨	✓	✓	✓					▨	▨	▨	▨	✓
12	✓	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨	✓						●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨						●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$2^{256+16+8}$

Current Probability

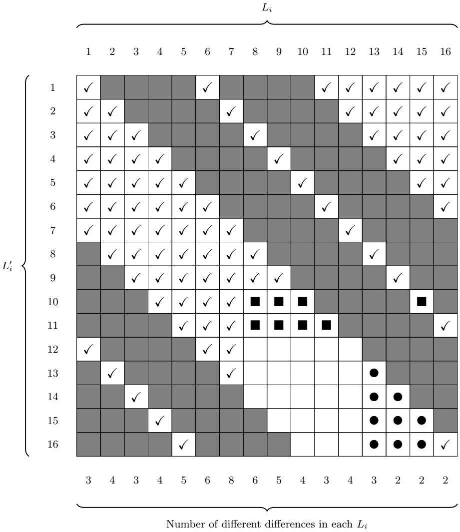
1

Next step: L'_{10}, L'_{11}.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓	✓	■	■	■	▨	▨	▨	▨	■	▨
11	▨	▨	▨	▨	✓	✓	✓	■	■	■	■	▨	▨	▨	▨	✓
12	✓	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨	✓						●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨						●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$2^{256+16+8}$

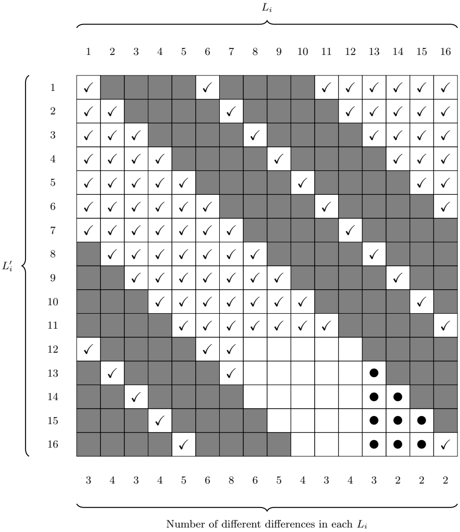
Current Probability

$2^{-8\cdot(1)}$

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨
11	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓
12	✓	▨	▨	▨	▨	✓	✓						▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨	✓						●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨						●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨					●	●	●	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨				●	●	●	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$$2^{256+16+8}$$

Current Probability

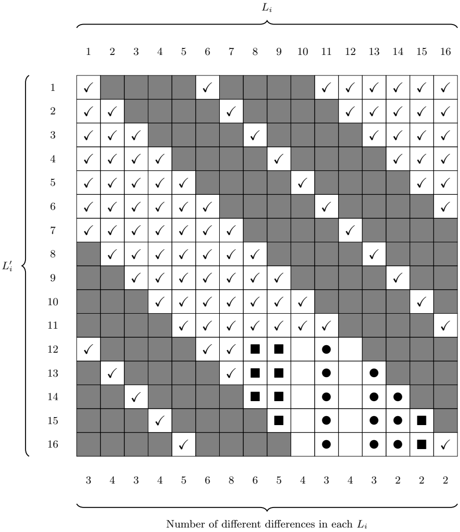
$$2^{-8\cdot(1)}$$

Next step: L_8, L_9, L_{11}, L_{15}.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨
11	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓
12	✓	▨	▨	▨	▨	✓	✓	■	■		●		▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨	✓	■	■		●		●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨	■	■		●		●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨	■		●		●	●	■	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨		●		●	●	■	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$$2^{256+16+8}$$

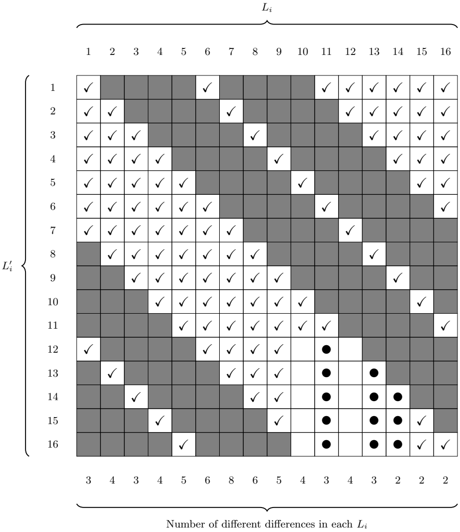
Current Probability

$$2^{-8\cdot(1+2)}$$

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨
11	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓
12	✓	▨	▨	▨	▨	✓	✓	✓	✓		●		▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨	✓	✓	✓		●		●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨	✓	✓		●		●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨	✓		●		●	●	✓	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨		●		●	●	✓	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$2^{256+16+8}$

Current Probability

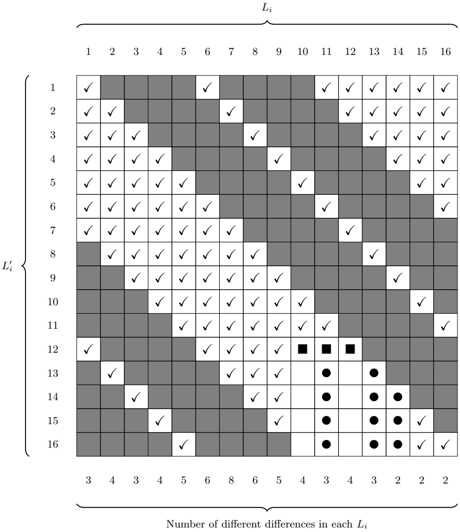
$2^{-8\cdot(1+2)}$

Next step: L'_{12}.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨
11	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓
12	✓	▨	▨	▨	▨	✓	✓	✓	✓	■	■	■	▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨	✓	✓	✓		●		●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨	✓	✓		●		●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨	✓		●		●	●	✓	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨		●		●	●	✓	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Current Complexity

$$2^{256+16+8}$$

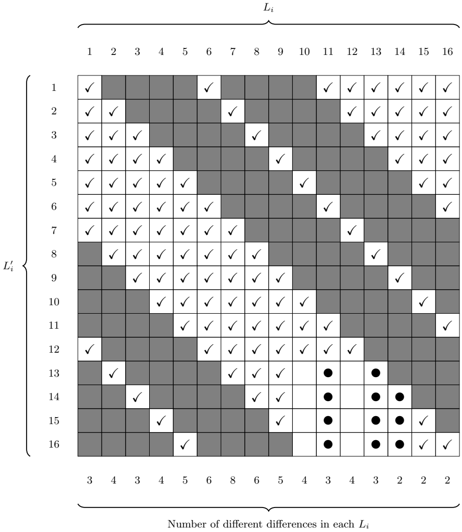
Current Probability

$$2^{-8\cdot(1+2+3)}$$

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨
11	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓
12	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨	✓	✓	✓		●		●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨	✓	✓		●		●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨	✓		●		●	●	✓	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨		●		●	●	✓	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$$2^{256+16+8}$$

Current Probability

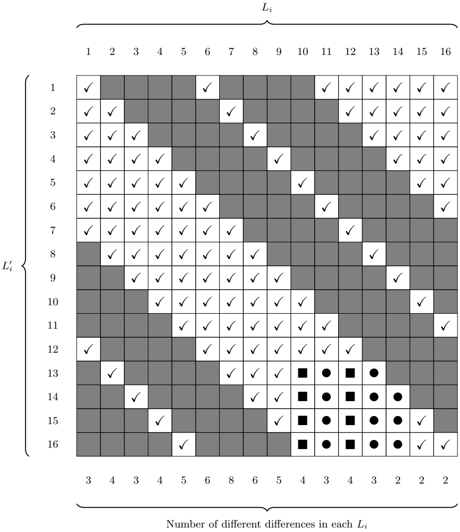
$$2^{-8\cdot(1+2+3)}$$

Next step: L_{10}, L_{12}.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



$\overbrace{\qquad\qquad}^{L_i}$

L'_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓					✓					✓	✓	✓	✓	✓	✓
2	✓	✓					✓					✓	✓	✓	✓	✓
3	✓	✓	✓					✓					✓	✓	✓	✓
4	✓	✓	✓	✓					✓					✓	✓	✓
5	✓	✓	✓	✓	✓					✓					✓	✓
6	✓	✓	✓	✓	✓	✓					✓					✓
7	✓	✓	✓	✓	✓	✓	✓					✓				
8		✓	✓	✓	✓	✓	✓	✓					✓			
9			✓	✓	✓	✓	✓	✓	✓					✓		
10				✓	✓	✓	✓	✓	✓	✓					✓	
11					✓	✓	✓	✓	✓	✓	✓					✓
12	✓					✓	✓	✓	✓	✓	✓	✓				
13		✓					✓	✓	✓	■	●	■	●			
14			✓					✓	✓	■	●	■	●	●		
15				✓					✓	■	●	■	●	●	✓	
16					✓					■	●	■	●	●	✓	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$$2^{256+16+8}$$

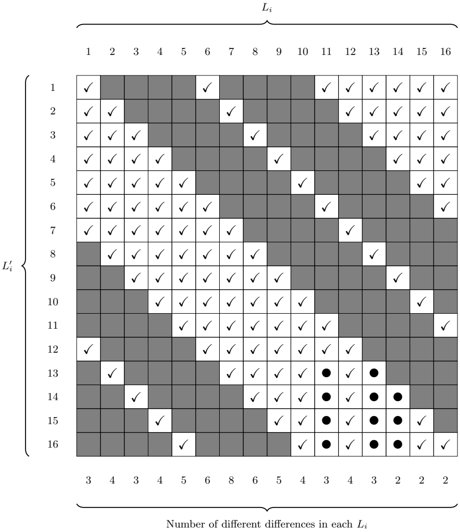
Current Probability

$$2^{-8\cdot(1+2+3)}$$

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓					✓					✓	✓	✓	✓	✓	✓
2	✓	✓					✓					✓	✓	✓	✓	✓
3	✓	✓	✓					✓					✓	✓	✓	✓
4	✓	✓	✓	✓					✓					✓	✓	✓
5	✓	✓	✓	✓	✓					✓					✓	✓
6	✓	✓	✓	✓	✓	✓					✓					✓
7	✓	✓	✓	✓	✓	✓	✓					✓				
8		✓	✓	✓	✓	✓	✓	✓					✓			
9			✓	✓	✓	✓	✓	✓	✓					✓		
10				✓	✓	✓	✓	✓	✓	✓					✓	
11					✓	✓	✓	✓	✓	✓	✓					✓
12	✓					✓	✓	✓	✓	✓	✓	✓				
13		✓					✓	✓	✓	✓	●	✓	●			
14			✓					✓	✓	✓	●	✓	●	●		
15				✓					✓	✓	●	✓	●	●	✓	
16					✓					✓	●	✓	●	●	✓	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$$2^{256+16+8}$$

Current Probability

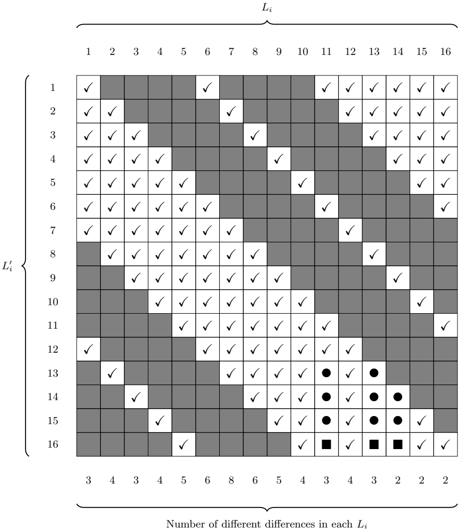
$$2^{-8\cdot(1+2+3)}$$

Next step: L'_2.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓					✓					✓	✓	✓	✓	✓	✓
2	✓	✓					✓					✓	✓	✓	✓	✓
3	✓	✓	✓					✓					✓	✓	✓	✓
4	✓	✓	✓	✓					✓					✓	✓	✓
5	✓	✓	✓	✓	✓					✓					✓	✓
6	✓	✓	✓	✓	✓	✓					✓					✓
7	✓	✓	✓	✓	✓	✓	✓					✓				
8		✓	✓	✓	✓	✓	✓	✓					✓			
9			✓	✓	✓	✓	✓	✓	✓					✓		
10				✓	✓	✓	✓	✓	✓	✓					✓	
11					✓	✓	✓	✓	✓	✓	✓					✓
12	✓					✓	✓	✓	✓	✓	✓	✓				
13		✓					✓	✓	✓	✓	●	✓	●			
14			✓					✓	✓	✓	●	✓	●	●		
15				✓					✓	✓	●	✓	●	●	✓	
16					✓					✓	■	✓	■	■	✓	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$$2^{256+16+8}$$

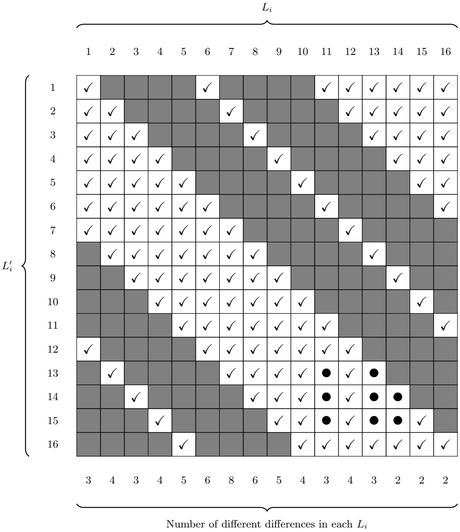
Current Probability

$$2^{-8\cdot(1+2+3+5)}$$

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Guess-and-Determine Algorithm



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓
2	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓
3	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓
4	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓
5	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓
6	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓
7	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨	▨
8	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨	▨
9	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨	▨
10	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓	▨
11	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨	✓
12	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓	▨	▨	▨	▨
13	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	●	✓	●	▨	▨	▨
14	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	●	✓	●	●	▨	▨
15	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	●	✓	●	●	✓	▨
16	▨	▨	▨	▨	✓	▨	▨	▨	▨	✓	✓	✓	✓	✓	✓	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Current Complexity

$$2^{256+16+8}$$

Current Probability

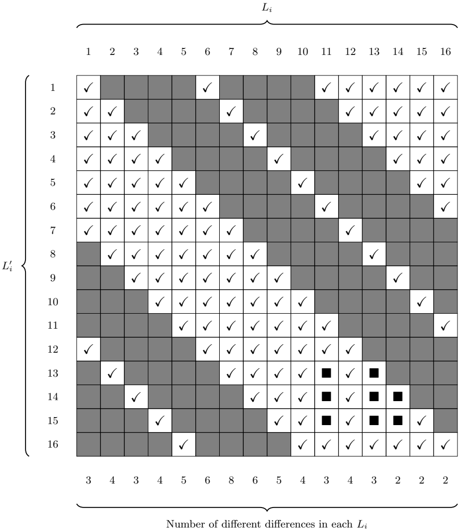
$$2^{-8\cdot(1+2+3+5)}$$

Next step: $L'_{13}, L'_{14}, L'_{15}$.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓					✓					✓	✓	✓	✓	✓	✓
2	✓	✓					✓					✓	✓	✓	✓	✓
3	✓	✓	✓					✓					✓	✓	✓	✓
4	✓	✓	✓	✓					✓					✓	✓	✓
5	✓	✓	✓	✓	✓					✓					✓	✓
6	✓	✓	✓	✓	✓	✓					✓					✓
7	✓	✓	✓	✓	✓	✓	✓					✓				
8		✓	✓	✓	✓	✓	✓	✓					✓			
9			✓	✓	✓	✓	✓	✓	✓					✓		
10				✓	✓	✓	✓	✓	✓	✓					✓	
11					✓	✓	✓	✓	✓	✓	✓					✓
12	✓					✓	✓	✓	✓	✓	✓	✓				
13		✓					✓	✓	✓	✓	■	✓	■			
14			✓					✓	✓	✓	■	✓	■	■		
15				✓					✓	✓	■	✓	■	■	✓	
16					✓					✓	✓	✓	✓	✓	✓	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Current Complexity

$$2^{256+16+8}$$

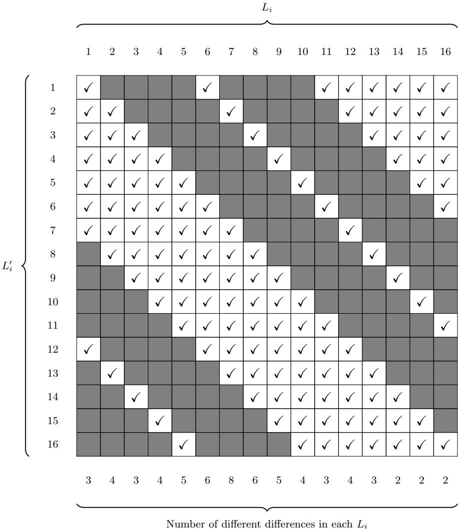
Current Probability

$$2^{-8\cdot(1+2+3+5+8+8+8)}$$

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- 🟩 Highlight current step

Guess-and-Determine Algorithm



L_i

L'_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	✓					✓					✓	✓	✓	✓	✓	✓
2	✓	✓					✓					✓	✓	✓	✓	✓
3	✓	✓	✓					✓					✓	✓	✓	✓
4	✓	✓	✓	✓					✓					✓	✓	✓
5	✓	✓	✓	✓	✓					✓					✓	✓
6	✓	✓	✓	✓	✓	✓					✓					✓
7	✓	✓	✓	✓	✓	✓	✓					✓				
8		✓	✓	✓	✓	✓	✓	✓					✓			
9			✓	✓	✓	✓	✓	✓	✓					✓		
10				✓	✓	✓	✓	✓	✓	✓					✓	
11					✓	✓	✓	✓	✓	✓	✓					✓
12	✓					✓	✓	✓	✓	✓	✓	✓				
13		✓					✓	✓	✓	✓	✓	✓	✓			
14			✓					✓	✓	✓	✓	✓	✓	✓		
15				✓					✓	✓	✓	✓	✓	✓	✓	
16					✓					✓	✓	✓	✓	✓	✓	✓
	3	4	3	4	5	6	8	6	5	4	3	4	3	2	2	2

Number of different differences in each L_i

Final Complexity

$$2^{256+16+8} = 2^{280}$$

Final Probability

$$2^{-8\cdot(1+2+3+5+8+8+8)} = 2^{-280}$$

The End.

Legend

- ✓ Known value and difference
- ● Known difference
- ★ Guessed value and difference
- ■ Highlight current step

Summing Up

Inbound Phase

In total we try: $2^{256+16+8} = 2^{280}$possibilities, and each gives a solution with probability

$$2^{-8\times(1+2+3+5+8+8+8)} = 2^{-280}.$$

Outbound Phase

$$\text{Again: } \mathbb{P}(\text{outbound}) = 2^{-2\times 56} = 2^{-112}.$$

Distinguisher

Finally, we distinguish the 10-round permutation in $2^{280+112} = 2^{392}$ operations and 2^{64} in memory.

This compares to a generic complexity of 2^{448} operations.

Conclusion

- We have provided new rebound results on building blocks of both versions of `Grøstl` that improve the previous number of analysed rounds.
- We propose a way to solve 3 fully active states in the middle.
- The results do not threaten the security of `Grøstl`, but we believe they will help better understanding `AES`-based constructions and their bounds regarding rebound techniques.

Conclusion

- We have provided new rebound results on building blocks of both versions of `Grøstl` that improve the previous number of analysed rounds.
- We propose a way to solve 3 fully active states in the middle.
- The results do not threaten the security of `Grøstl`, but we believe they will help better understanding `AES`-based constructions and their bounds regarding rebound techniques.

Thank you!