Reflection Cryptanalysis of PRINCE-like Ciphers

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Outline

Description of PRINCE-like cipher

- Low-latency SPN block cipher was proposed at ASIACRYPT2012.

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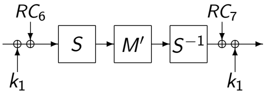
- Independently of the value of α, the designers showed that PRINCE is secure against known attacks.

Description of PRINCE-like Cipher



The 2 midmost rounds

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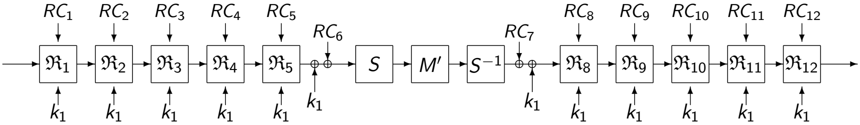

Total 12 rounds

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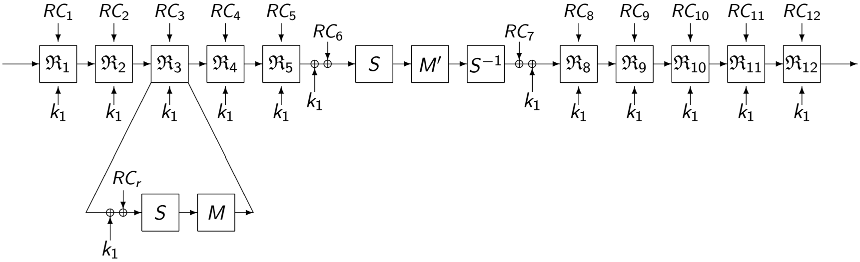

The first rounds

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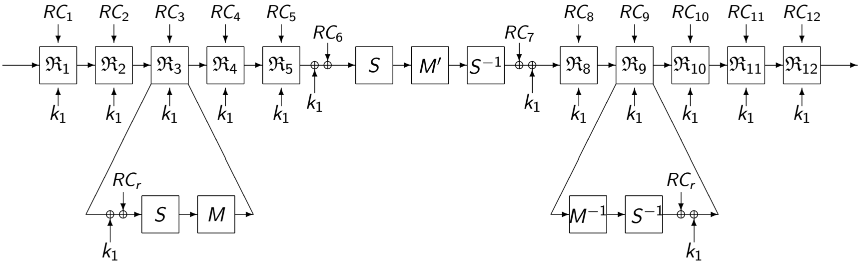

The last rounds

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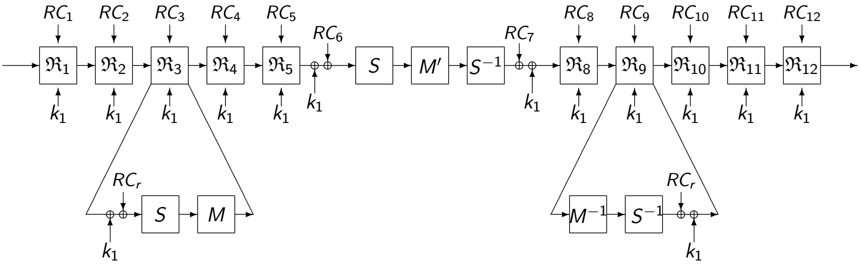

Related constants:

$$RC_{2R-r+1} = RC_r \oplus \alpha, \text{ for all } r = 1, \ldots, 2R$$

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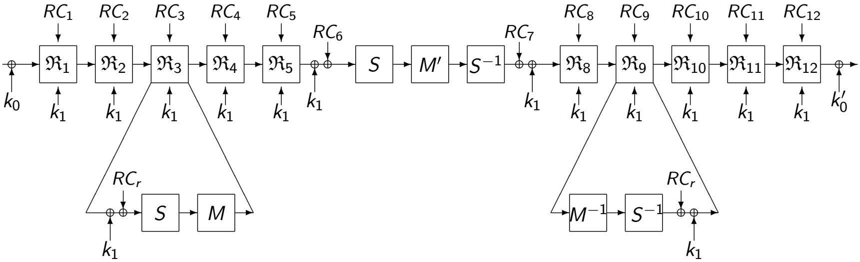

The whitening key

Description of PRINCE

- PRINCE-like cipher with $n = 64$.
- Constant is defined as $\alpha = 0xc0ac29b7c97c50dd$.
- The S-layer is a non-linear layer where each nibble is processed by the same Sbox.

Description of PRINCE

- M' is an involutory 64×64 block diagonal matrix $(\hat{M}_0, \hat{M}_1, \hat{M}_1, \hat{M}_0)$.

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$$\hat{M}_0 = \begin{pmatrix} M_0 & M_1 & M_2 & M_3 \\ M_1 & M_2 & M_3 & M_0 \\ M_2 & M_3 & M_0 & M_1 \\ M_3 & M_0 & M_1 & M_2 \end{pmatrix}, \quad \hat{M}_1 = \begin{pmatrix} M_1 & M_2 & M_3 & M_0 \\ M_2 & M_3 & M_0 & M_1 \\ M_3 & M_0 & M_1 & M_2 \\ M_0 & M_1 & M_2 & M_3 \end{pmatrix}.$$

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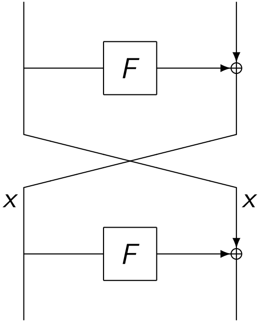
- The second linear matrix M for PRINCE is obtained by composition of M' and a permutation SR of nibbles by setting $M = SR \circ M'$.

Previous Works: Reflection Attack

- It has been applied on some ciphers and hash functions with Feistel construction (Kara 2008, Bouillaguet et al. 2010).



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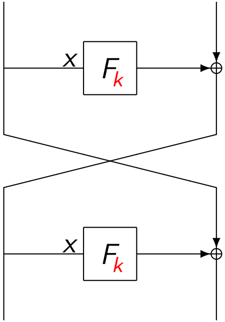

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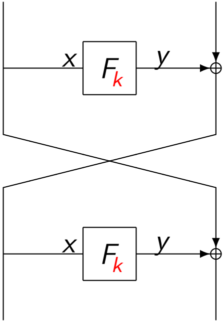

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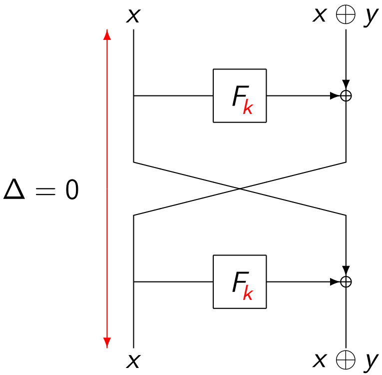

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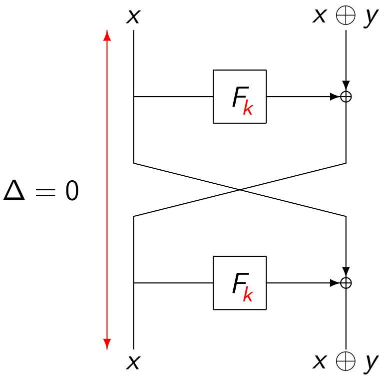

This work

Using probabilistic reflection property instead of deterministic approach.

Fixed Points

Definition

Let $f : A \to A$ be a function on a set A. A point $x \in A$ is called a fixed point of the function f if and only if $f(x) = x$.

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Let $f : \mathbb{F}_2^n \to \mathbb{F}_2^n$ be a linear involution. Then the number of fixed points of f is greater than or equal to $2^{n/2}$.

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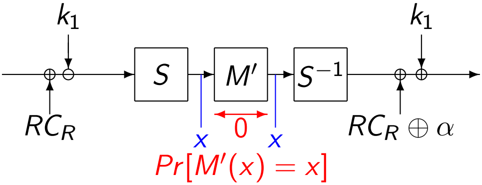
Idea

Take advantage of α-reflection property and the fact that always fixed points exist in midmost rounds of PRINCE-like ciphers.

Characteristic $\mathcal{I}_1$



$$\mathcal{P}_{\mathcal{I}_1} = \mathcal{P}_{F_{M'}} = \frac{|F_{M'}|}{2^n}.$$

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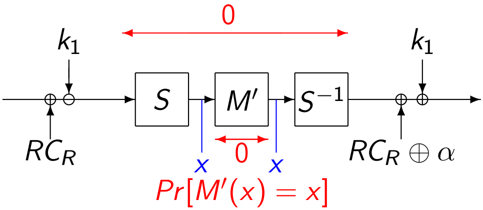

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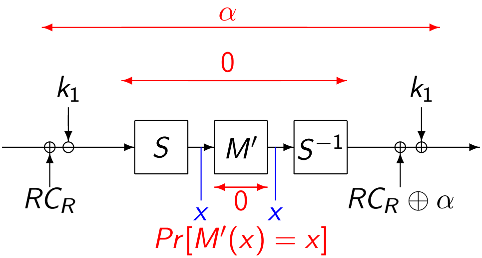
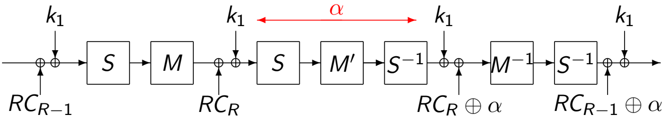


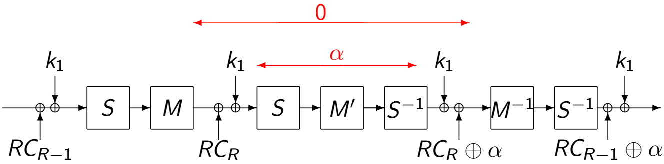
$$\mathcal{P}_{\mathcal{I}_1} = \mathcal{P}_{F_{M'}} = \frac{|F_{M'}|}{2^n}.$$

Characteristic $\mathcal{I}_2$



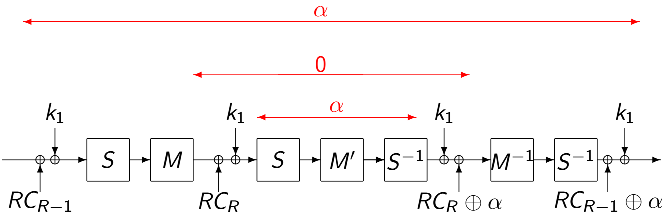
$$\mathcal{P}_{\mathcal{I}_2} = 2^{-n} \# \left\{ x \in \mathbb{F}_2^n \,|\, S^{-1}(M'(S(x))) \oplus x = \alpha \right\}.$$

Characteristic $\mathcal{I}_2$



$$\mathcal{P}_{\mathcal{I}_2} = 2^{-n} \# \left\{ x \in \mathbb{F}_2^n \mid S^{-1}(M'(S(x))) \oplus x = \alpha \right\}.$$

Characteristic $\mathcal{I}_2$



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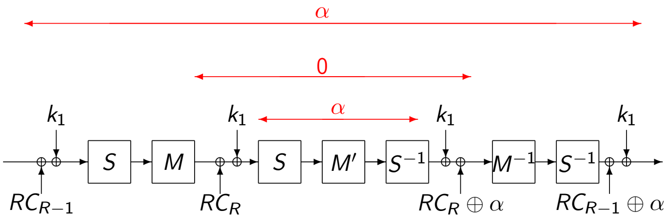
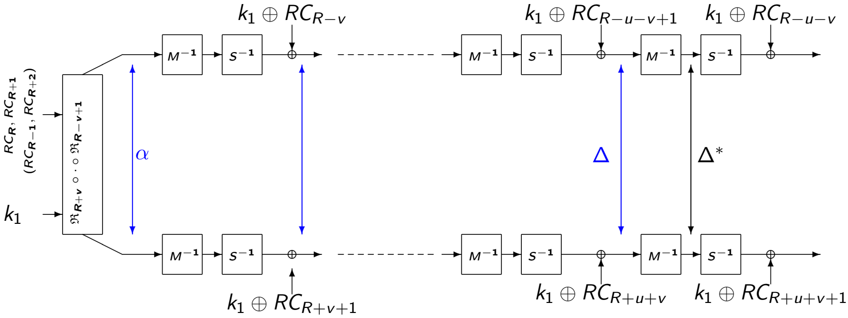
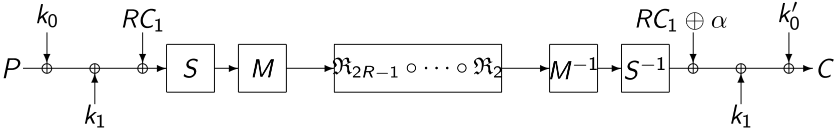


If $\mathcal{P}_{\mathcal{I}_2} = 0$ then we have impossible differential.

External Characteristic $\mathcal{P}_{\mathcal{C}_r}$



Key Recovery



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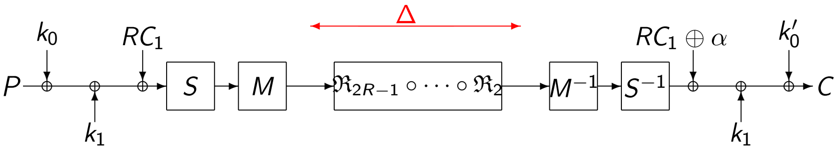

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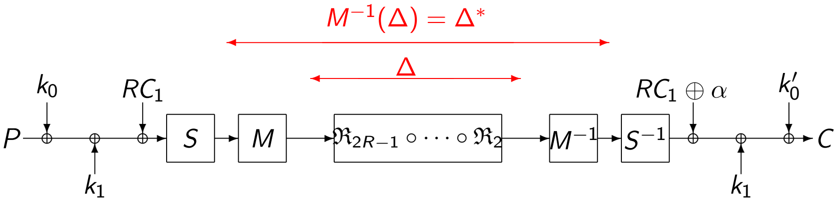
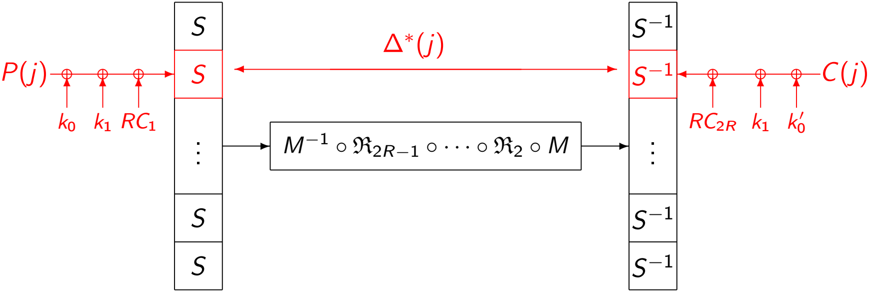


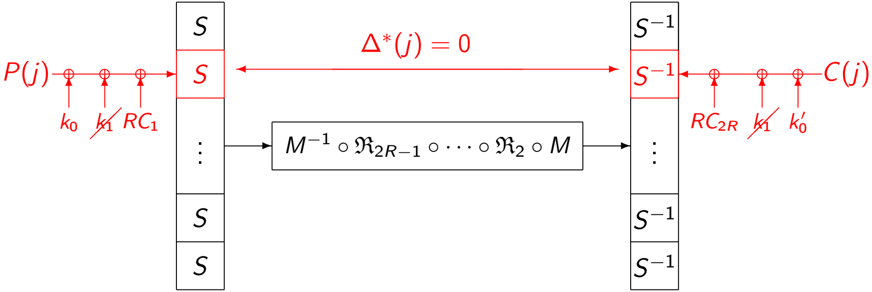
Key Recovery Nibble by Nibble



$$\Delta^*(j) = S(P(j) \oplus k_0(j) \oplus k_1(j) \oplus RC_1(j))$$

$$\oplus S(C(j) \oplus k_0'(j) \oplus k_1(j) \oplus RC_{2R}(j))$$

Key Recovery for Passive Nibble



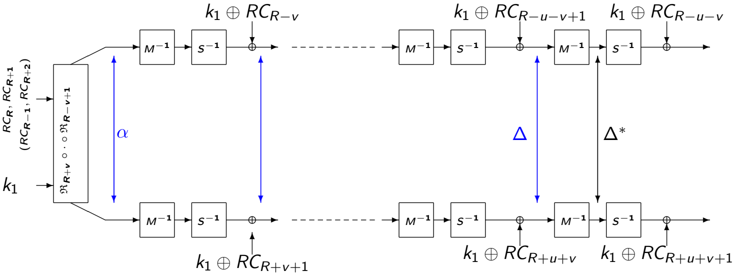
$$P(j) \oplus k_0(j) \oplus C(j) \oplus k'_0(j) \oplus \alpha(j) = 0,$$

- The difference after passing through the S-boxes is still zero.
- The value of $k_1(j)$ need not be known.

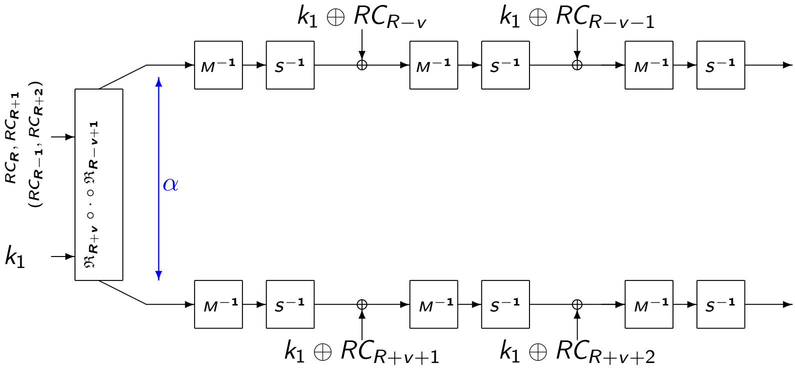
Maximizing Probability $\mathcal{P}_{\mathcal{C}}$ of Characteristic

To maximize $\mathcal{P}_{\mathcal{C}}$ we can either use

- Cancellation idea.
- Branch and Bound algorithm.



Cancellation Idea



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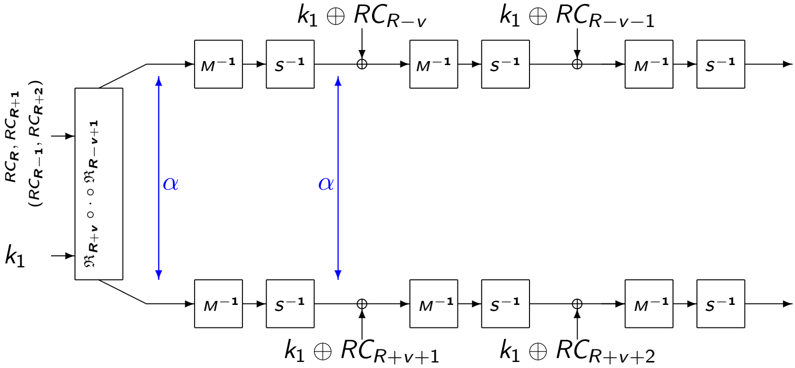

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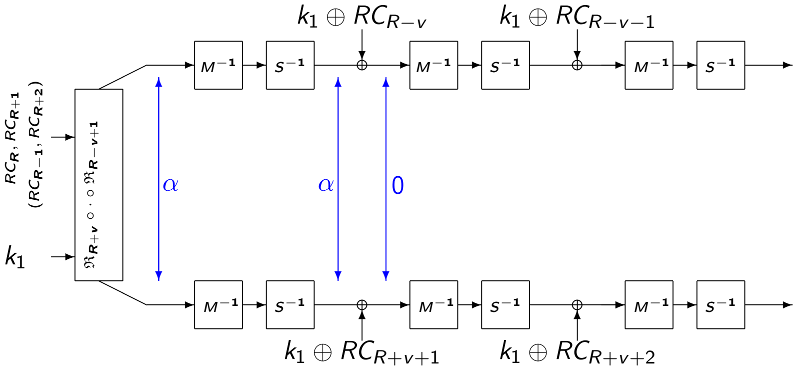

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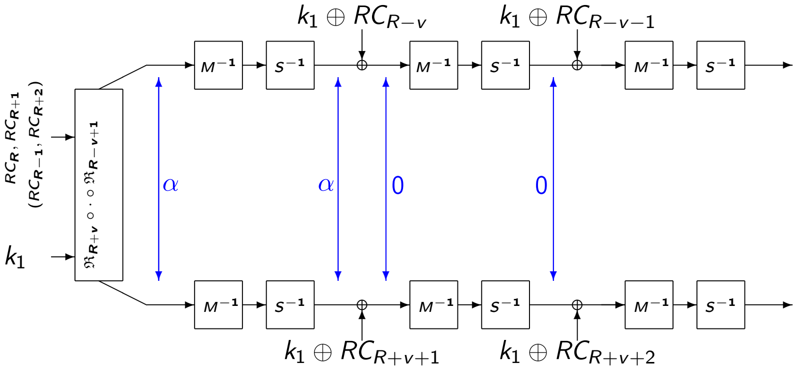

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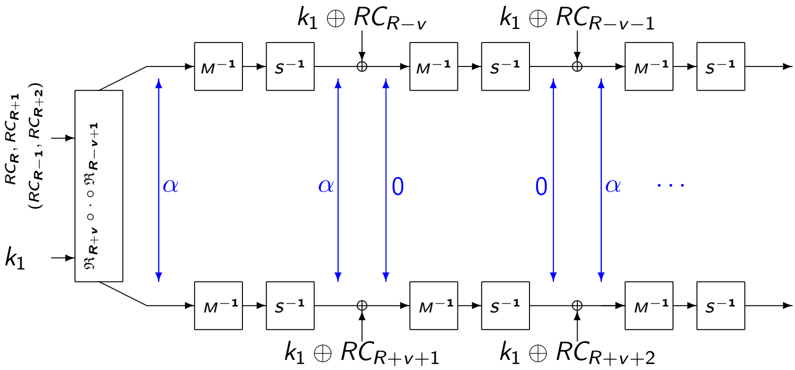

With $\mathcal{P} = \Pr_{\mathbf{X}} \left[S(\mathbf{X}) \oplus S(\mathbf{X} \oplus \alpha) = M^{-1}(\alpha) \right]$ there is an iterative characteristic over four rounds of a PRINCE-like cipher.

Best α with Cancellation Idea on 12 rounds

α	Δ^*	$w(\Delta^*)$	$\mathcal{P}_{\mathcal{C}_4}$	Data Compl.	Time Compl.
0x8400400800000000	0x8800400400000000	4	2^{-22}	$2^{57.95}$	$2^{71.37}$
0x8040000040800000	0x8080000040400000	4	2^{-22}	$2^{57.95}$	$2^{71.37}$
0x0000408000008040	0x0000404000008080	4	2^{-22}	$2^{57.95}$	$2^{71.37}$
0x0000000048008004	0x0000000044008008	4	2^{-22}	$2^{57.95}$	$2^{71.37}$
0x0000440040040000	0x0000440040040000	4	2^{-24}	$2^{60.27}$	$2^{73.69}$
0x8008000000008800	0x8008000000008800	4	2^{-24}	$2^{60.27}$	$2^{73.69}$

Examples of α with Branch and Bound Algorithm on 12 Rounds

α	Δ^*	$w(\Delta^*)$	$\mathcal{P}_{\mathcal{C}_4}$	Data Compl.	Time Compl.
0x0108088088010018	0x0000001008000495	5	2^{-26}	$2^{62.78}$	$2^{80.2}$
0x0088188080018010	0x00000100c09d0008	5	2^{-26}	$2^{62.78}$	$2^{80.2}$
0x0108088088010018	0x000000100800d8cc	6	2^{-26}	$2^{62.83}$	$2^{84.25}$
0x0001111011010011	0x1101100110000100	7	2^{-28}	$2^{63.45}(a = 32)$	$2^{88.87}$

Number of non-zero nibbles of α

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$$\alpha = \begin{bmatrix} \texttt{0x7} & \texttt{0x1} & \texttt{0xc} & \texttt{0xb} \\ \texttt{0x9} & \texttt{0x5} & \texttt{0x9} & \texttt{0x3} \\ \texttt{0x9} & \texttt{0xa} & \texttt{0x5} & \texttt{0x9} \\ \texttt{0x3} & \texttt{0x6} & \texttt{0x8} & \texttt{0xd} \end{bmatrix},$$

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Truncated Attack

Assume α is such that $M^{-1}(\alpha) = \begin{bmatrix} * & 0 & 0 & 0 \\ 0 & 0 & 0 & * \\ 0 & 0 & * & 0 \\ 0 & * & 0 & 0 \end{bmatrix}$ where $*$ can be any arbitrary value. For six rounds $\mathfrak{R}_{R-2} \circ \cdots \circ \mathfrak{R}_{R+3}$, the following truncated characteristic:

$$Y^O_{R+3} \oplus X^I_{R-2} = \begin{bmatrix} * & 0 & 0 & 0 \\ * & 0 & 0 & * \\ * & 0 & * & 0 \\ * & * & 0 & 0 \end{bmatrix} \oplus \alpha,$$

holds with probability $\mathcal{P}_{F_{M'}} = \frac{|F_{M'}|}{2^n} = 2^{-32}$.

Truncated Attack

Similar characteristics can be obtained for α such that:

$$M^{-1}(\alpha) = \begin{bmatrix} 0 & * & 0 & 0 \\ * & 0 & 0 & 0 \\ 0 & 0 & 0 & * \\ 0 & 0 & * & 0 \end{bmatrix} \text{ or } M^{-1}(\alpha) = \begin{bmatrix} 0 & 0 & * & 0 \\ 0 & * & 0 & 0 \\ * & 0 & 0 & 0 \\ 0 & 0 & 0 & * \end{bmatrix} \text{ or}$$

$$M^{-1}(\alpha) = \begin{bmatrix} 0 & 0 & 0 & * \\ 0 & 0 & * & 0 \\ 0 & * & 0 & 0 \\ * & 0 & 0 & 0 \end{bmatrix}.$$

- This truncated characteristic over six rounds exists for $4 \times (2^{16} - 1) \approx 2^{18}$ values of α,
- Key recovery attack on 8 rounds can be done by data complexity $2^{35.8}$ and time complexity of $2^{96.8}$ memory accesses in addition of 2^{88} full encryption.

Conclusions

- We introduced new generic distinguishers on PRINCE-like ciphers.
- The security of PRINCE-like ciphers depends strongly on the choice of the value of α.
- We identified special classes of α for which 4, 6, 8 or 10 rounds can be distinguished from random.
- The weakest class allows an efficient key-recovery attack on 12 rounds of the cipher.
- Our best attack on PRINCE with original α breaks a reduced 6-round version.
- New design criteria for the selection of the value of α for PRINCE-like ciphers are obtained.

Thanks for your attention!