

# Higher-Order Side Channel Security and Mask Refreshing

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FSE 2013 – March 2013



# Side Channel Analysis

- Side Channel Attacks (SCA) appear 15 years ago
  - ▶ 1996 : Timing Attacks
  - ▶ 1998 : Power Analysis
  - ▶ 2000 : Electromagnetic Analysis
- Numerous attacks
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  - ▶ 1999 : (multi-bit) DPA Messerges99
  - ▶ 2000 : Higher-order SCA Messerges2000
  - ▶ 2002 : Template SCA ChariRaoRohatgi2002
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  - ▶ 2005 : Stochastic SCA SchindlerLemkePaar2006
  - ▶ 2008 : Mutual Information SCA GierlichsBatinaTuyls2008
  - ▶ etc.



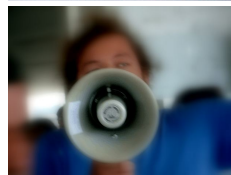
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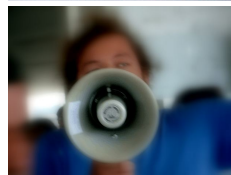
# SCA Countermeasures

- **Masking** [IBM Team at CRYPTO 1999].
  - ▶ Efficient against SCA in practice.
  - ▶ Difficult to implement for non-linear transformations.
- **Shuffling** [Researchers from Graz University at ACNS 2006].
  - ▶ Less efficient against SCA in practice.
  - ▶ Easy to implement for every transformation.
- **Whitening** [Kocher Jaffe June, CRYPTO 1999].
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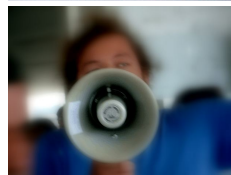
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# Masking/Sharing Countermeasures

Idea : consists in securing the implementation using **secret sharing techniques**.

- First Ideas in GoubinPatarin99 and ChariJutlaRaoRohatgi99.
- Soundness based on the following remark :

[Chari-Jutla-Rao-Rohatgi CRYPTO'99]

- ▶ Bit  $x$  masked  $\mapsto x_0, x_1, \dots, x_d$
- ▶ Leakage :  $L_i \sim x_i + \mathcal{N}(\mu, \sigma^2)$
- ▶ # of leakage samples to test  $((L_i)_i | x = 0) = ((L_i)_i | x = 1)$  :

$$q \geq O(1)\sigma^d$$

- Until now, security proofs are not unconditional and are "limited" to so-called **probing adversaries**.





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# Probing Adversary

- Notion introduced in IshaiSahaiWagner, CRYPTO 2003
- A  $d^{\text{th}}$ -order probing adversary is allowed to observe **at most  $d$**  intermediate results during the overall algorithm processing.
  - ▶ Hardware interpretation :  $d$  is the maximum of wires observed in the circuit.
  - ▶ Software interpretation :  $d$  is the maximum of different timings during the processing.
- $d^{\text{th}}$ -order probing adversary =  $d^{\text{th}}$ -order SCA as introduced in Messerges99.
- Countermeasures proved to be secure against a  $d^{\text{th}}$ -order probing adv. :
  - ▶  $d = 1$  : KocherJaffeJune99, BlömerGuajardoKrummel04, ProuffRivain07.
  - ▶  $d = 2$  : RivainDottaxProuff08.
  - ▶  $d \geq 1$  : IshaiSahaiWagner03, ProuffRoche11, GenelleProuffQuisquater11, CarletGoubinProuffQuisquaterRivain12.



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# Higher-Order Masking Schemes

Achieving security in the probing adversary model

## Definition

A *dth-order masking scheme* for an encryption algorithm  $c \leftarrow \mathcal{E}(m, k)$  is an algorithm

$$(c_0, c_1, \dots, c_d) \leftarrow \mathcal{E}'((m_0, m_1, \dots, m_d), (k_0, k_1, \dots, k_d))$$

- **Completeness** : there exists  $R$  s.t. :

$$R(c_0, \dots, c_d) = \mathcal{E}(m, k)$$

- **Security** :  $\forall \{iv_1, iv_2, \dots, iv_d\} \subseteq \{\text{intermediate var. of } \mathcal{E}'\}$  :

$$\Pr(k \mid iv_1, iv_2, \dots, iv_d) = \Pr(k)$$

For SPN (eg. DES, AES) the main issue is **masking the S-box**.



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# Masking a S-box

Original work of Ishai, Sahai and Wagner

Main idea : split the S-box computation into elementary operations and protect each of them individually.

- Original idea limited to  $GF(2)$  IshaiSahaiWagner2003
- Extended to any field in RivainProuff2010 and FaustRabinReyzinTromerVaikuntanathan2011.
- Data are split by bitwise addition :  $x \longrightarrow x_0, \dots, x_d$  s.t.  $x_i \leftarrow \$, i > 0$ , and  $x_0 = \bigoplus_i x_i$ .
- Masking of Linear Transformations  $L$  is **easy** :

$$L(x) \rightarrow \underbrace{L(x_0), L(x_1), \dots, L(x_d)}_{L(x_0) \oplus L(x_1) \oplus \dots \oplus L(x_d) = L(x)}$$

- Masking of non-linear transformations is an **issue** since the operations cannot be done on each shares separately.
  - ▶  $\rightarrow$  Problem reduces to secure multiplications !





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# Masking Multiplications ✕

Ishai-Sahai-Wagner Scheme (ISW)

- Outlines of the scheme :

- ▶ Input :  $(a_i)_i, (b_i)_i$  s.t.  $\bigoplus_i a_i = a, \bigoplus_i b_i = b$
- ▶ Output :  $(c_i)_i$  s.t.  $\bigoplus_i c_i = a \times b$

$$\bigoplus_i c_i = \left(\bigoplus_i a_i\right) \times \left(\bigoplus_i b_i\right) = \bigoplus_{i,j} a_i \times b_j$$

- Example ( $d = 2$ ) :

- Ishai *et al.* prove  $(d/2)$ th-order security

- ▶ Extended to get  $d$ th-order security in RivainProuff10



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$$\begin{pmatrix} a_0 b_0 & (a_0 b_1 \oplus r_{0,1}) \oplus a_1 b_0 & (a_0 b_2 \oplus r_{0,2}) \oplus a_2 b_0 \\ r_{0,1} & a_1 b_1 & (a_1 b_2 \oplus r_{1,2}) \oplus a_2 b_1 \\ r_{0,2} & r_{1,2} & a_2 b_2 \end{pmatrix}$$

$c_1$

- Ishai *et al.* prove  $(d/2)$ th-order security

- ▶ Extended to get  $d$ th-order security in RivainProuff10



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Ishai-Sahai-Wagner Scheme (ISW)

- Outlines of the scheme :

- ▶ Input :  $(a_i)_i, (b_i)_i$  s.t.  $\bigoplus_i a_i = a, \bigoplus_i b_i = b$
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# Application to Secure Power Functions

... with a focus on the *AES power function*  $x \mapsto x^{254}$

Let  $\text{Exp} : x \mapsto x^r$  be a power function defined over a finite field  $\text{GF}(2^n)$ .

- Split  $\text{Exp}$  into a sequence of multiplications and squarings.
- Squaring is a  $\text{GF}(2)$ -linear operation  $\rightarrow$  **easy to mask** :
  - ▶ masked square :  $x^2 \rightarrow x_0^2, x_1^2, \dots, x_d^2$
- Multiplications masked with ISW Scheme
- To reduce the overall cost of the securing, favour squaring over multiplication in the  $\text{Exp}$  evaluation method :
  - ▶ amount to look at small **addition chains** for  $r$
- For AES non-linear function ( $r = 254$ ), Rivain and Prouff proves that the evaluation can be done with **4** multiplications only (**optimal**).



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# Masking the S-box

RivainProuff10

Algorithmic description :

**Input** : shares  $\mathbf{x}_i$  s.t.  $\bigoplus_i \mathbf{x}_i = \mathbf{x}$

**Output** : shares  $y_i$  s.t.  $\bigoplus_i y_i = \mathbf{x}^{254}$

1.  $(z_i)_i \leftarrow (x_i^2)_i$

$$[\bigoplus_i z_i = x^2]$$

2. RefreshMasks( $(z_i)_i$ )

3.  $(y_i)_i \leftarrow \text{ISW}((z_i)_i, (x_i)_i)$

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# Security

- Security proved against a  $d^{\text{th}}$ -order probing adversary
- RefreshMasks assumed to be out of the scope of the proof.
- A simple (and assumed to be secure) algorithm is proposed to refresh the masks :

**Input** : shares  $z_i$  s.t.  $\bigoplus_i z_i = z$

**Output** : new shares  $z'_i$  s.t.  $\bigoplus_i z'_i = z$

1. for  $i = 1$  to  $d$  do
2.     $tmp \leftarrow \text{rand}(n)$
3.     $z_0 \leftarrow z_0 \oplus tmp$
4.     $z'_i \leftarrow z_i \oplus tmp$



# The Flaw

Let us focus on the three first steps of Rivain-Prouff's scheme.

1.  $(z_i)_i \leftarrow (x_i^2)_i$
2.  $(z'_i)_i \leftarrow \text{RefreshMasks}((z_i)_i)$
3.  $(y_i)_i \leftarrow \text{ISW}((z'_i)_i, (x_i)_i)$

- By construction, at the  $d/2^{\text{th}}$  iteration of RefreshMasks :



- By definition, ISW involves the following processings (cross-products) :

$$z'_i \times x_{i+d/2}$$

for all  $i \in [1; d/2]$



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- By construction, at the  $d/2^{\text{th}}$  iteration of RefreshMasks :

$$z_0 = z \oplus \bigoplus_{1 \leq i \leq d/2} z'_i \oplus \bigoplus_{d/2+1 \leq i \leq d} z_i$$

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# The Flaw

$$z_0 = z \oplus \bigoplus_{1 \leq i \leq d/2} z'_i \oplus \bigoplus_{d/2+1 \leq i \leq d} x_i^2 \rightarrow \ell_0$$

$$z'_i \times x_{i+d/2} \quad \forall i \in [1; d/2] \rightarrow \ell_i$$

- The  $d/2$  leakage values  $\ell_i$  bring information on all the shares  $z'_i$  and  $x_{i+d/2}$  for  $i \leq d/2$ .
- This information is combined with  $\ell_0$  to retrieve information on (a.k.a. **unmask**)  $z$ .
  - ▶ Indeed  $\Pr[z \mid (\ell_i)_i, \ell_0] \neq \Pr[z]$ .





# First (natural) Countermeasure

- Replace the RefreshMasks call by a call to ISW s.t. :
  - ▶ the first input is the sharing (of  $x$ ) to refresh and
  - ▶ the second input is a sharing of 1.
- By definition, ISW will indeed outputs a **new sharing** of  $x \times 1$ .
- We get :

1.  $(z_i)_i \leftarrow (x_i^2)_i$
2.  $(z_i)_i \leftarrow \text{ISW}((z_i)_i, (1_i)_i)$   $(1_i)_i$  sharing of 1
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- Problem : security difficult to prove !



# Second Countermeasure Proposal

- Principle : Replace every processing of  $h(x) = x \cdot x^{2^j}$  s.t.
  1.  $(z_i)_i \leftarrow (x_i^{2^j})_i$   $(z_i)_i$  sharing of  $x^{2^j}$
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  3.  $(y_i)_i \leftarrow \text{ISW}((z_i)_i, (x_i)_i)$   $(y_i)_i$  sharing of  $x \cdot x^{2^j}$

by a single processing of a new algorithm ISW'

- Core idea :

$$\begin{aligned}y &= \bigoplus_i a_i \cdot \bigoplus_i a_i^{2^j} \\&= \bigoplus_i a_i^{2^j+1} \oplus \bigoplus_{i < k} (a_i \cdot a_k^{2^j} \oplus a_k \cdot a_i^{2^j}) \\&= \bigoplus_i h(a_i) \oplus \bigoplus_{i < k} f(a_i, a_k)\end{aligned}$$

involve the new function  $f(x, y) = x \cdot y^{2^j} \oplus x^{2^j} \cdot y$

- ▶  $f$  is bilinear, thus we have

$$(Property *) \quad f(x, y) = h(x \oplus y) \oplus h(x) \oplus h(y)$$



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# Masking of Power functions $x \mapsto x^{2^j+1}$

Outlines of the new scheme ISW'

- I/O :

- ▶ Input :  $(a_i)_i$  s.t.  $\bigoplus_i a_i = a$

- ▶ Output :  $(c_i)_i$  s.t.  $\bigoplus_i c_i = h(a) = a \times a^{2^j}$

- Example ( $d = 2$ ) :

- Security against  $d^{\text{th}}$  order probing adversary is given in the paper.



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$$\begin{pmatrix} a_0^{2^j+1} & a_0 \cdot a_1^{2^j} & a_0 \cdot a_2^{2^j} \\ a_1 \cdot a_0^{2^j} & a_1^{2^j+1} & a_1 \cdot a_2^{2^j} \\ a_2 \cdot a_0^{2^j} & a_2 \cdot a_1^{2^j} & a_2^{2^j+1} \end{pmatrix}$$

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$$\begin{pmatrix} a_0^{2^j+1} & a_0 \cdot a_1^{2^j} \oplus a_1 \cdot a_0^{2^j} & a_0 \cdot a_2^{2^j} + a_2 \cdot a_0^{2^j} \\ 0 & a_1^{2^j+1} & a_1 \cdot a_2^{2^j} \oplus a_2 \cdot a_1^{2^j} \\ 0 & 0 & a_2^{2^j+1} \end{pmatrix}$$

- Security against  $d^{\text{th}}$  order probing adversary is given in the paper.



# Masking of Power functions $x \mapsto x^{2^j+1}$

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We eventually get :

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It is not only more secure than the first Rivain-Prouff proposal, but also **more efficient** → see timings in the paper.



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- Global security of the Masking Scheme yet to prove :

e.g.  $y = x^{14}$

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- can we do better?  
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