

# Malleable Proof Systems and Applications

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Melissa Chase (MSR Redmond)

Markulf Kohlweiss (MSR Cambridge)

Anna Lysyanskaya (Brown University)

**Sarah Meiklejohn (UC San Diego)**

# Non-malleable cryptography

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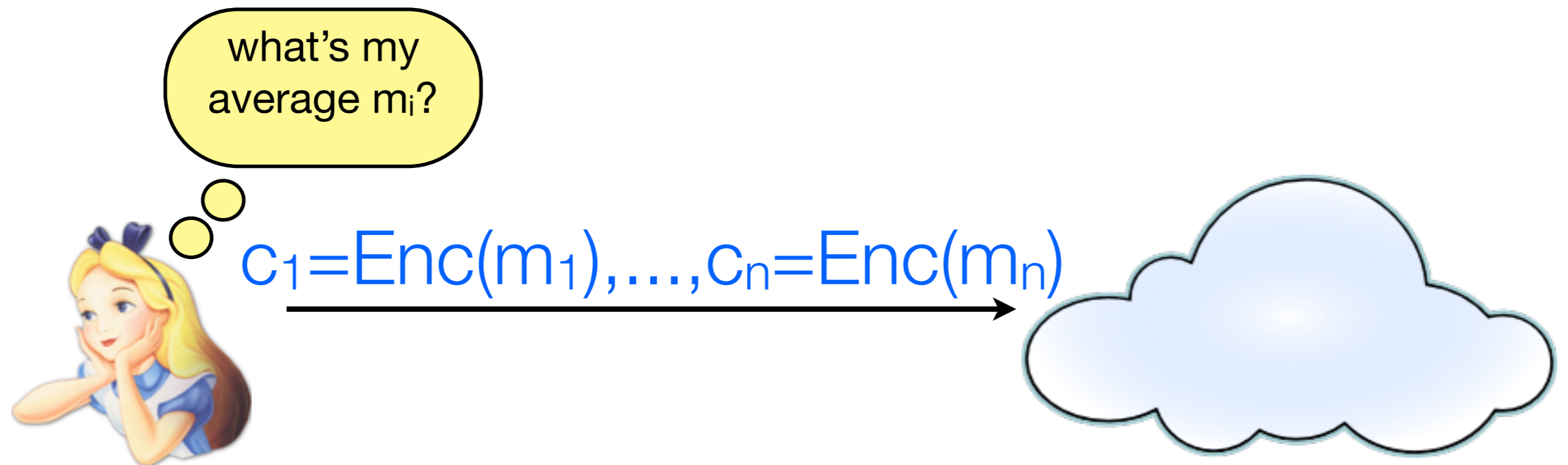
$$c_1 = \text{Enc}(m_1), \dots, c_n = \text{Enc}(m_n)$$



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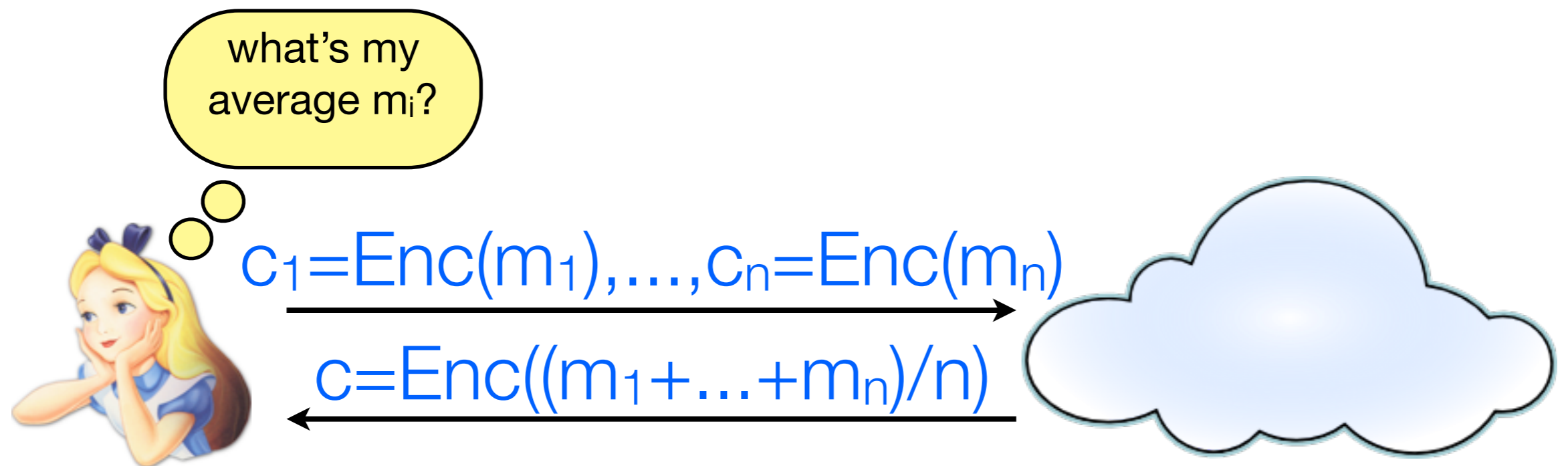
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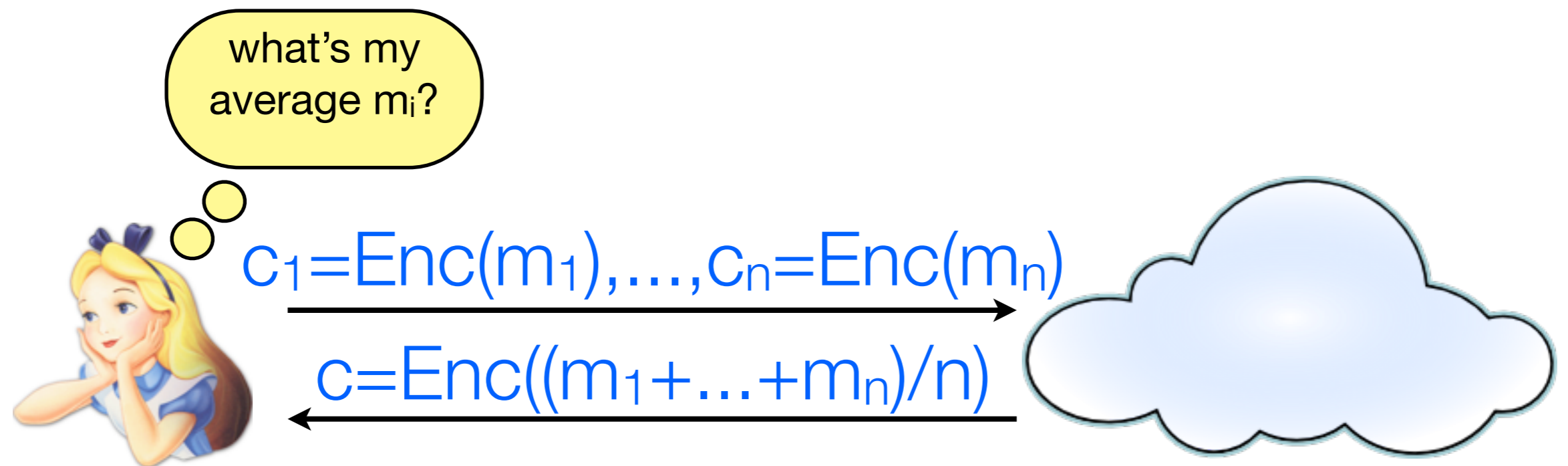
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Has applications in cloud storage, outsourcing computation, search on encrypted data, etc.

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In this work:

- Introduce notions of **uncontrolled** and **controlled** malleability for proofs
- Give two applications: **CM-CCA security** and **compact verifiable shuffles**
- Examine malleability within existing proof systems

# Outline

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Definitions

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cm-NIZK construction

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## Definitions

Zero knowledge  
Malleability  
Controlled malleability  
Derivation privacy

cm-NIZK construction

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Example: take a proof  $\pi_1$  that  $b_1$  is a bit and a proof  $\pi_2$  that  $b_2$  is a bit, and “maul” them somehow to get a proof that  $b_1 \cdot b_2$  is a bit

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More generally, a proof is **malleable with respect to  $T$**  if there exists an algorithm **Eval** that on input  $(T, \{x_i, \pi_i\})$ , outputs a proof  $\pi$  for  $T(\{x_i\})$

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- E.g.,  $T = \times$ ,  $x_i = \text{“}b_i \text{ is a bit”}$

If we want zero knowledge, need to make sure proofs are malleable only with respect to operations under which the language is **closed**

- E.g., with bits, we run into trouble if we try to use  $T = +$

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- Even more, **simulation-sound extractability** [G06] says that in fact we can always pull out a witness from any proof output by the adversary
- Our definition goes one step further: either we can pull out a witness, or it was derived from a simulated proof under a transformation in  $\mathcal{J}$

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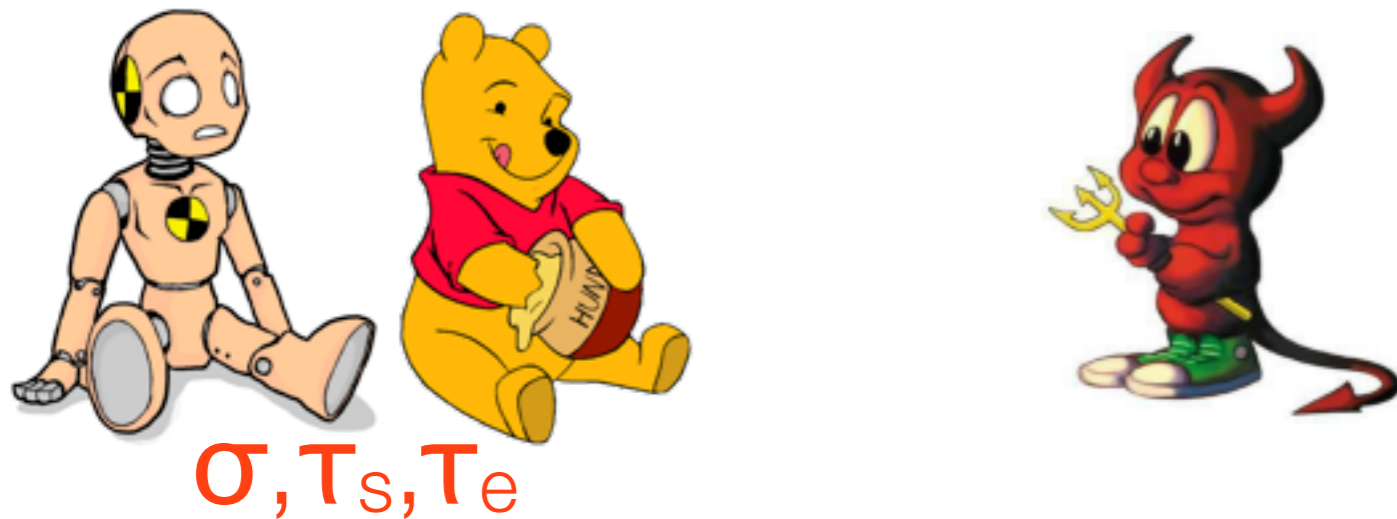




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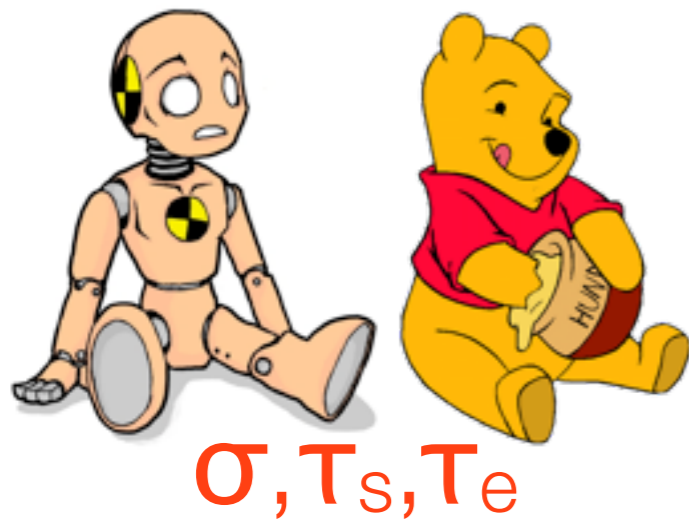
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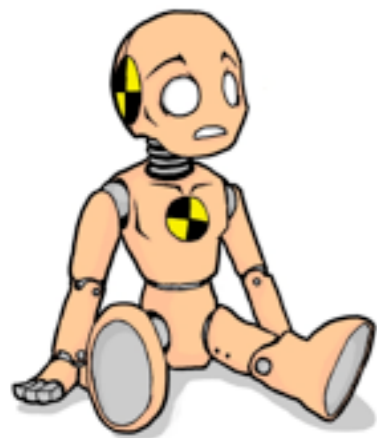
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$\sigma, \tau_s,$



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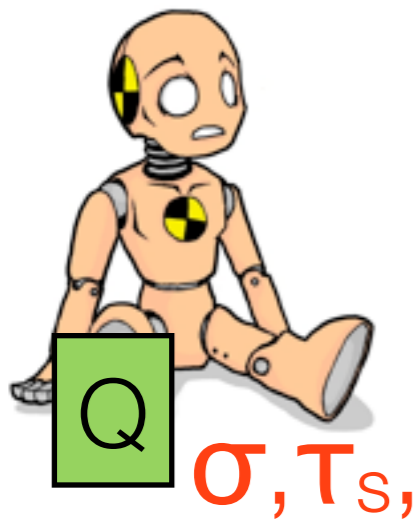


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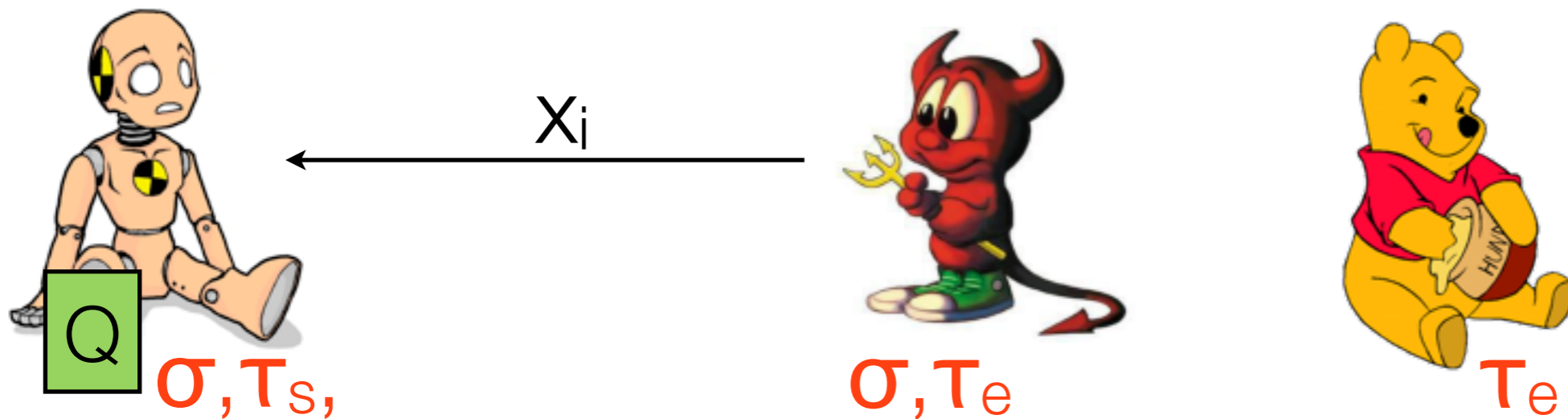


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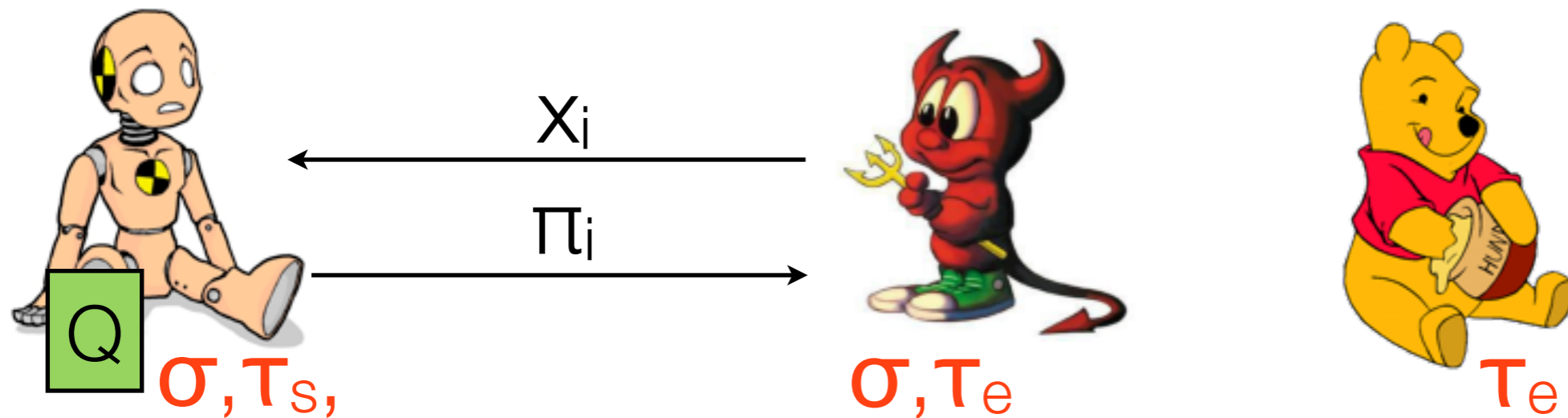
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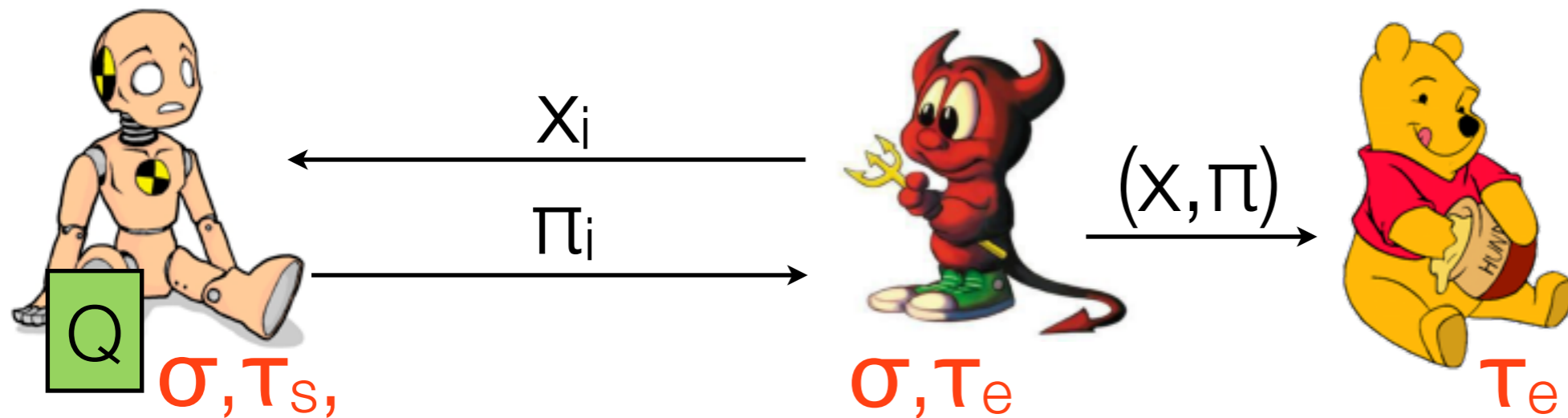
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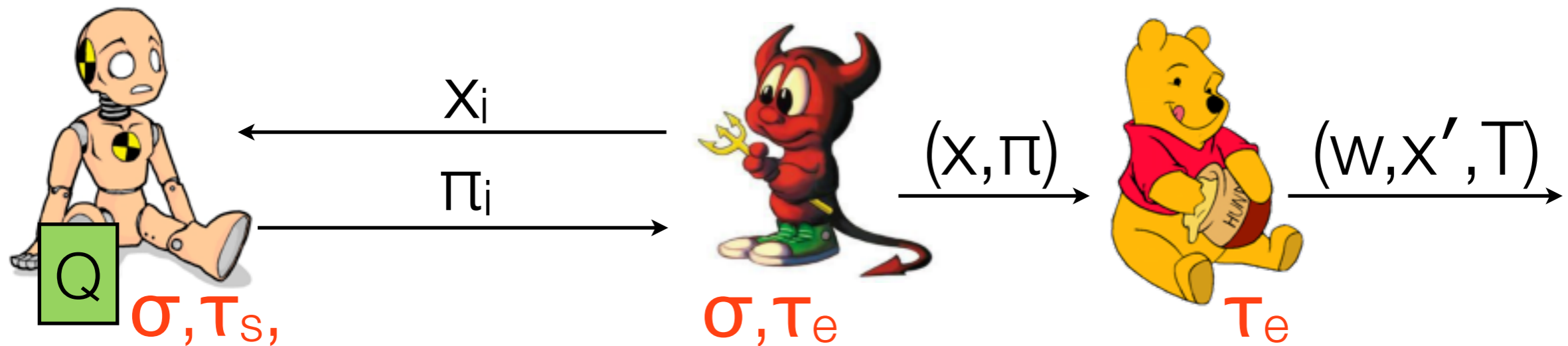
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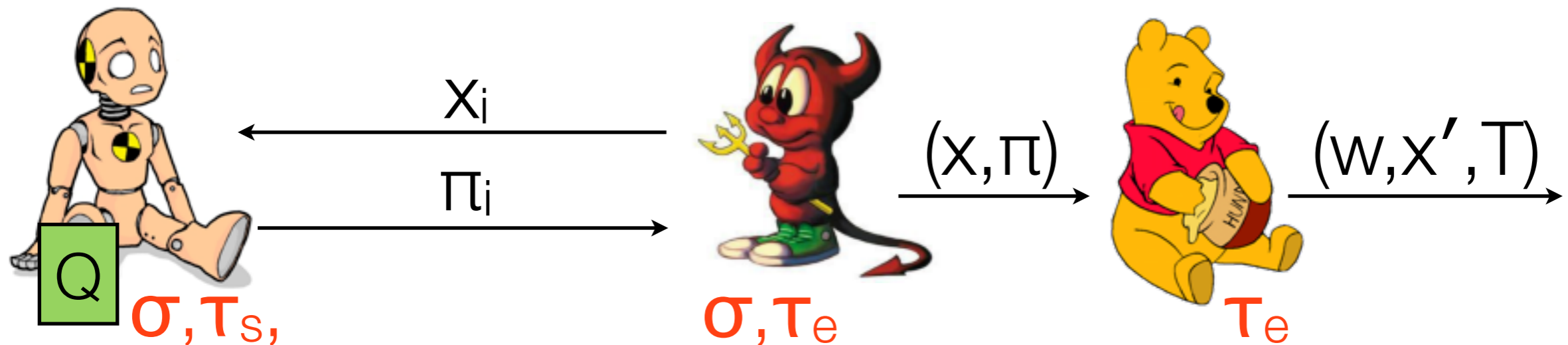
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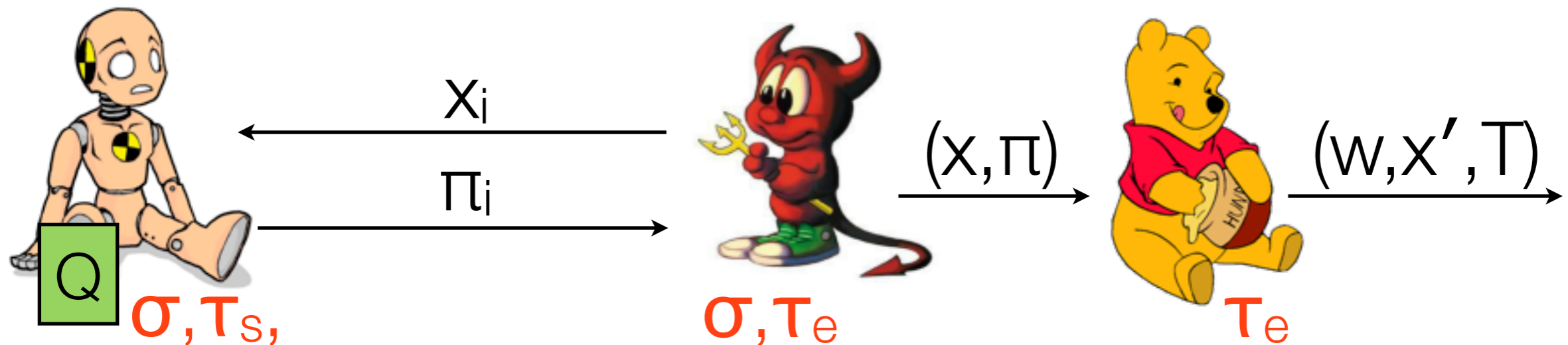
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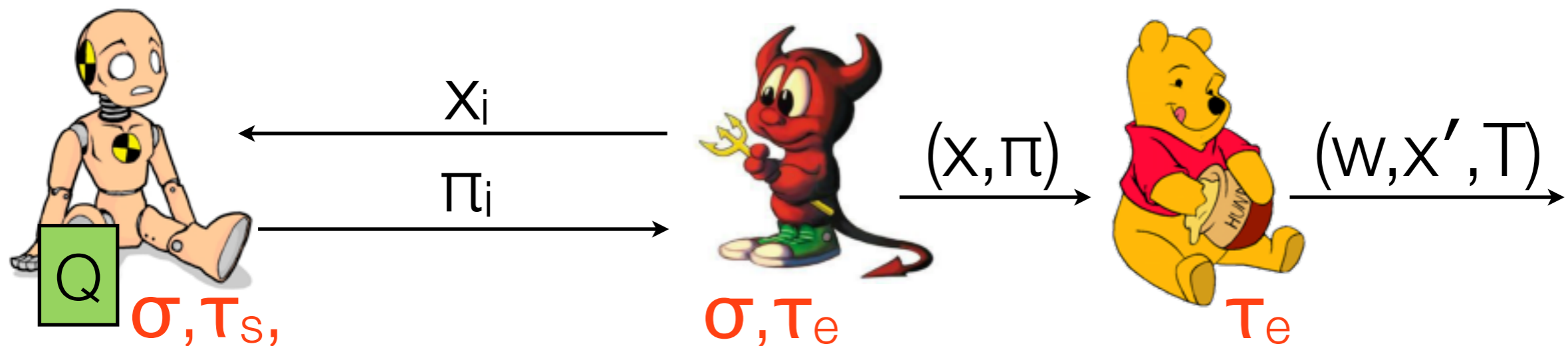


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(like function privacy for encryption)

If a proof is zero knowledge, CM-SSE, and strongly derivation private, then we call it a **cm-NIZK**

# Outline

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Definitions

**cm-NIZK construction**

Generic construction  
Efficient instantiation

Applications

Conclusions

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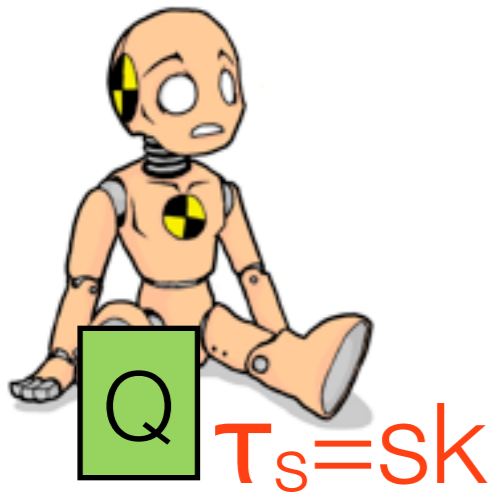


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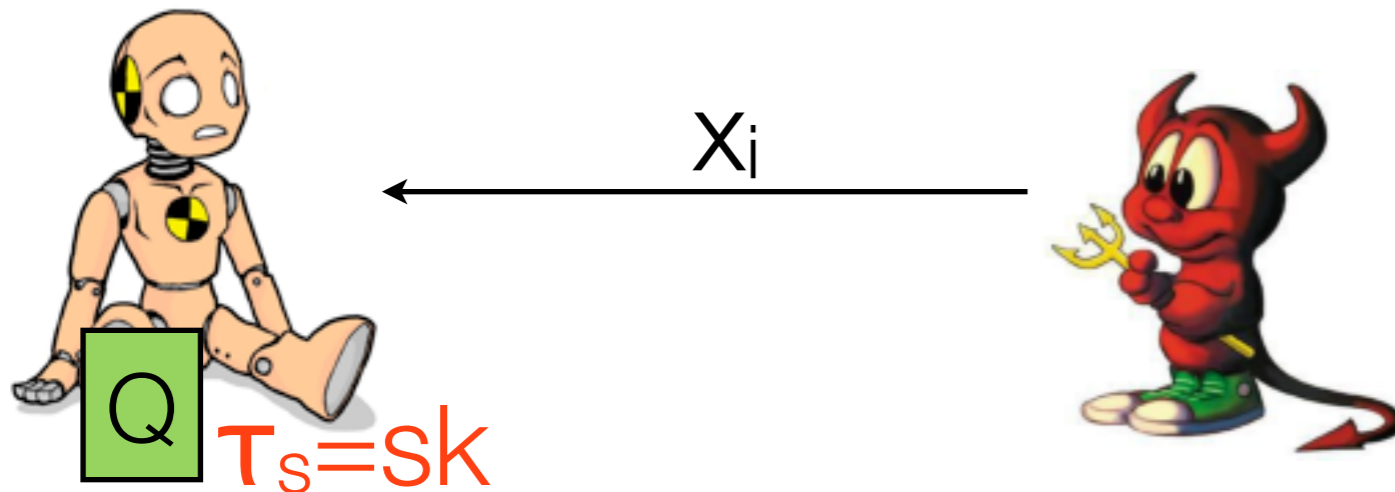


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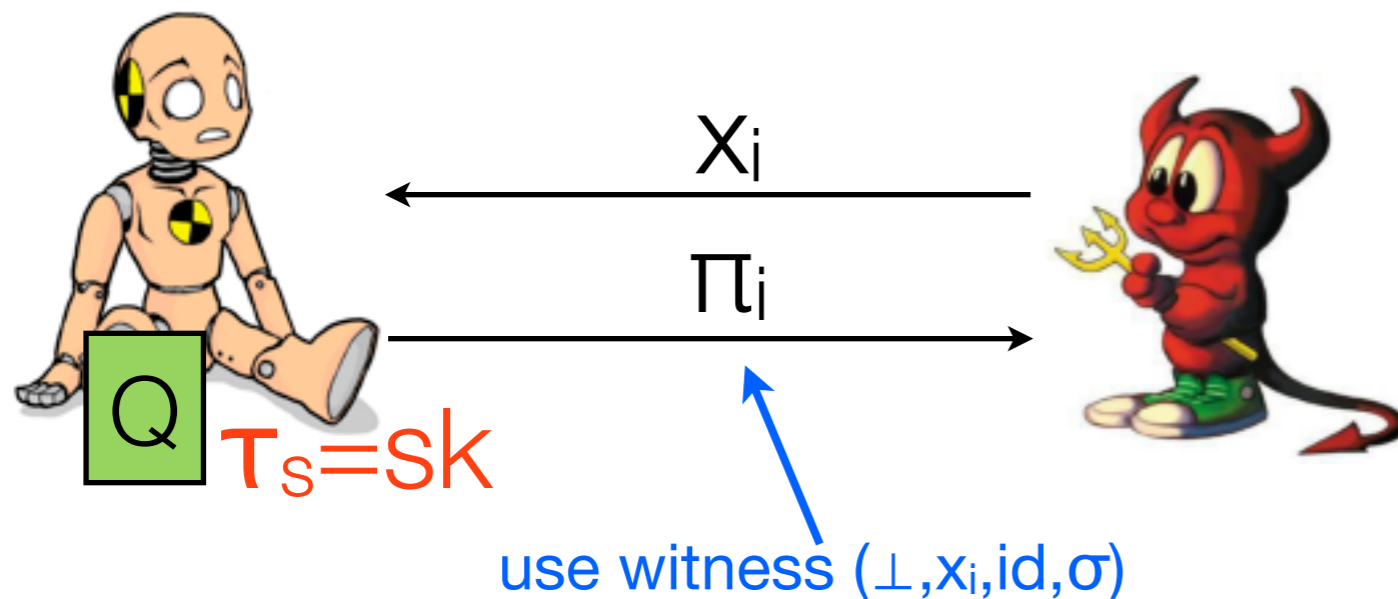


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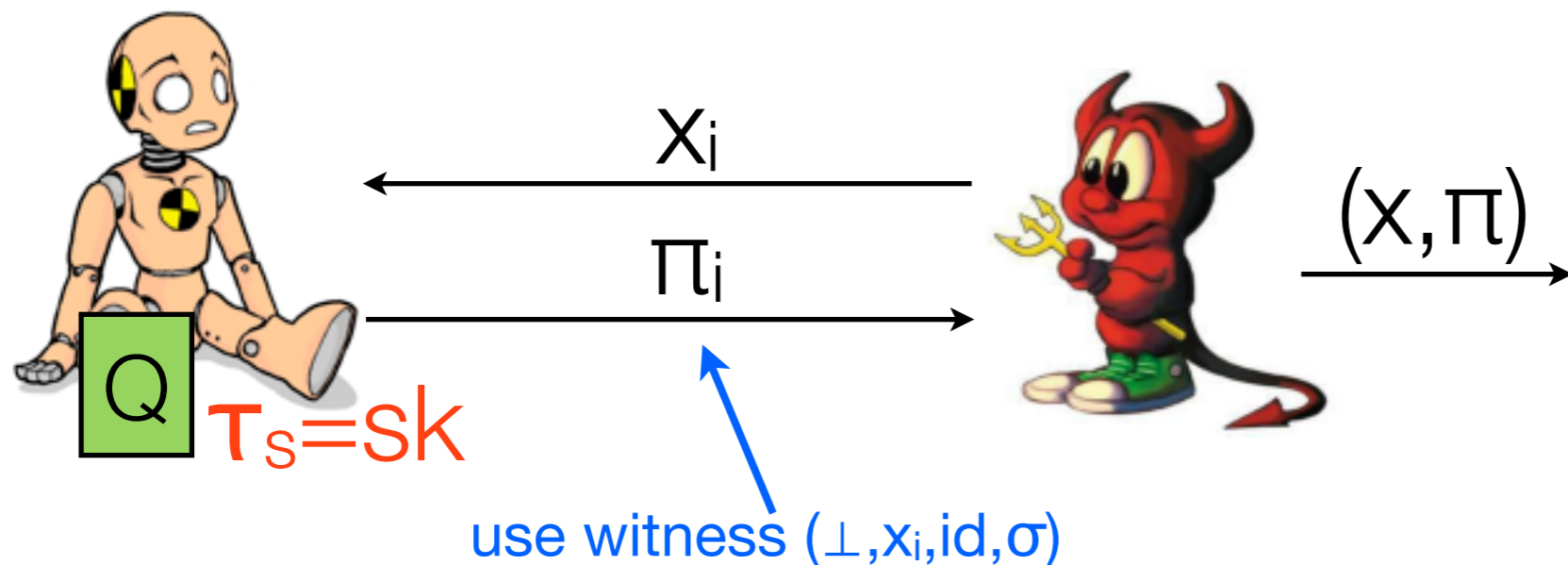


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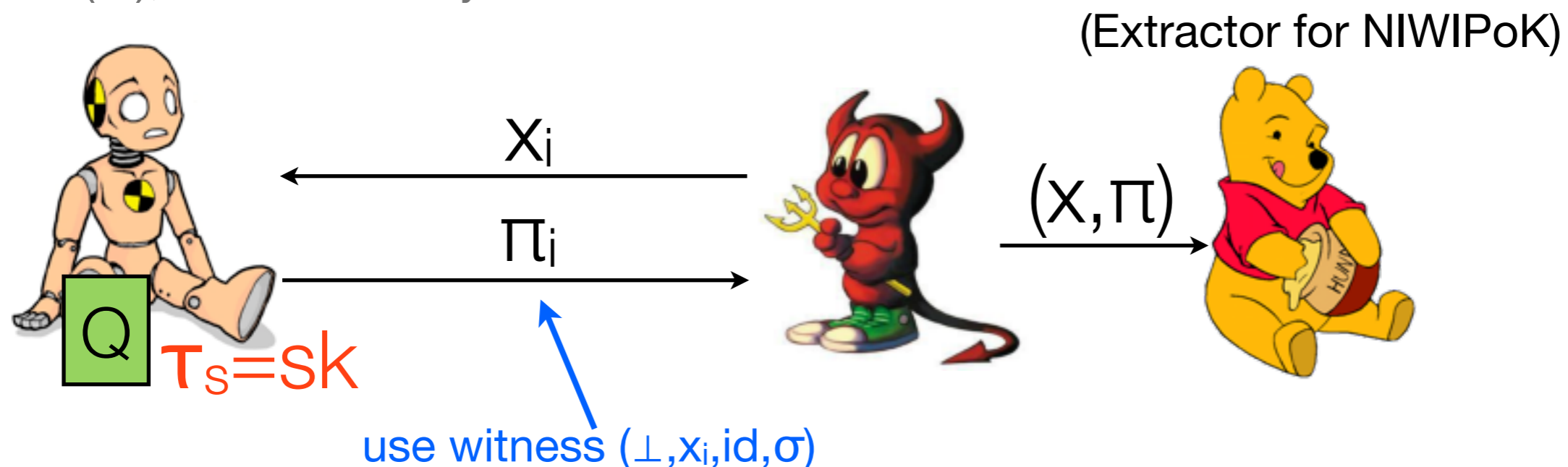
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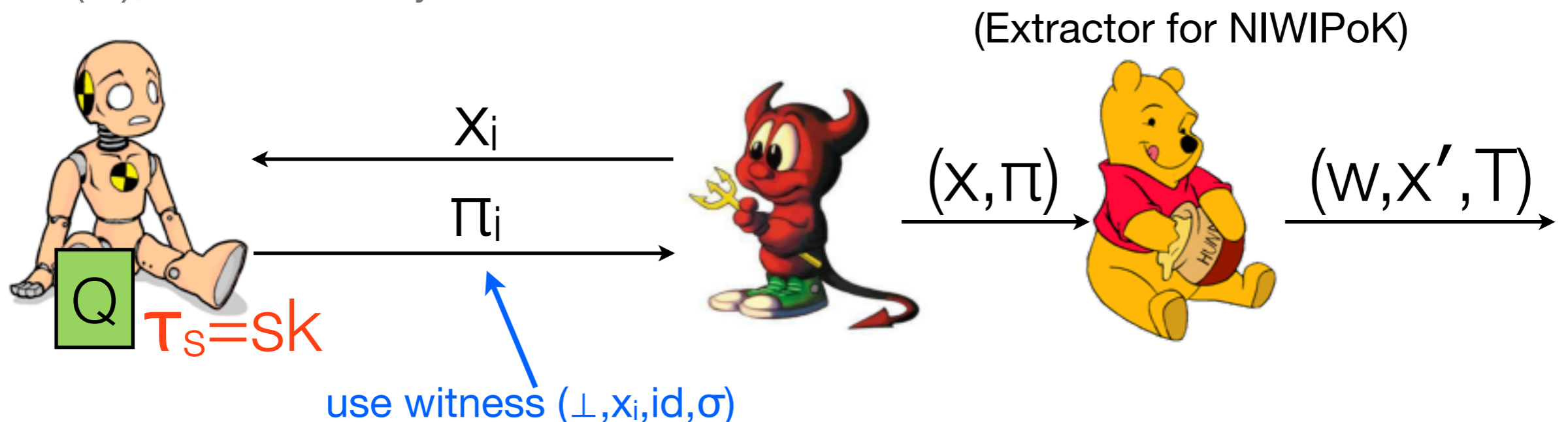
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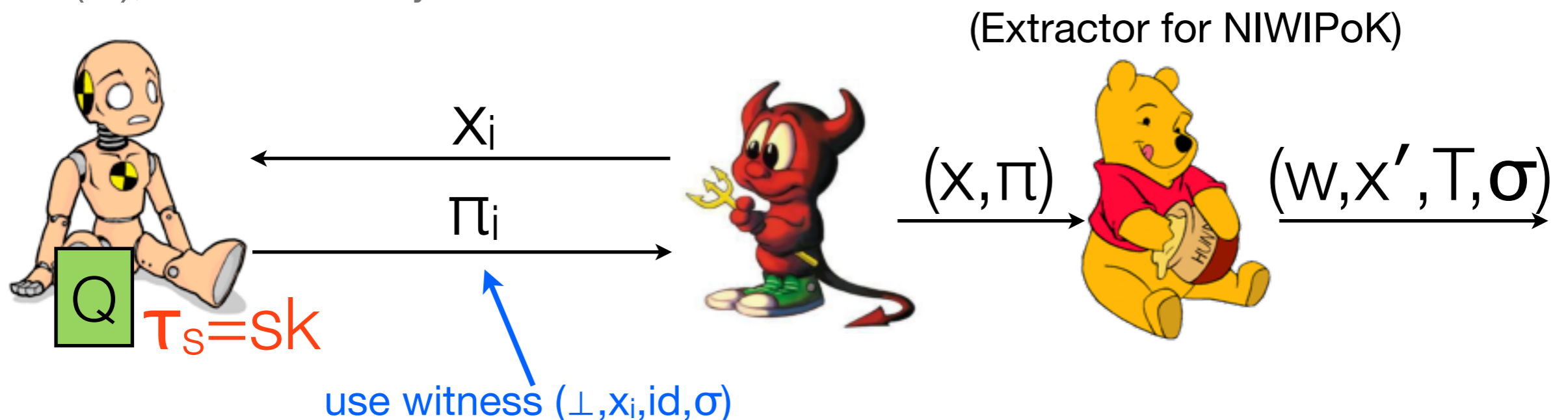
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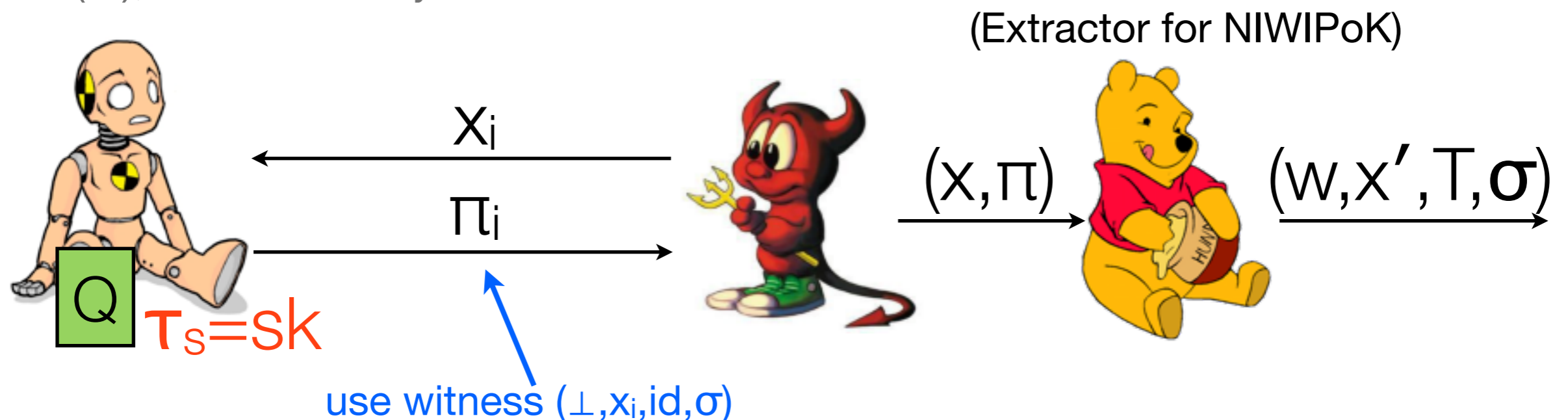
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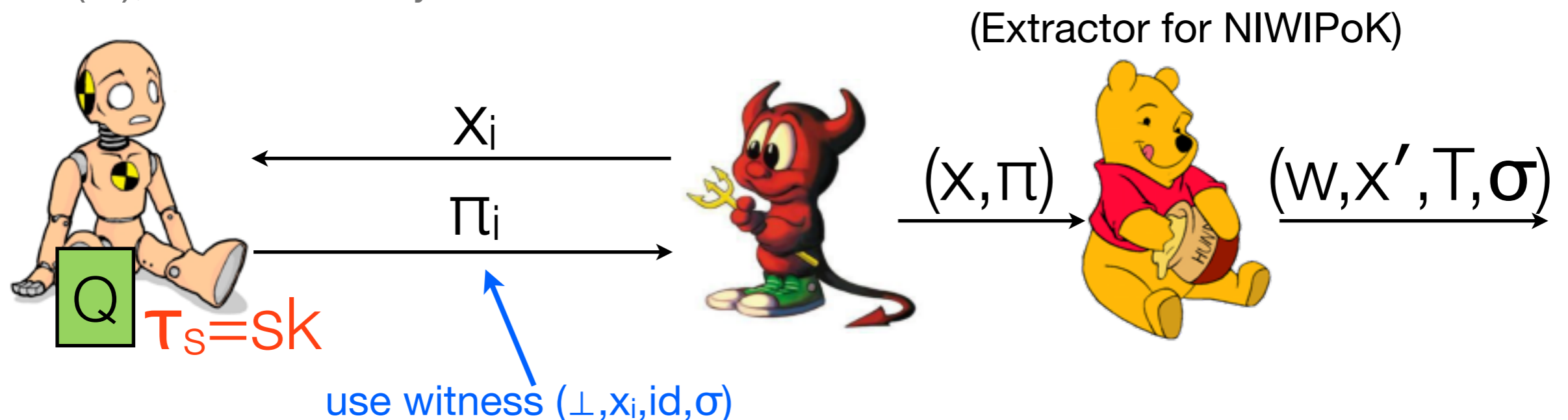
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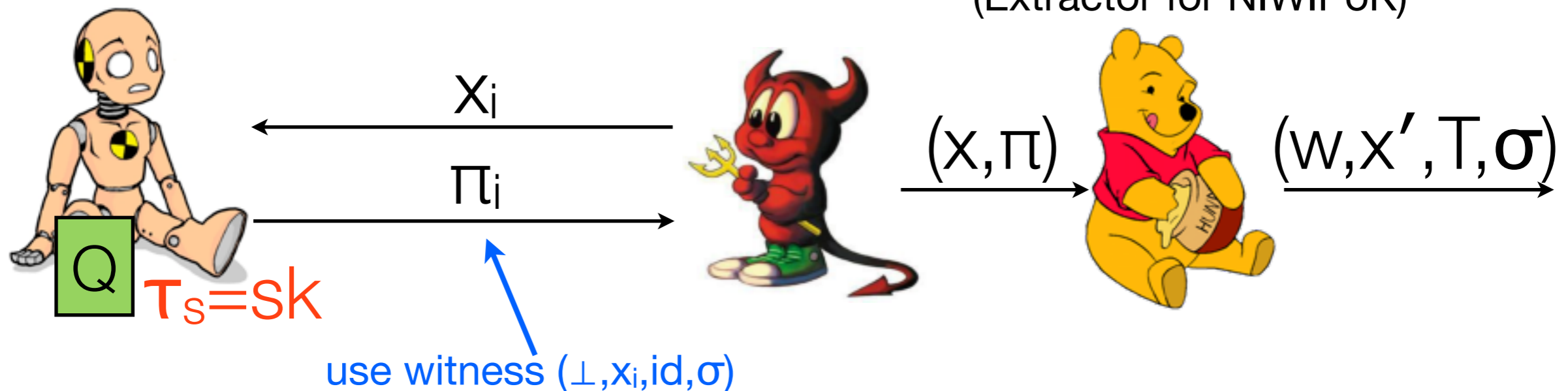
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(Extractor for NIWIPoK)



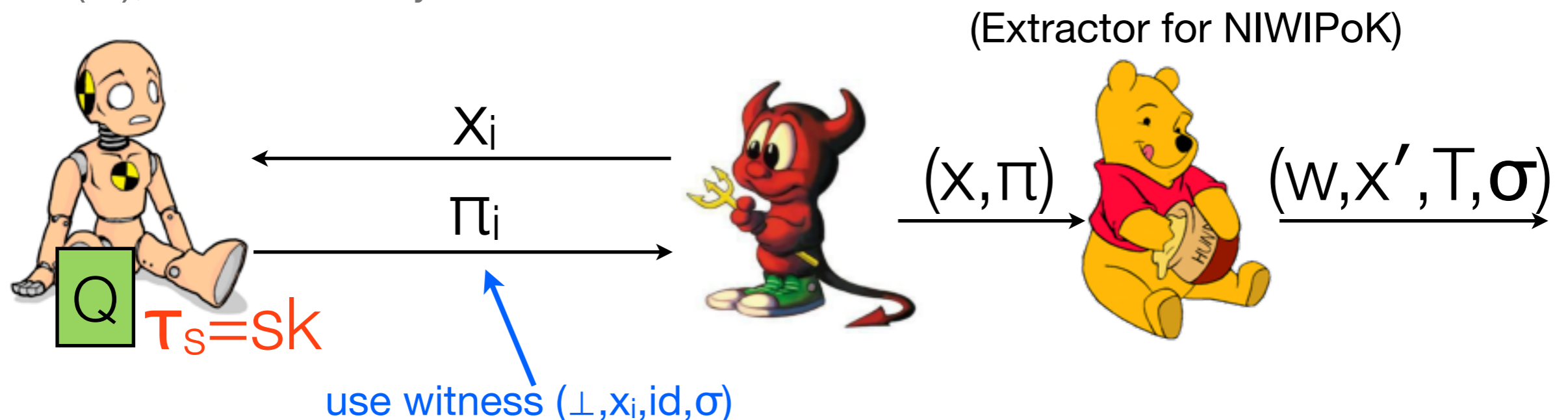
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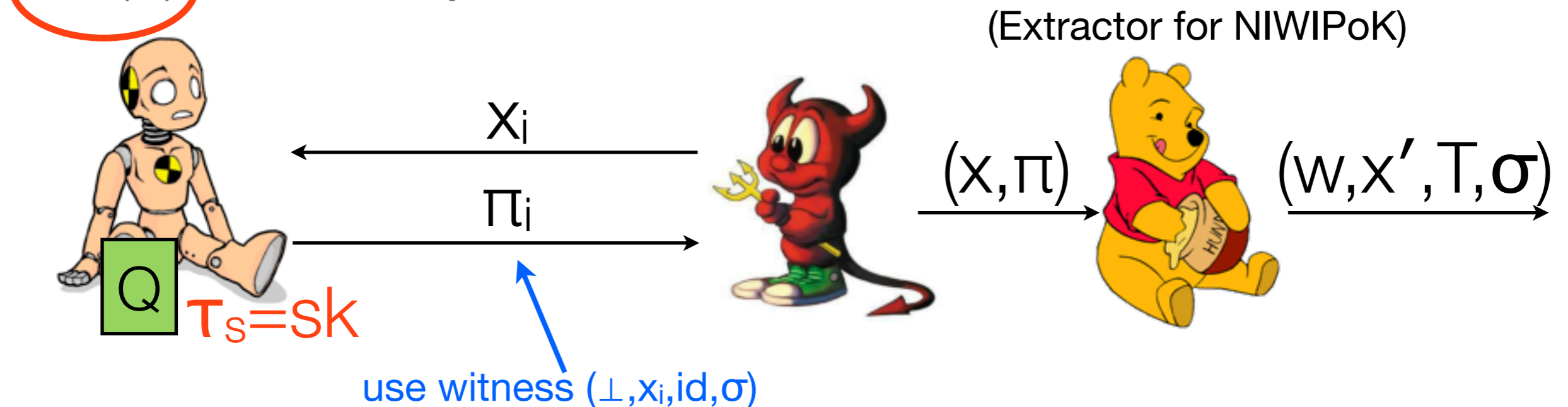


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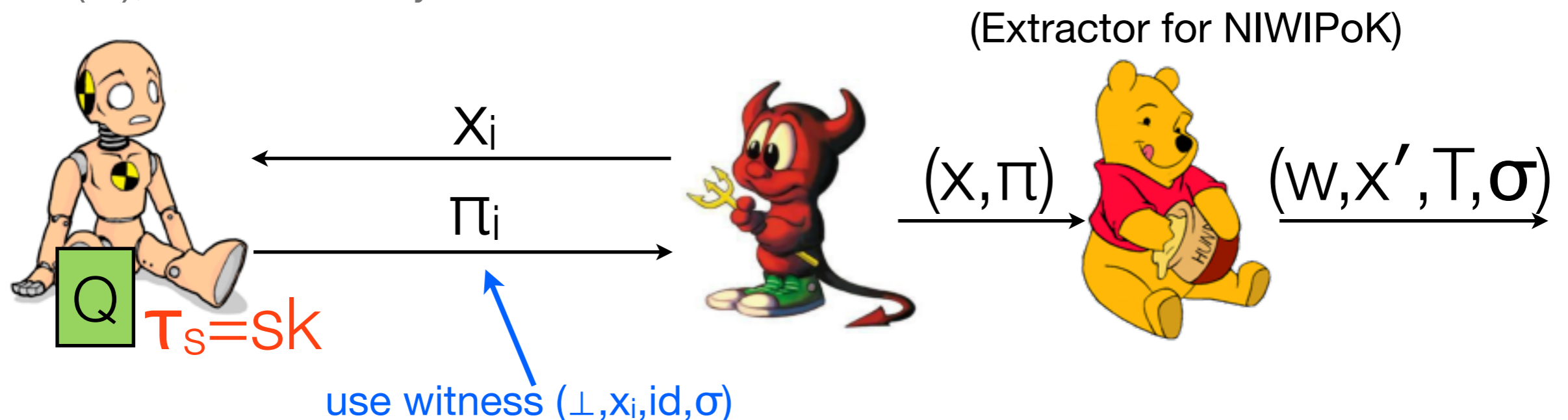


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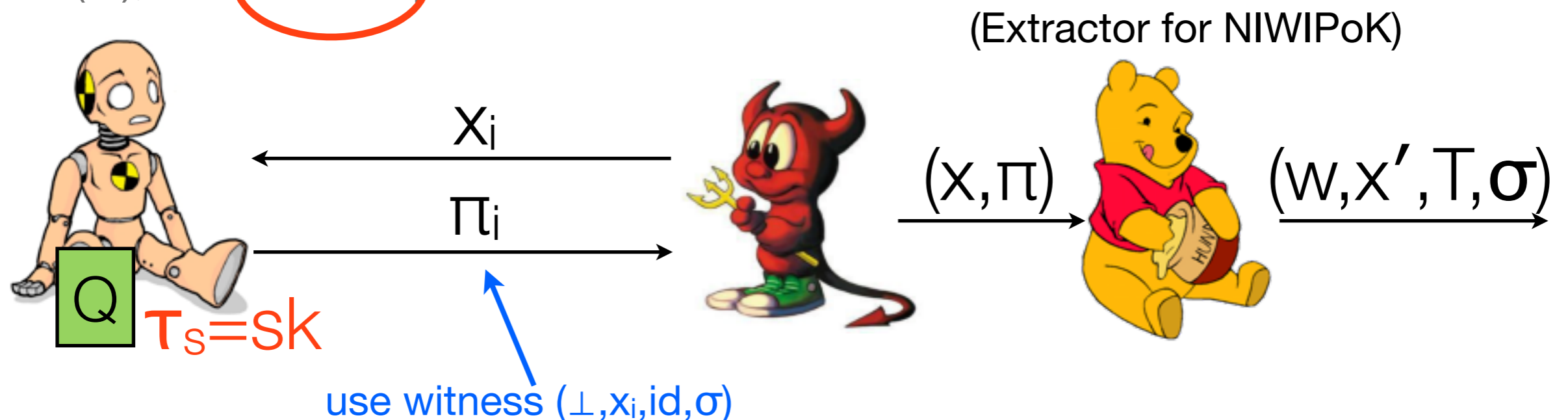


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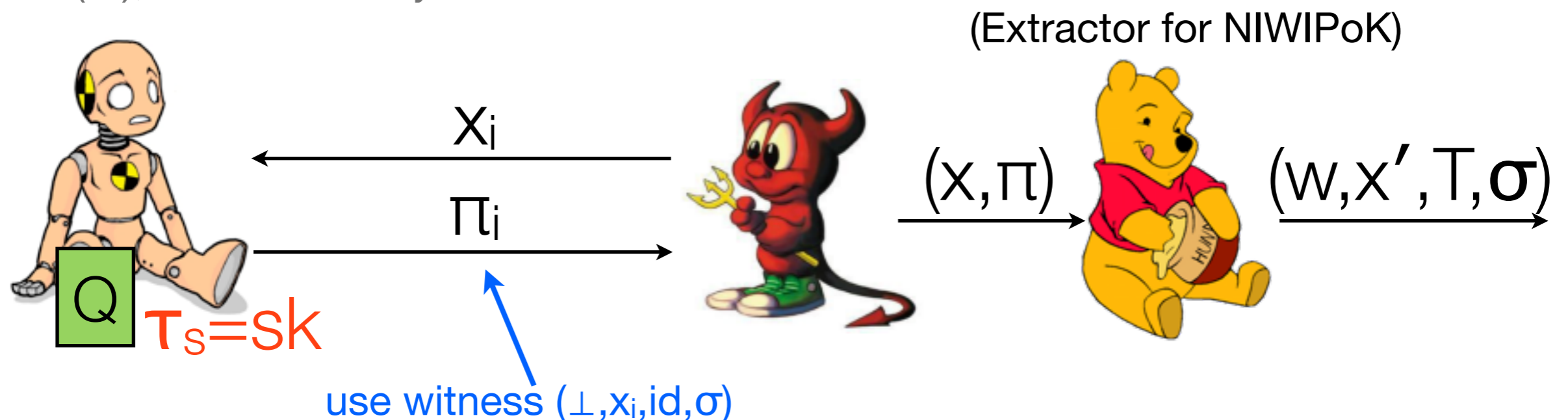


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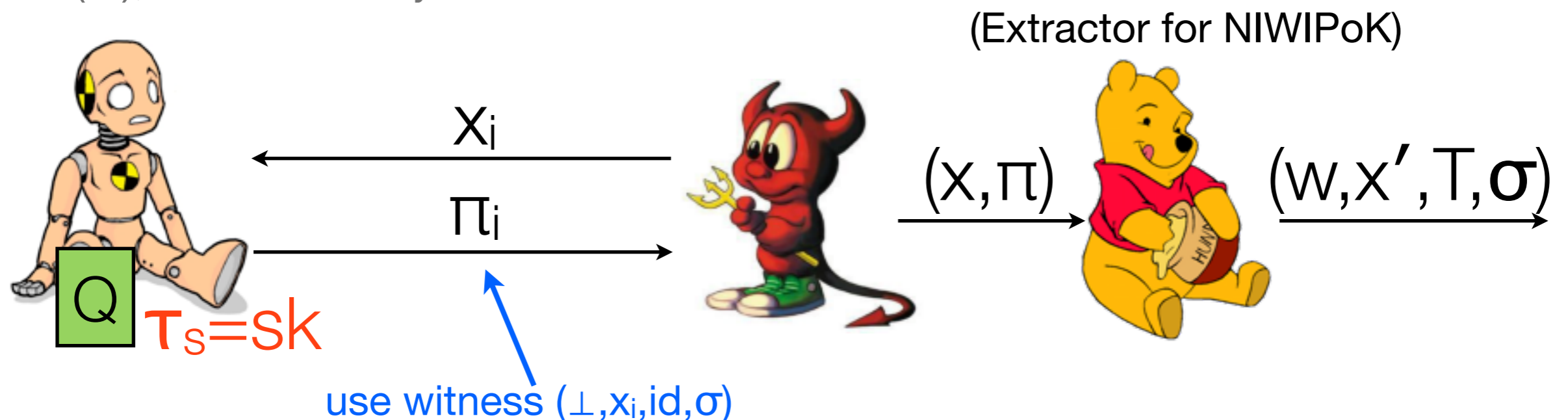


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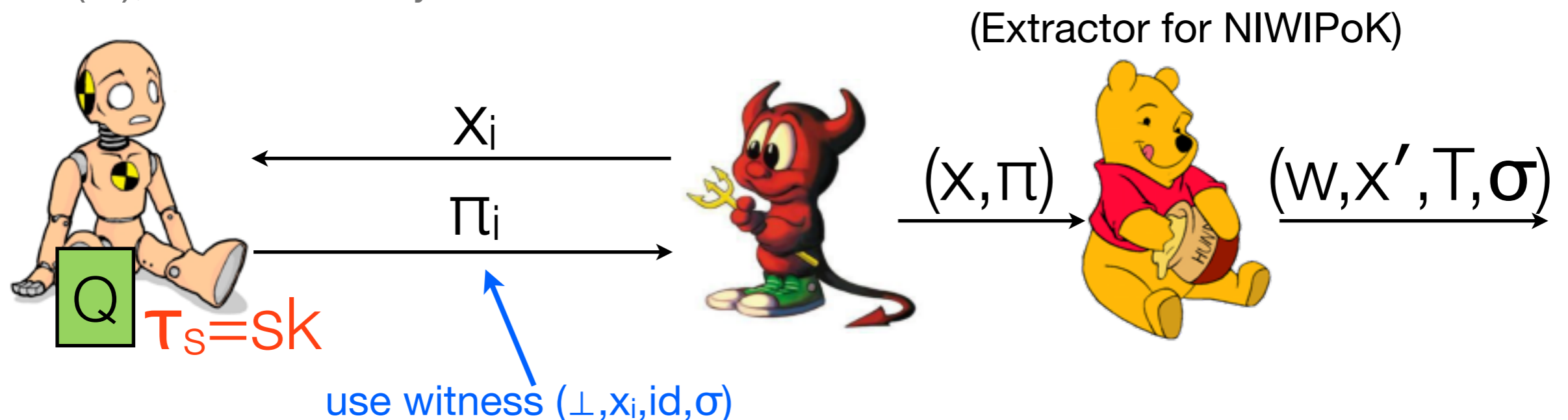
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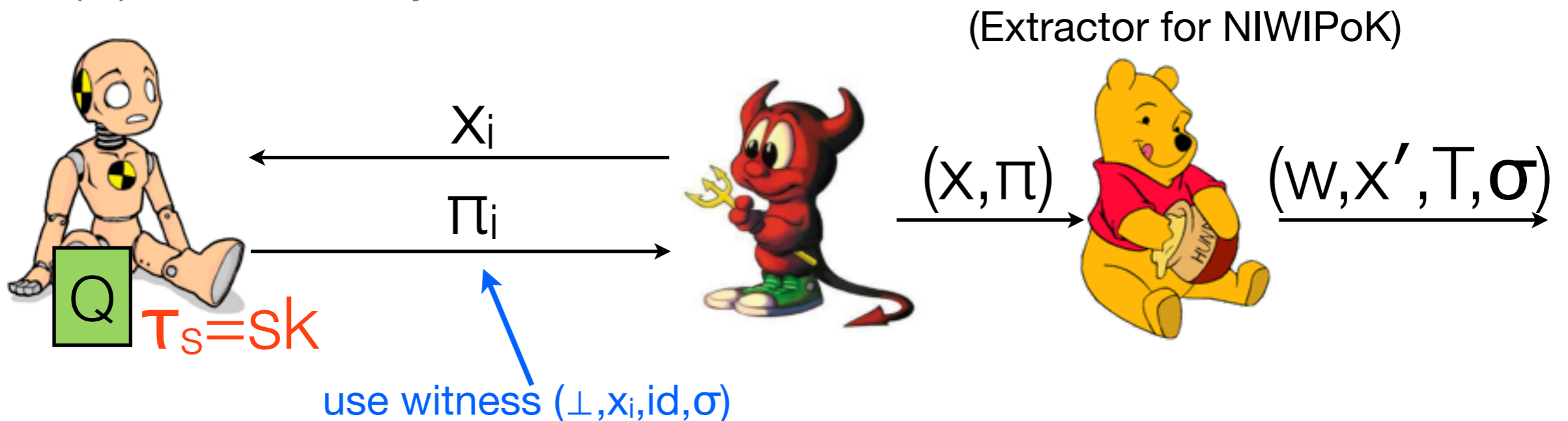


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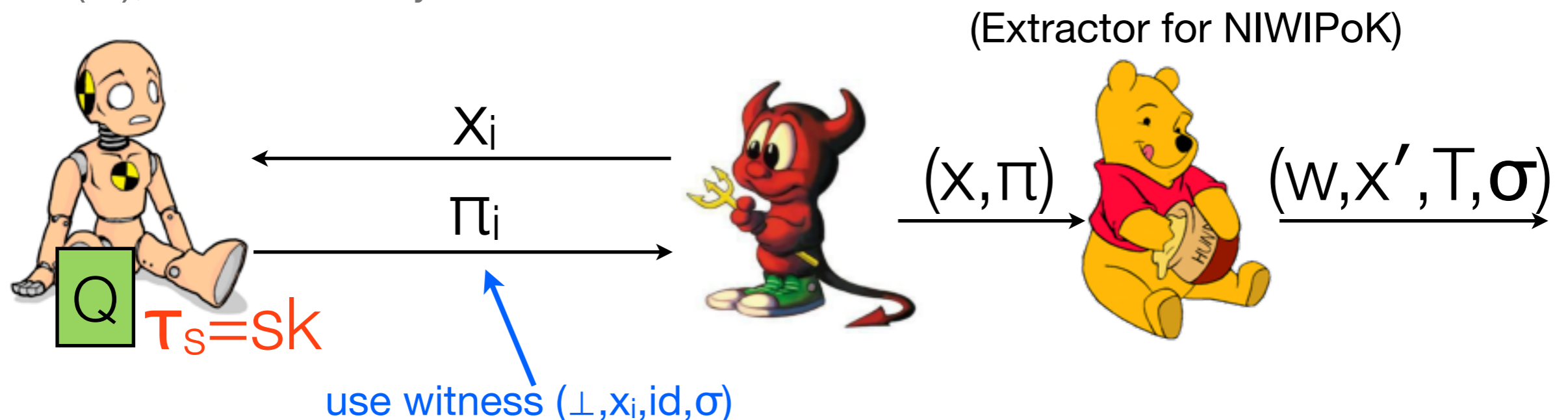


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In the paper, we examine the many ways in which [GS proofs are malleable](#)

# Outline

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Definitions

cm-NIZK construction

## Applications

Boosting encryption security  
Compactly verifiable shuffles

Conclusions

# CM-CCA security

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Expand our notion of controlled malleability from proofs to encryption to get **CM-CCA security** (inspired by HCCA [PR08] and related to targeted malleability [BSW12])

# CM-CCA security

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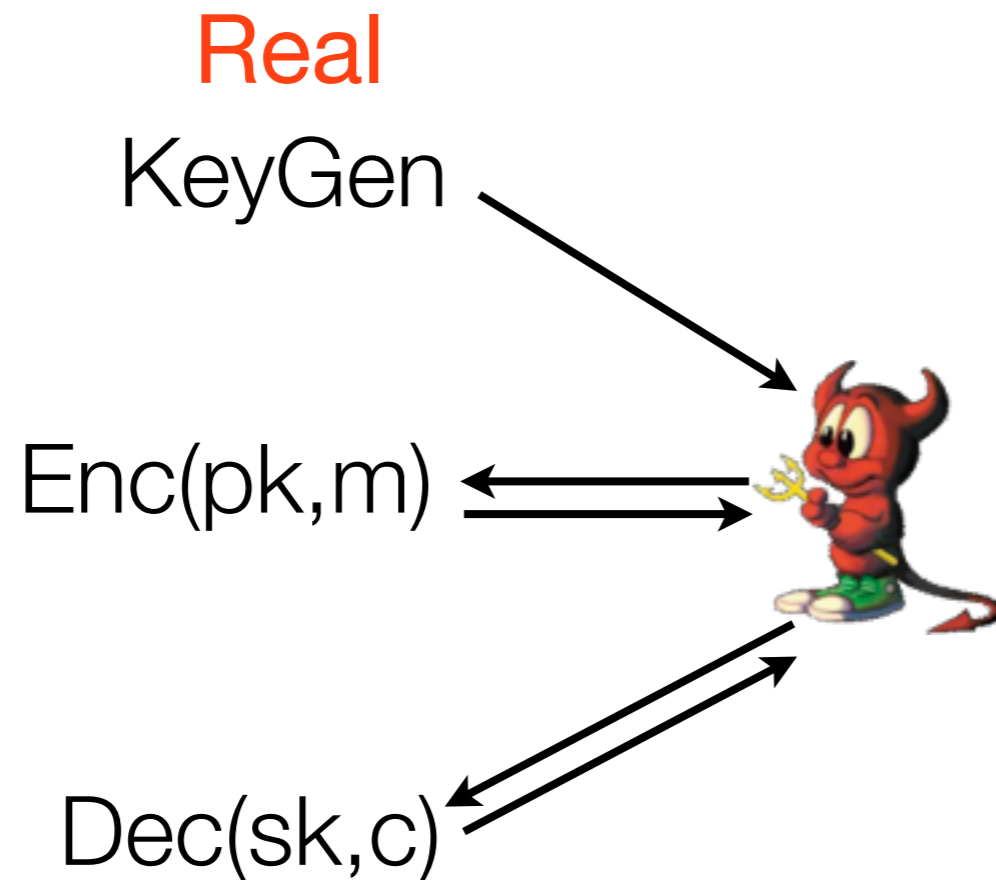
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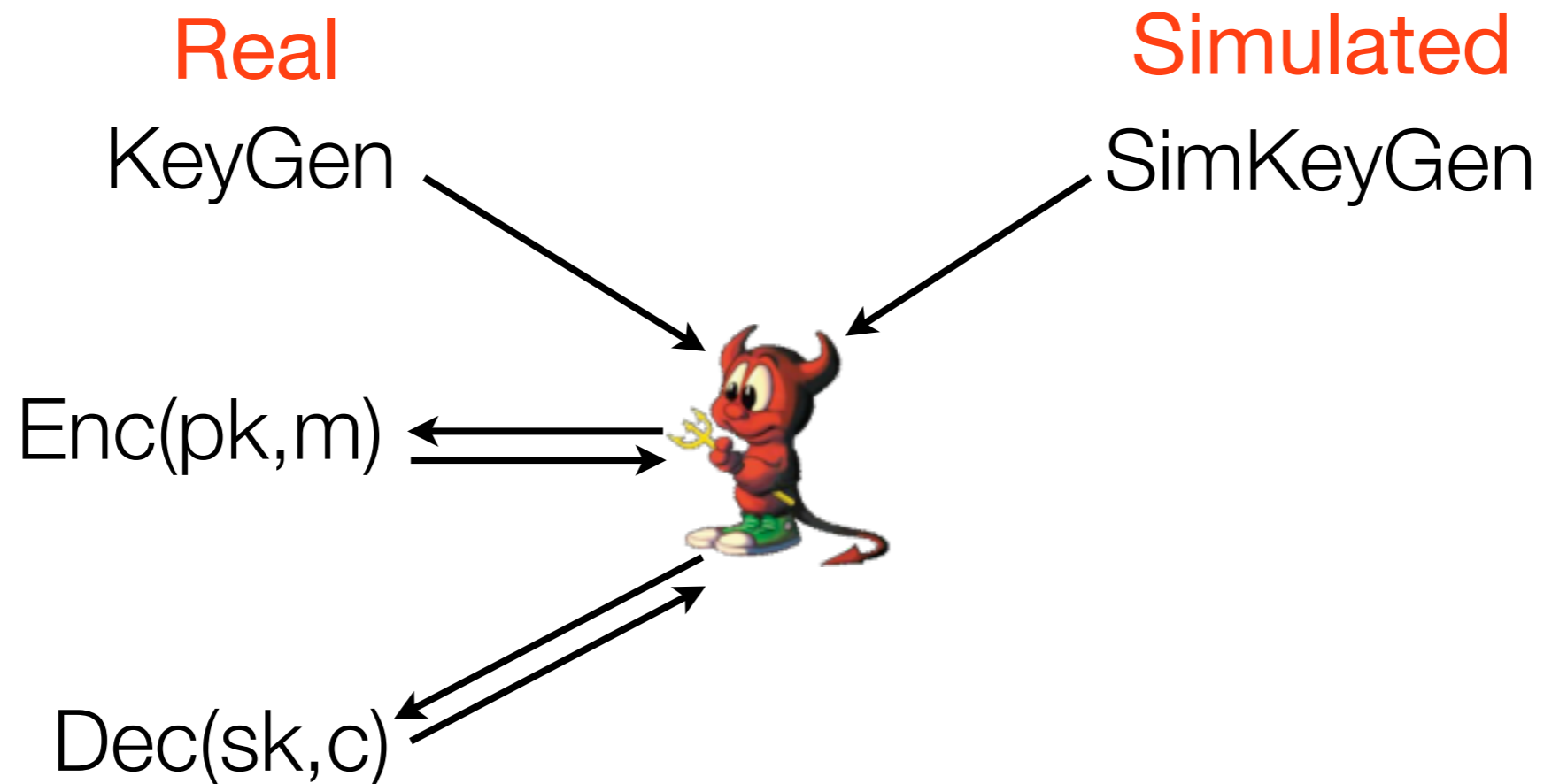
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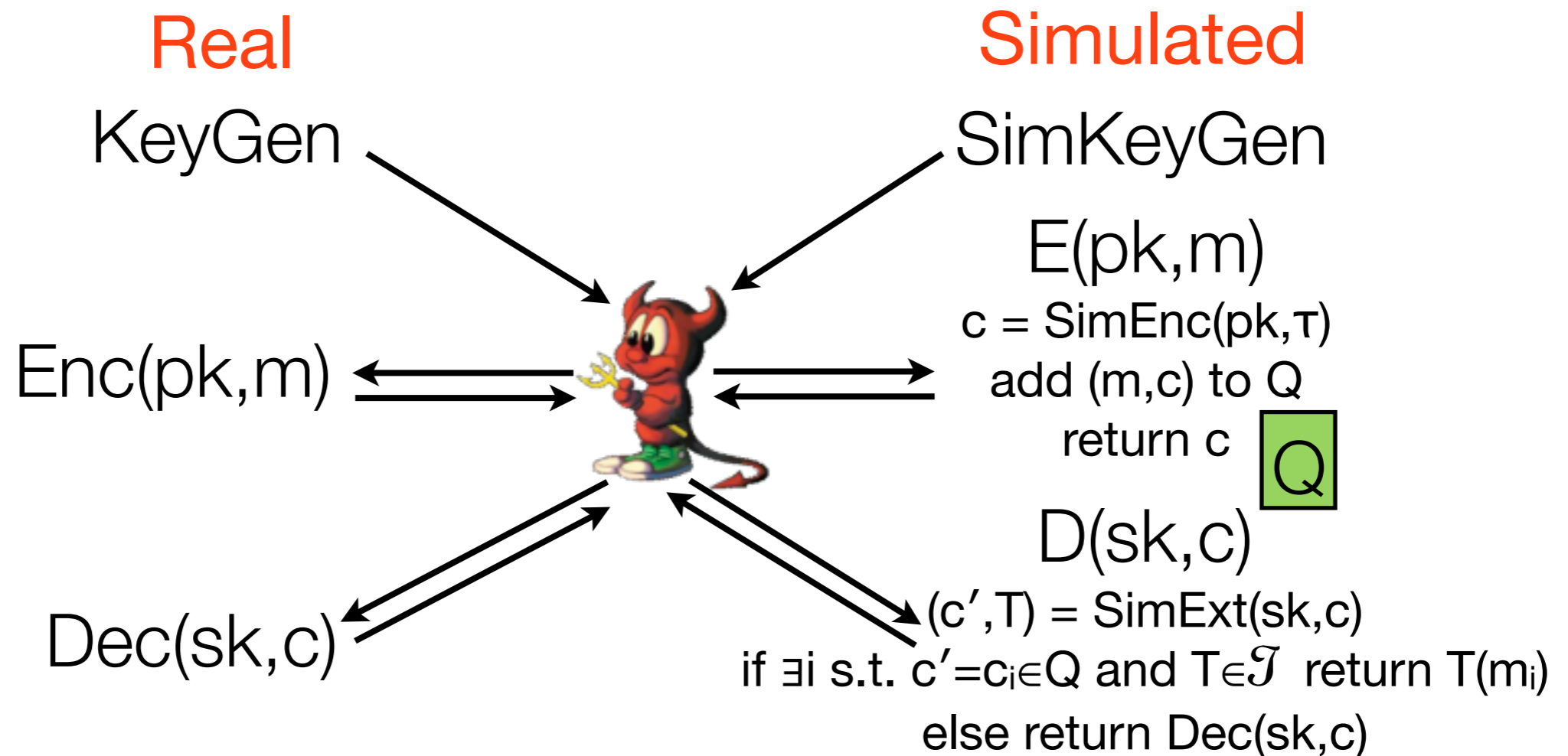
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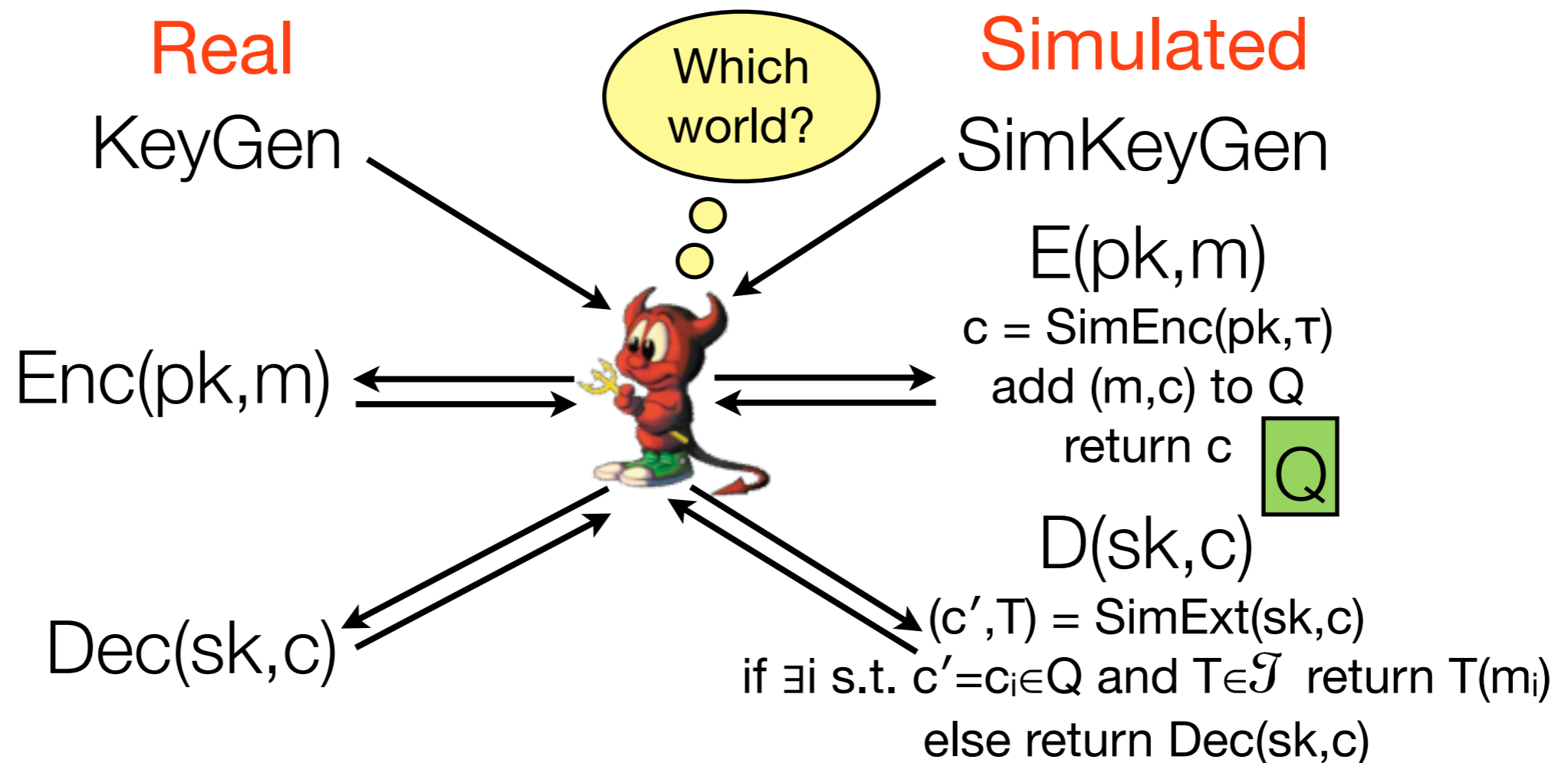
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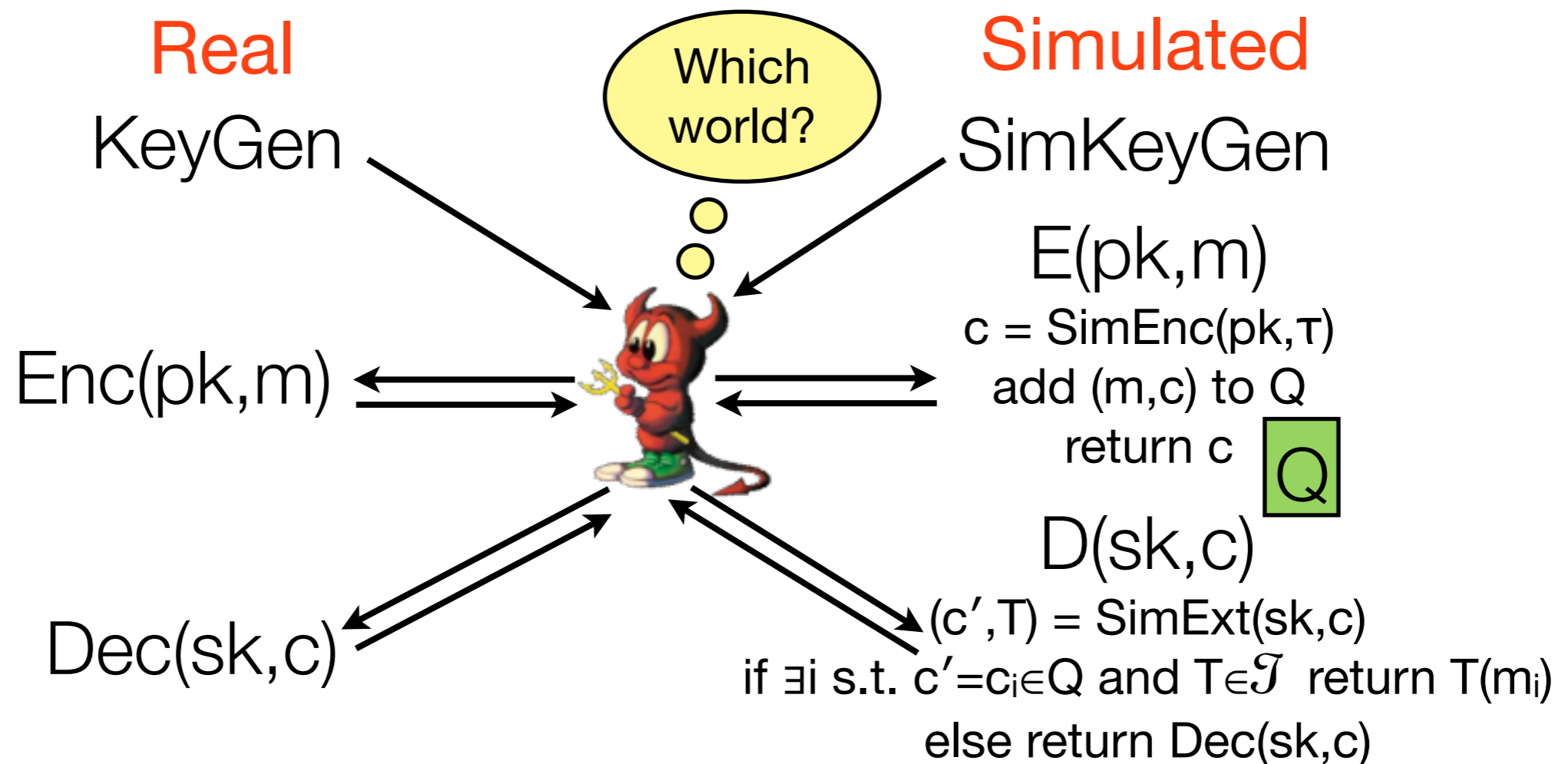
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Give a generic construction for achieving CM-CCA-secure encryption: just define **Enc(pk,m) = (c,π)**, where **c** is IND-CPA-secure and **π** is a cm-NIZK

# A shuffle

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C<sub>1</sub>  
C<sub>2</sub>  
C<sub>3</sub>  
C<sub>4</sub>  
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Users encrypt their individual values to yield a public set of ciphertexts  $\{c_i\}$

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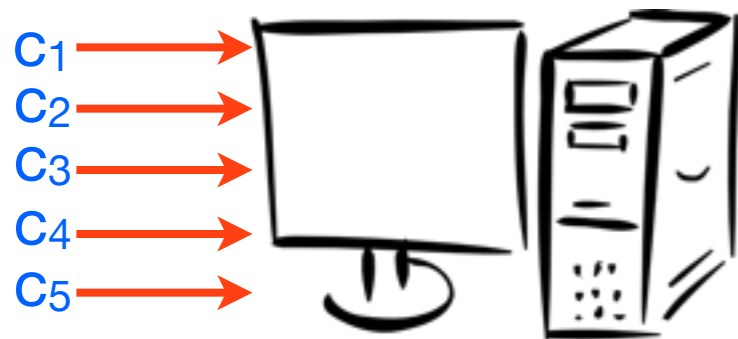


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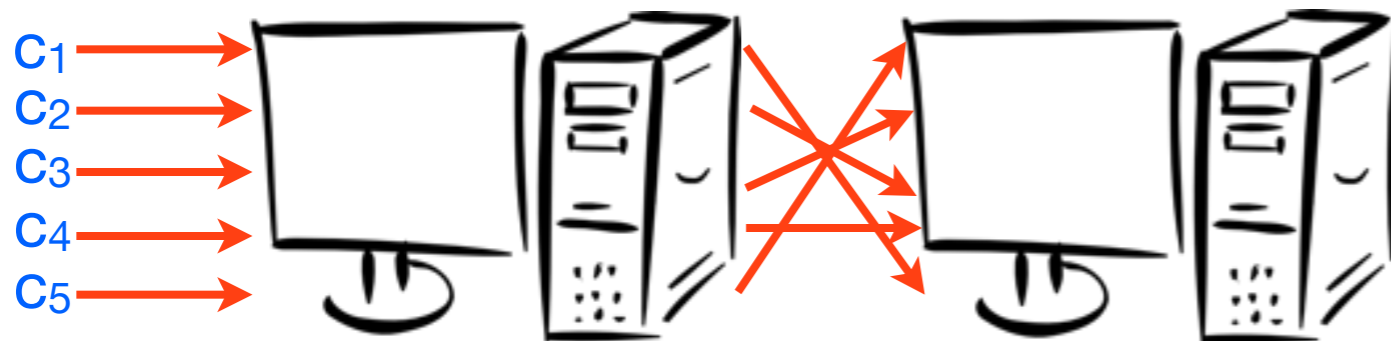


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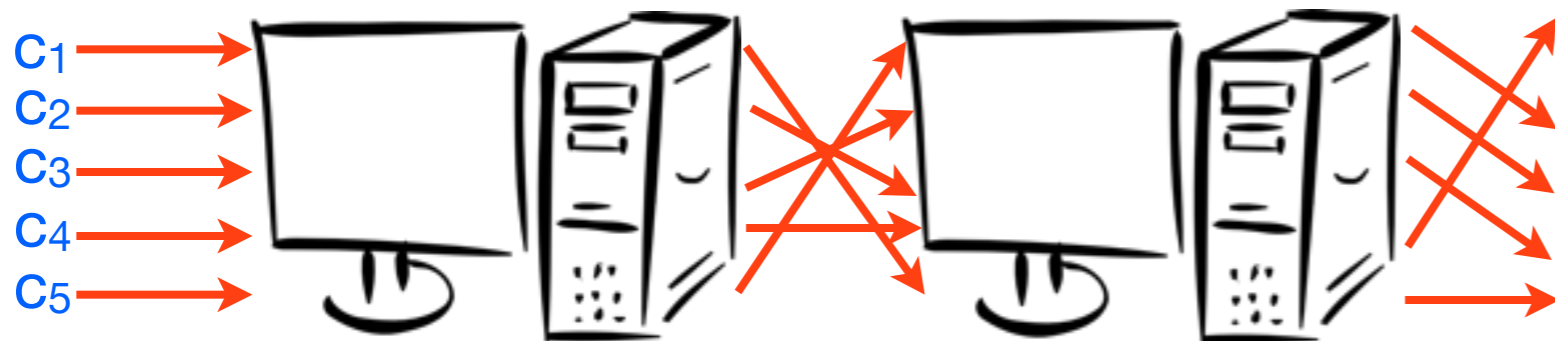


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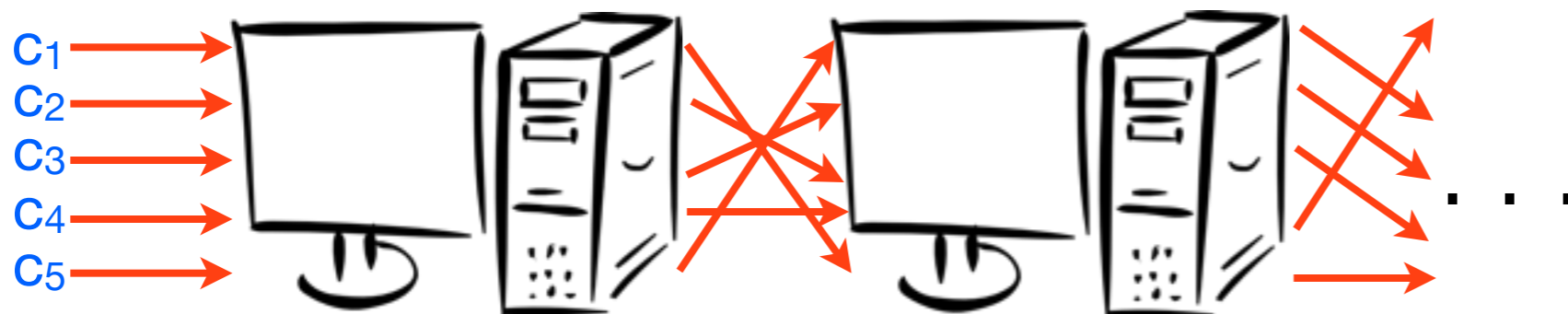


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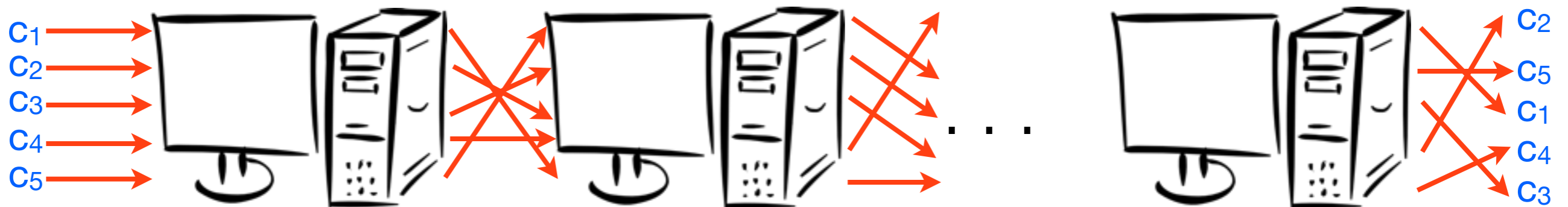
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Because values are shuffled, decryption won't reveal whose vote is whose

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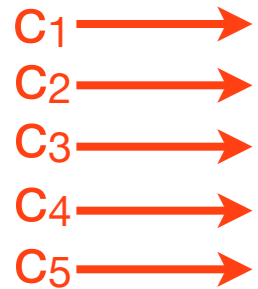
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**Problem:** How do we know these mix servers are behaving honestly?

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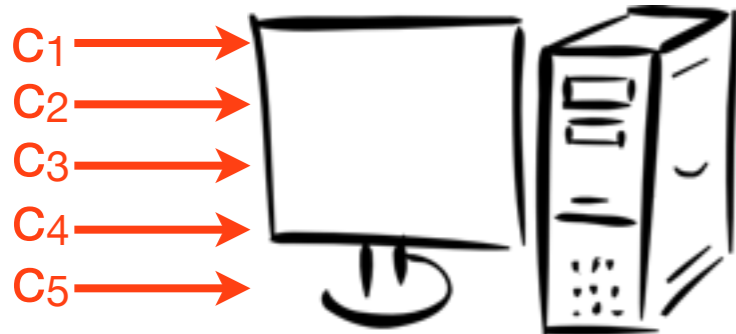
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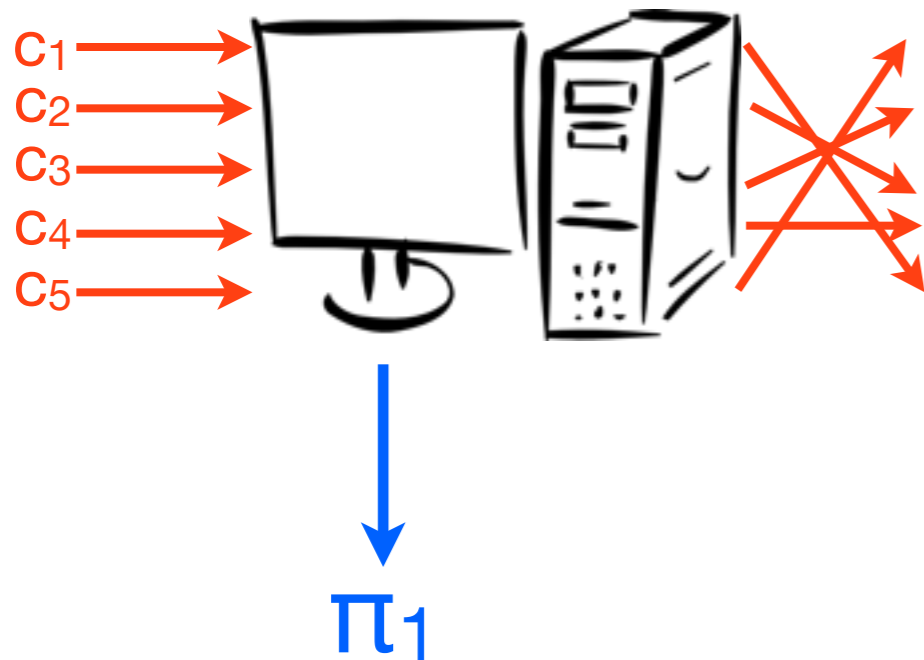




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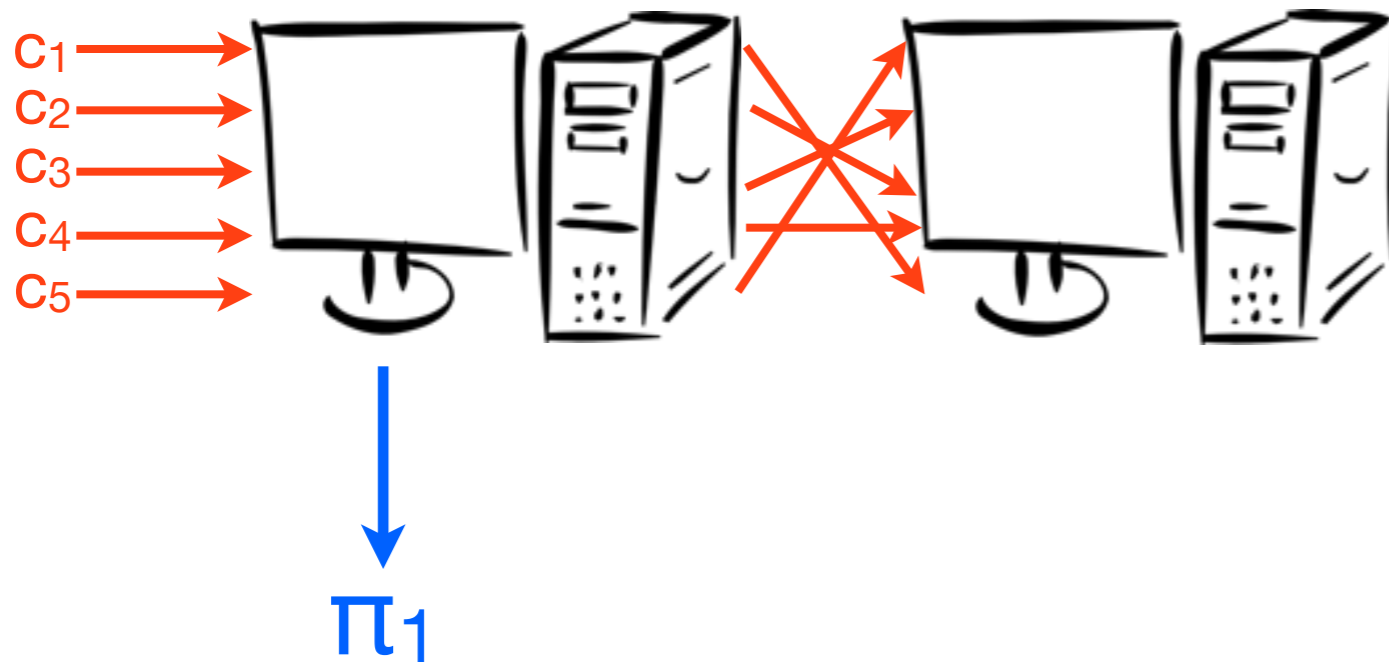
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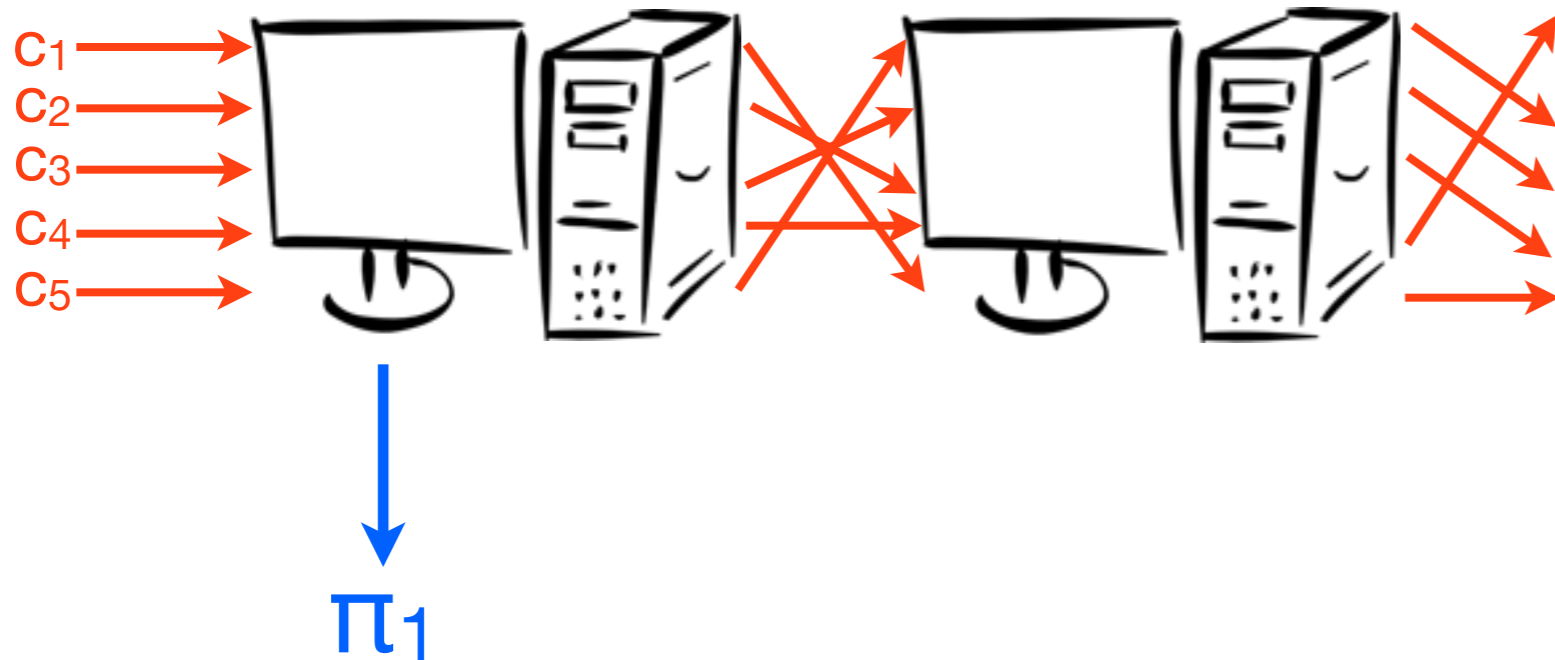
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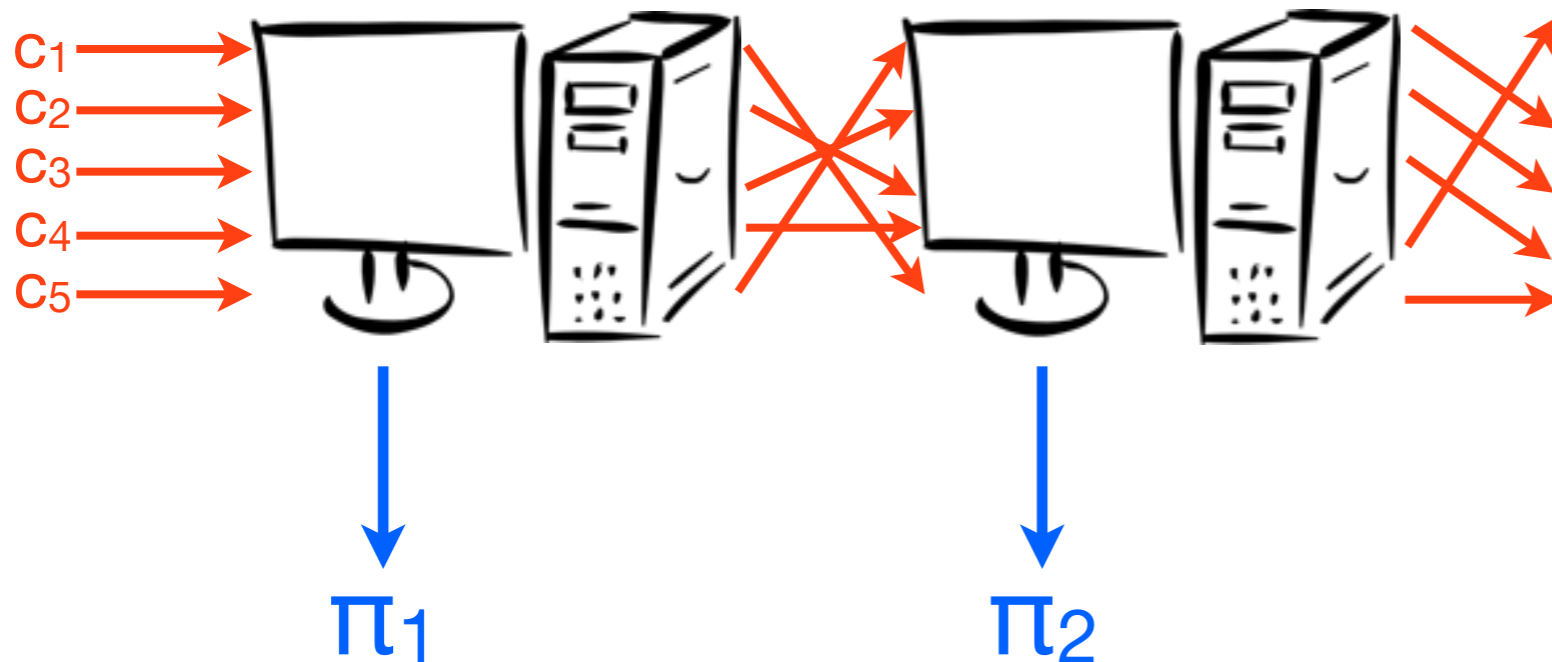
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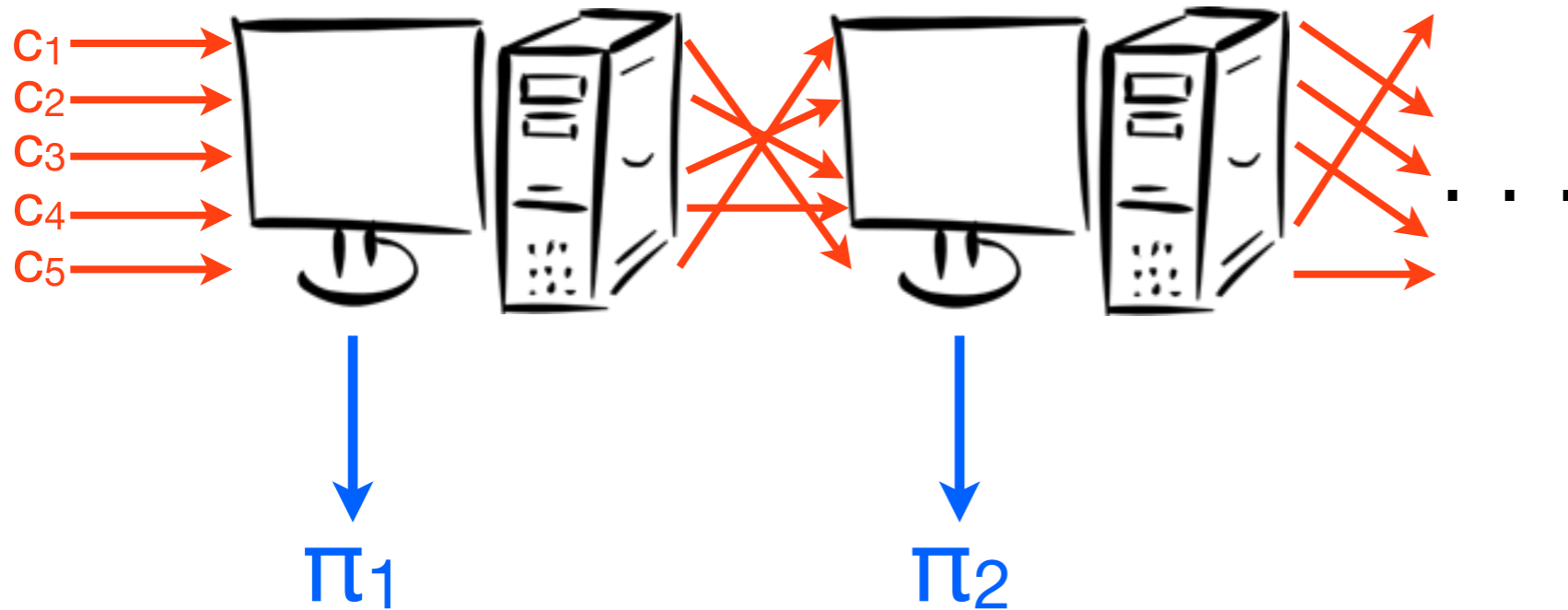
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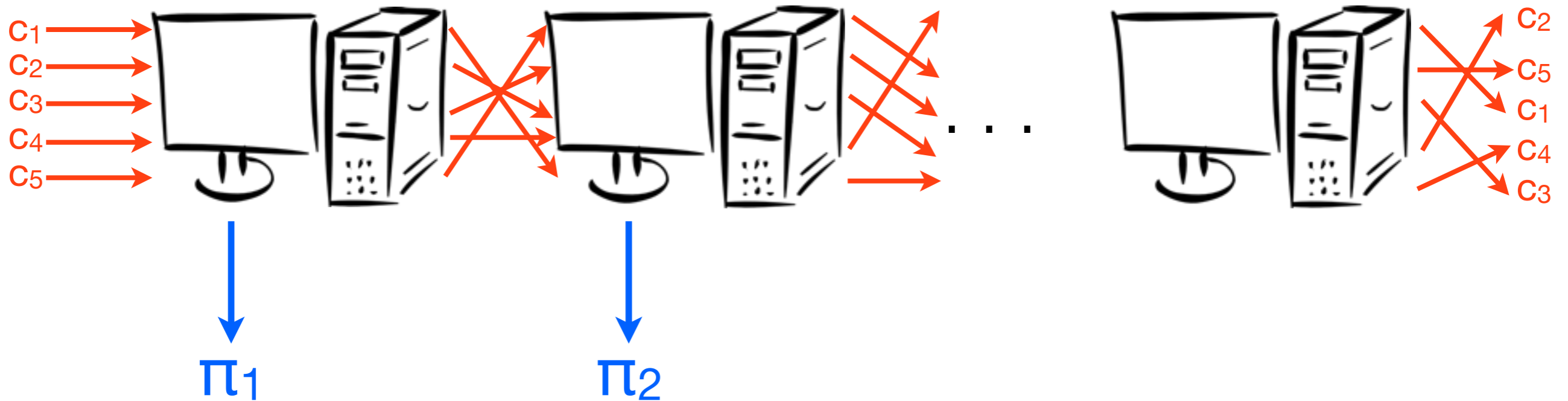
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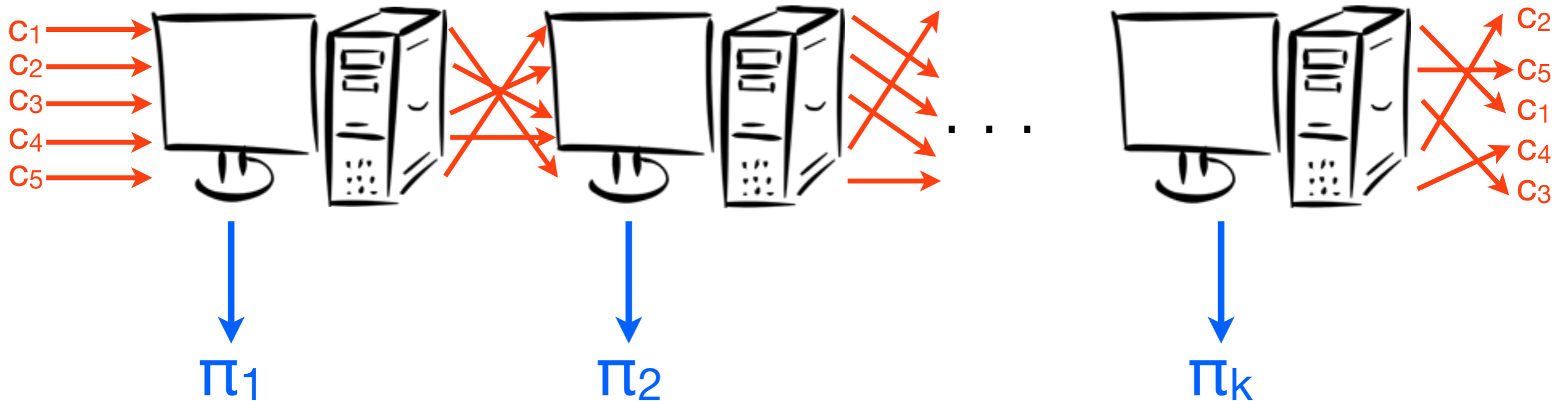
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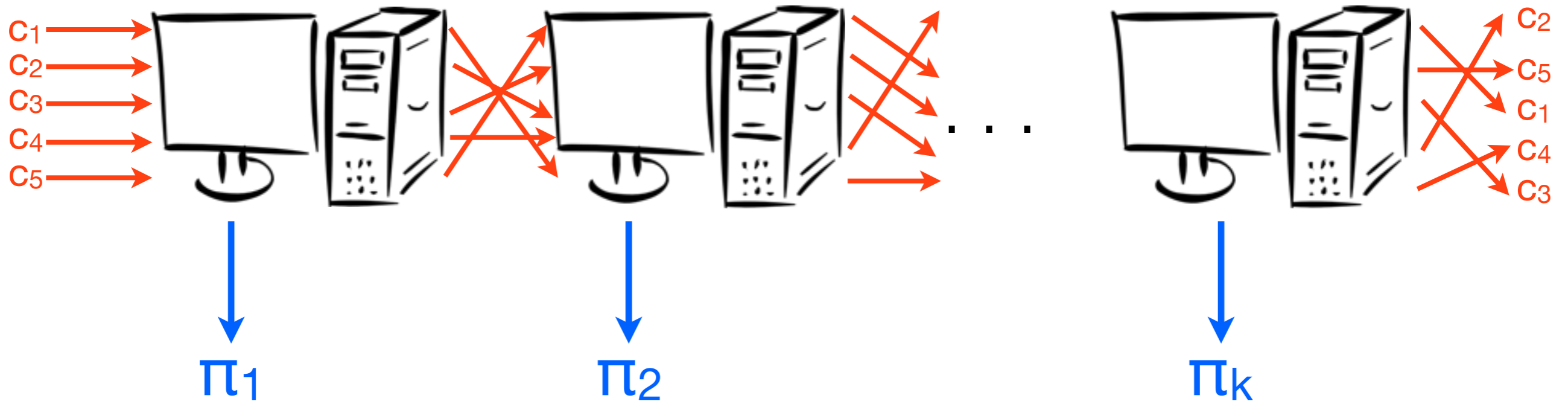
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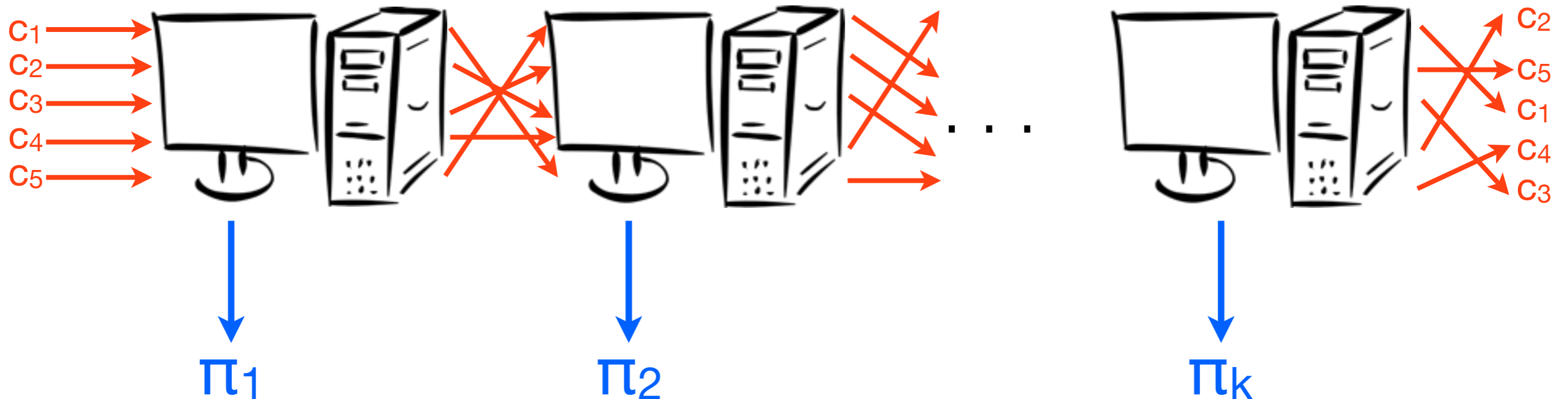


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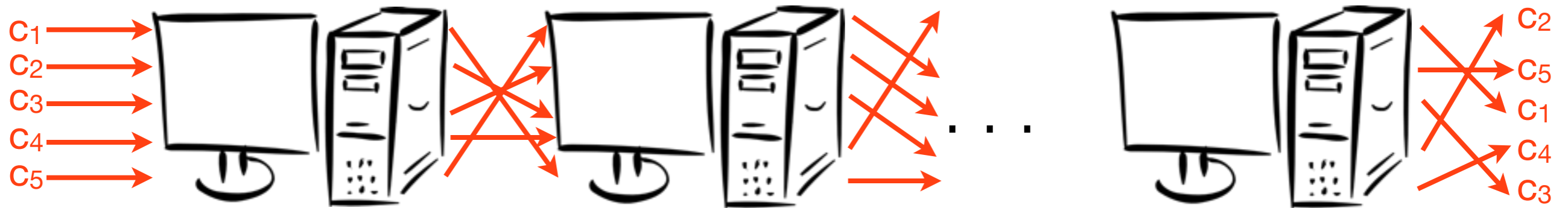


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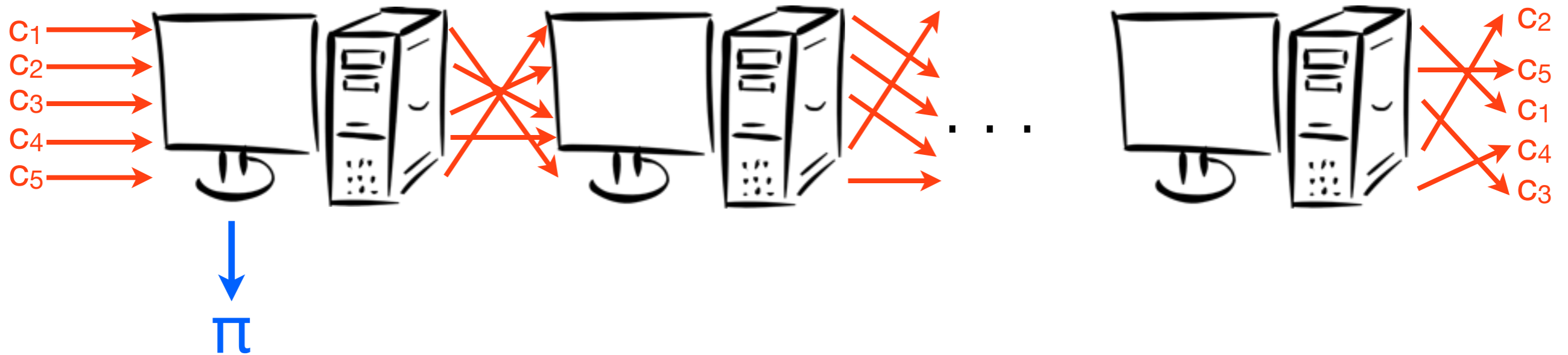
**New problem:** The size of this proof grows with the number of mix servers

# Using malleability to shrink the overall proof size

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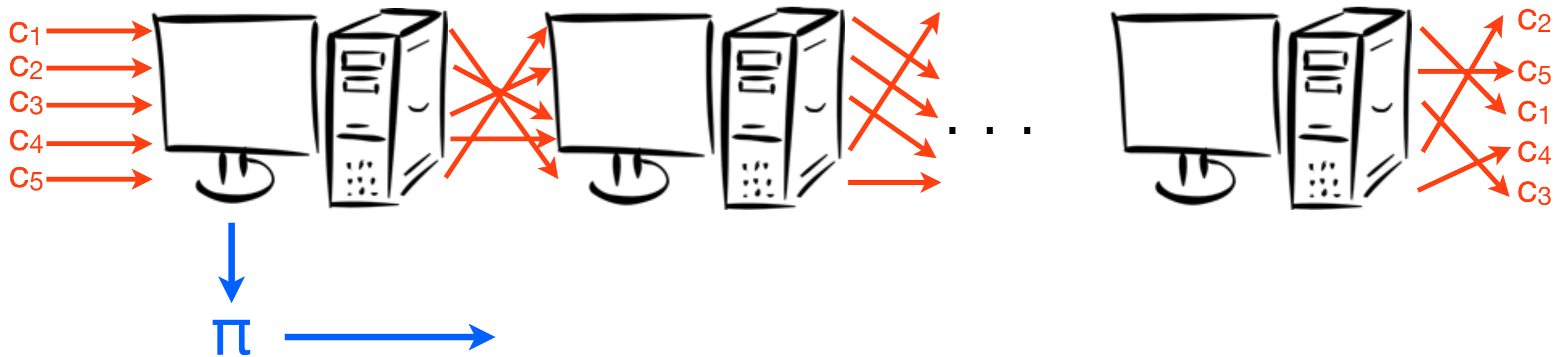


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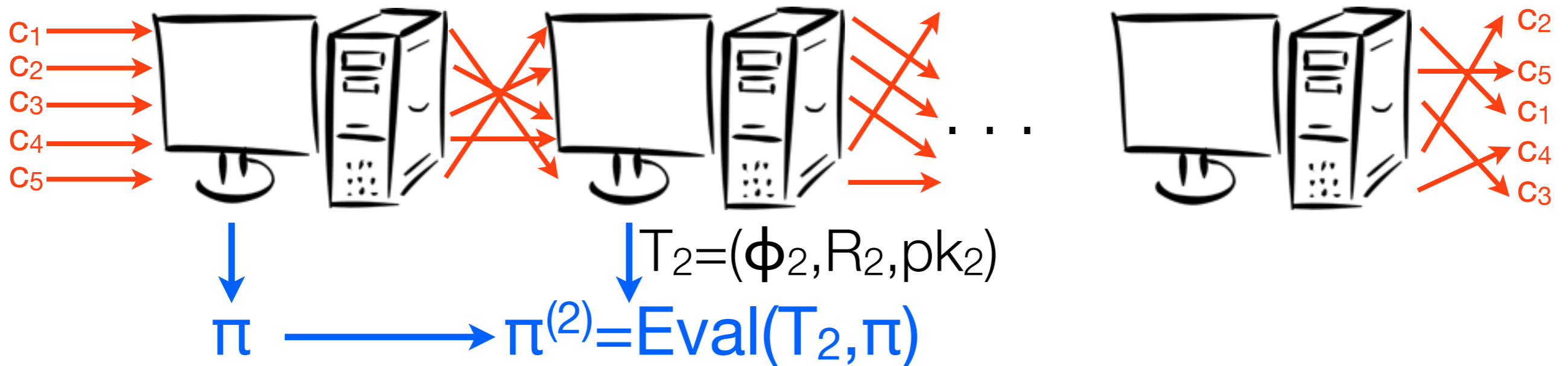
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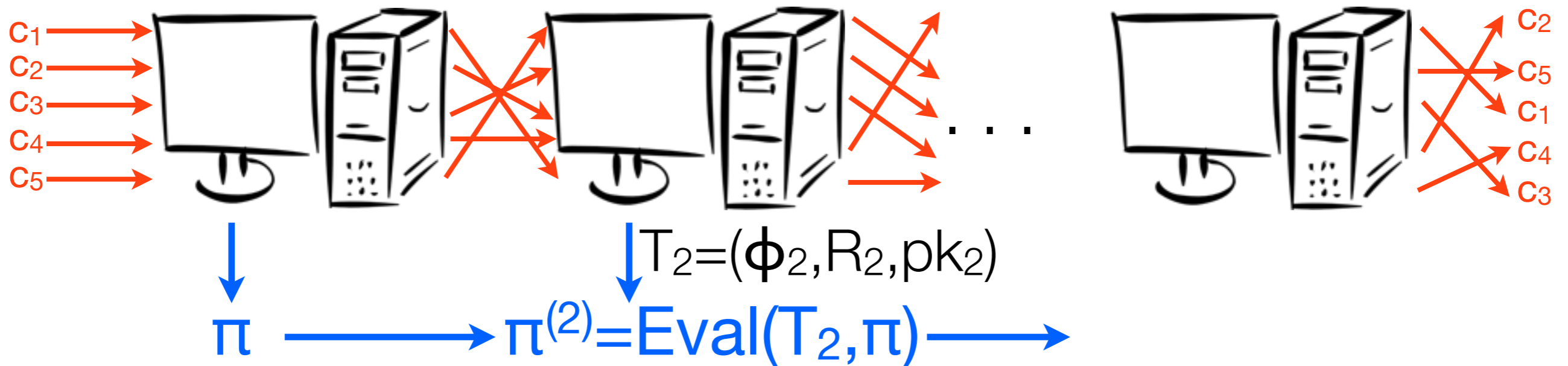
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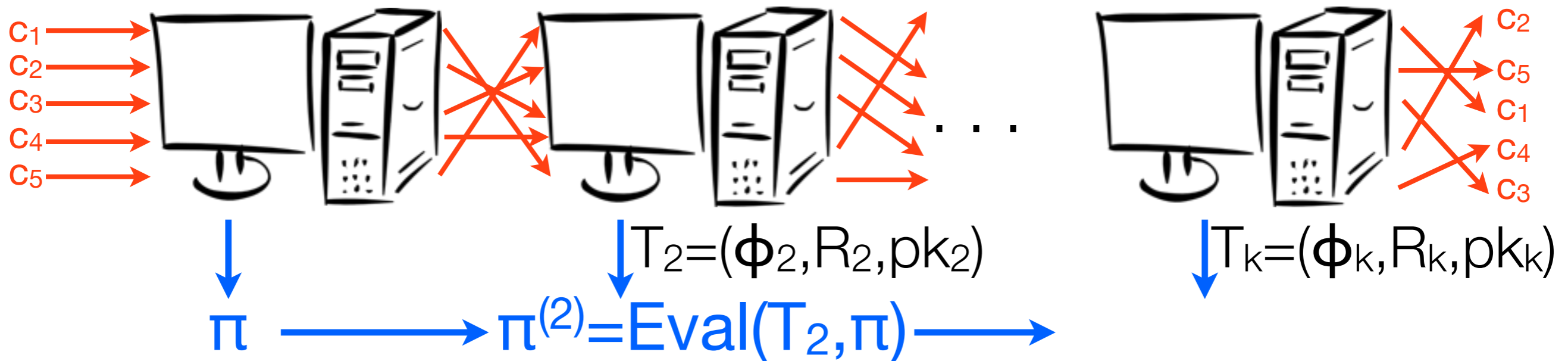
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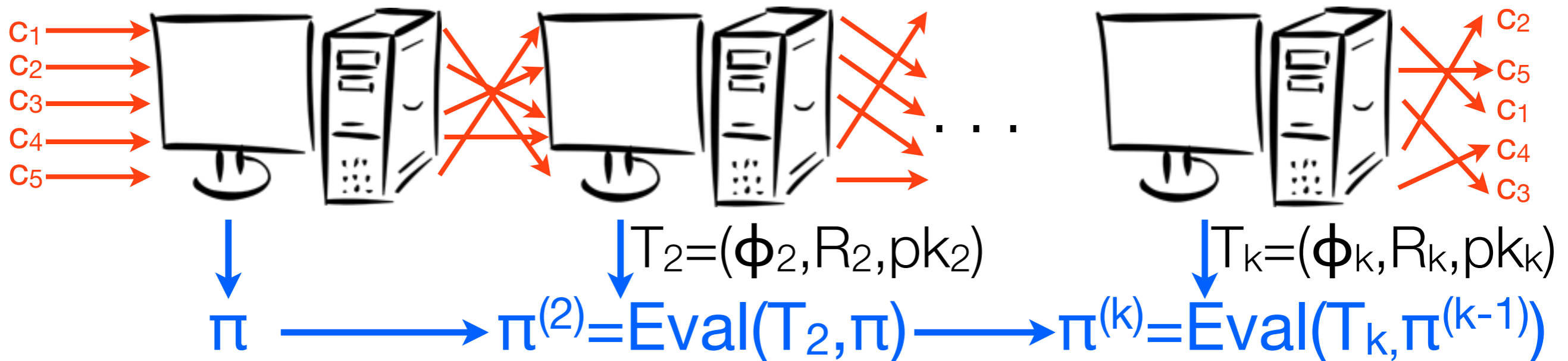
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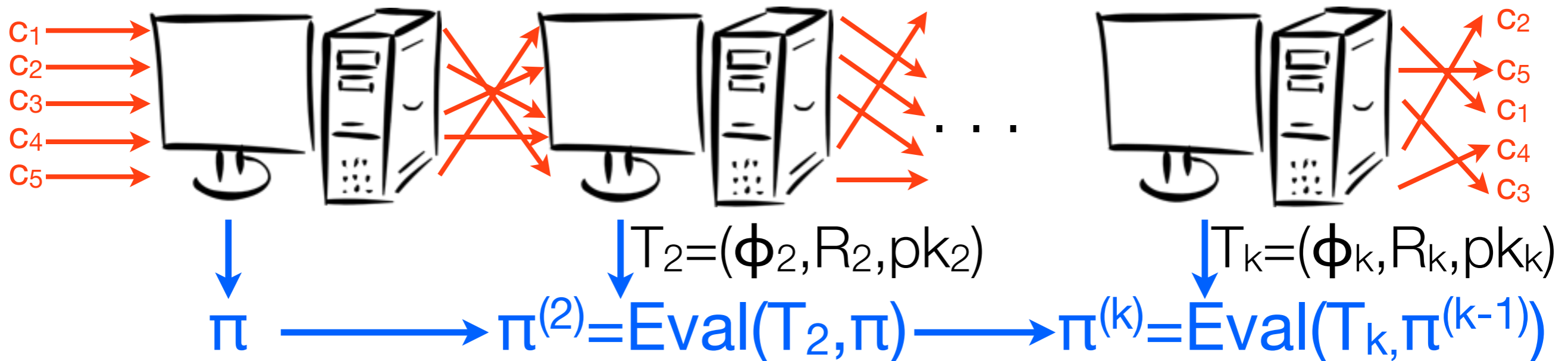


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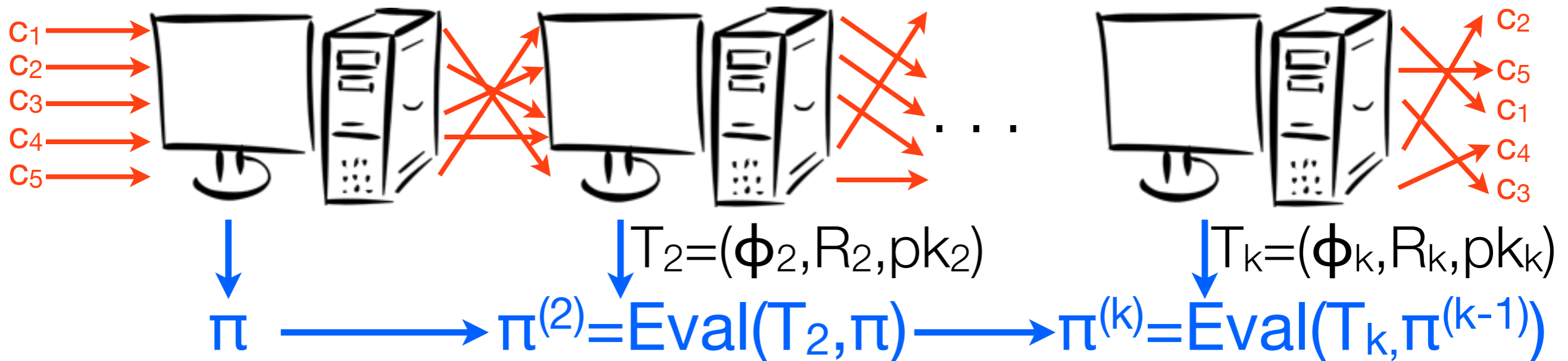


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- This bound isn't just theoretical: in this paper we get  $O(n^2+k)$  but in a recent result we use new methods to achieve  $O(n+k)$

# Outline

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Definitions

cm-NIZK construction

Applications

**Conclusions**

# Conclusions and open problems

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**Thanks!**  
**Any questions?**