Trapdoors for Lattices: Simpler, Tighter, Faster, Smaller

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Lattice-Based Cryptography

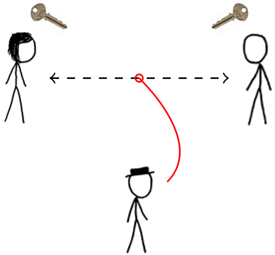
$y = g^x \bmod p$

$m^e \bmod N$

$N = p \cdot q$

$e(g^a, g^b)$

$\Longrightarrow$



(Images courtesy xkcd.org)

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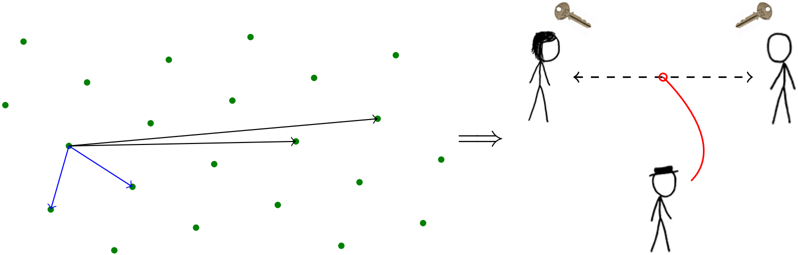

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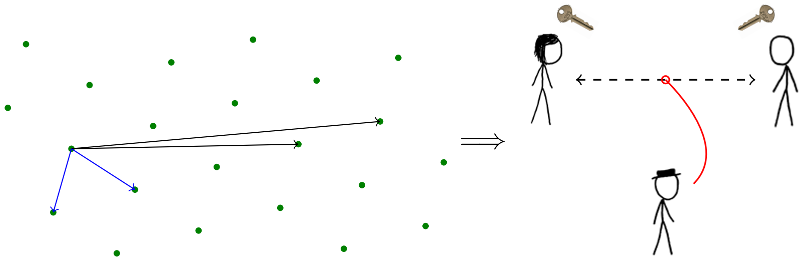

Why?

- Simple & efficient: linear, highly parallel operations
- Resist quantum attacks (so far)
- Secure under worst-case hardness assumptions [Ajtai'96,. . .]
- Solve 'holy grail' problems like FHE [Gentry'09,. . .]

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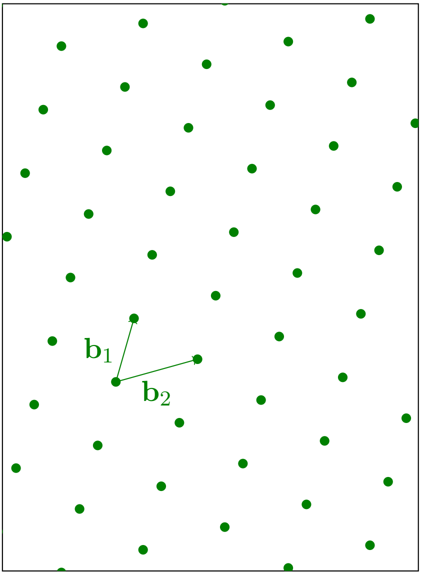
Point Lattices



A lattice is the set of all integer linear combinations of (linearly independent) basis vectors $\mathbf{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\} \subset \mathbb{R}^d$:

$$\Lambda = \sum_{i=1}^{n} \mathbf{b}_i \cdot \mathbb{Z}$$

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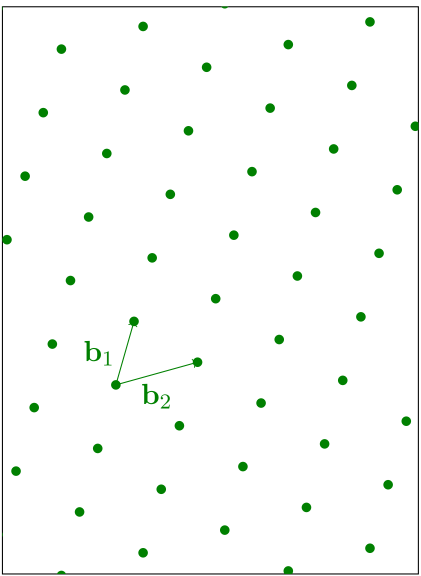

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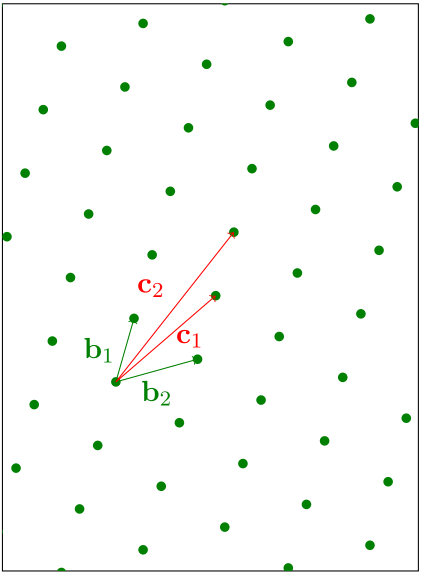

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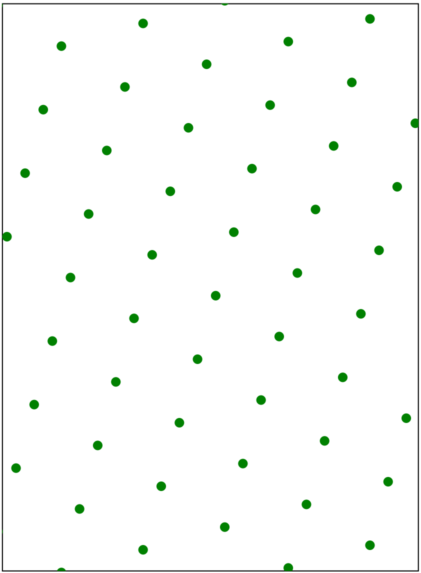

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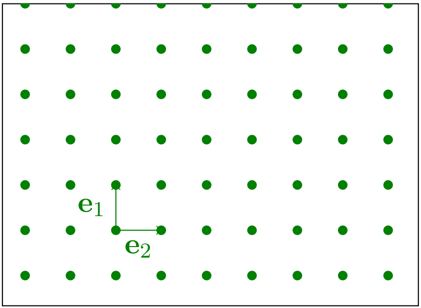
Definition (Lattice)

Discrete additive subgroup of $\mathbb{R}^d$

E.g. $\Lambda = \{\mathbf{x} \in \mathbb{Z}^d \colon \mathbf{A}\mathbf{x} = \mathbf{0}\}$

Point Lattices: Examples



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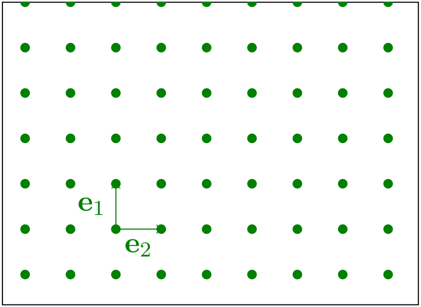

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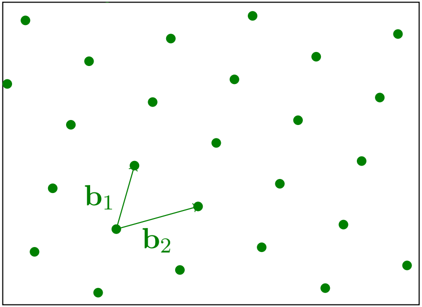

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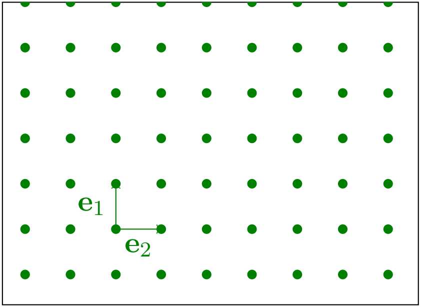

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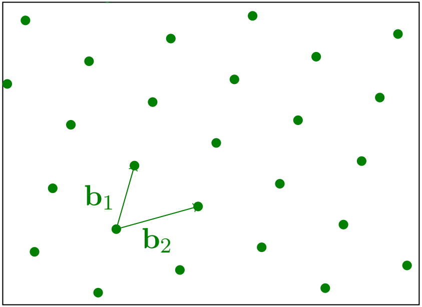

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Remark

All lattices have the same group structure, but different geometry

Lattice-Based One-Way Functions

- Public key $\begin{bmatrix} \cdots & \mathbf{A} & \cdots \end{bmatrix} \in \mathbb{Z}_q^{n \times m}$ for $q = \mathsf{poly}(n)$, $m = \Omega(n \log q)$.

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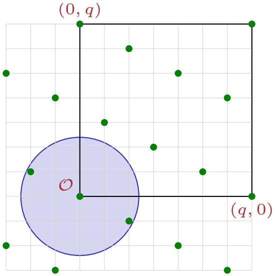

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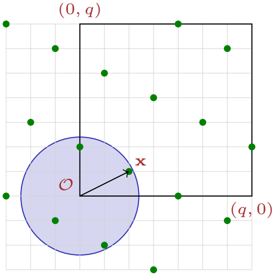

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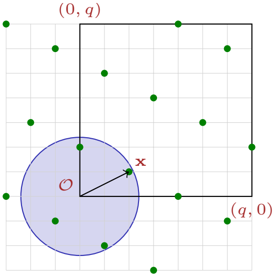

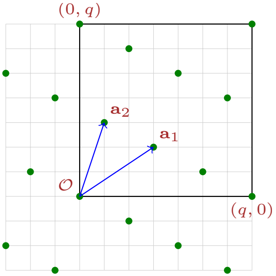

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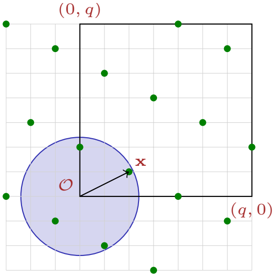

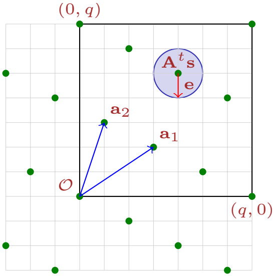

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  - ⋆ $f_{\mathbf{A}}$ and $g_{\mathbf{A}}$ are essentially equivalent functions
  - ⋆ See e.g. "Duality in lattice cryptography" [M'10]
  - ⋆ Main difference: $\mathbf{e}$ is even shorter than $\mathbf{x}$
  - ⋆ Notational convention:

Function	$\mathbf{x}/\mathbf{e}$	Injective	Surjective
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- $f_{\mathbf{A}}$, $g_{\mathbf{A}}$ in forward direction yield CRHFs, CPA-secure encryption

...and not much else.

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find the unique preimage $\mathbf{s}$

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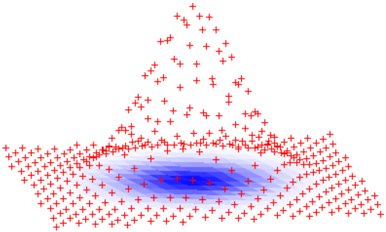

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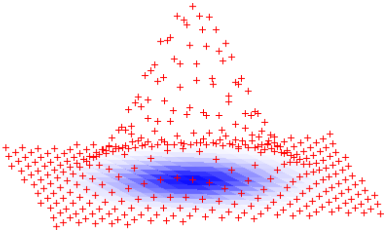
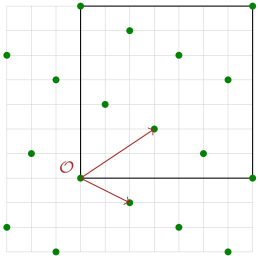
- <u>How?</u> Use a "strong trapdoor" for $\mathbf{A}$: a short basis of $\Lambda^{\perp}(\mathbf{A})$

[Babai'86,GGH'97,Klein'01,GPV'08,P'10]





Applications of Strong Trapdoors

Applications of f^{-1}, g^{-1}

- "Hash and Sign" signatures in Random oracle (RO) model [GPV'08]
- Standard model (no RO) signatures [CHKP'10,R'10,B'10]
- SM CCA-secure encryption [PW'08,P'09]
- SM (Hierarchical) IBE [GPV'08,CHKP'10,ABB'10a,ABB'10b]
- Many more: OT, NISZK, homom enc/sigs, deniable enc, func enc, . . .
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Some Drawbacks. . .

✗ Generating $\mathbf{A}$ w/ short basis is complicated and slow [Ajtai'99,AP'09]

✗ Known inversion algorithms trade quality for efficiency

	tight, iterative, fp	looser, parallel, offline
$g_{\mathbf{A}}^{-1}$	[Babai'86]	[Babai'86]
$f_{\mathbf{A}}^{-1}$	[Klein'01,GPV'08]	[P'10]

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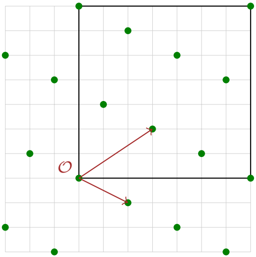

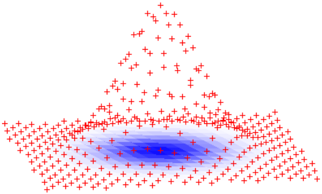

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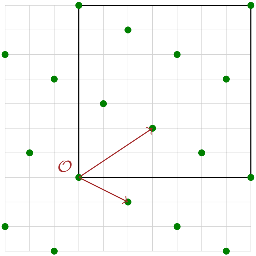

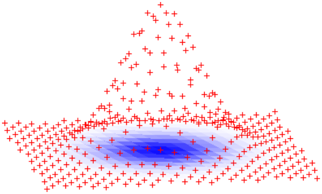

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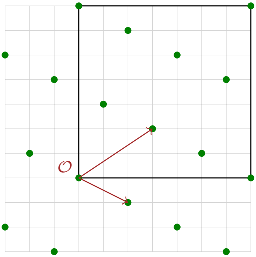

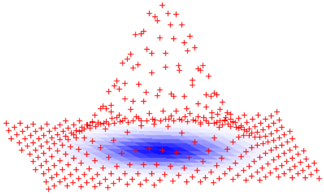

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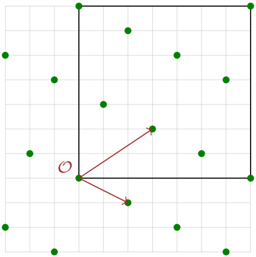

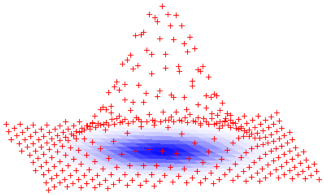

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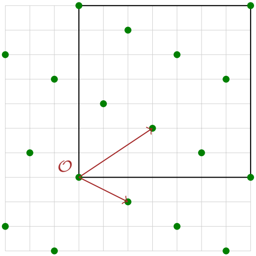

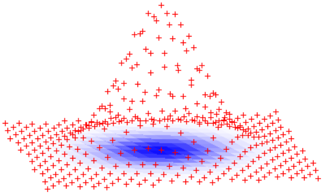

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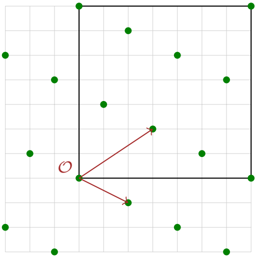
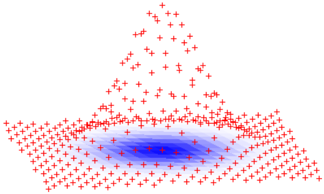
Taming the Parameters

$$n\left\{\underbrace{\begin{bmatrix} \cdots & \mathbf{A} & \cdots \end{bmatrix}}_{m}\right.$$

$$f_{\mathbf{A}}(\mathbf{x}) = \mathbf{A}\mathbf{x}$$





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✔ Better dimension m & quality σ

$\implies$ "win-win-win" in security-keysize-runtime

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- ✔ More efficient applications (beyond "black-box" improvements)

Concrete Parameter Improvements

	Before [AP'09]	**Now** (fast f^{-1})	**Improvement**
Dim m	slow f^{-1}: $> 5n \log q$ fast f^{-1}: $> n \log^2 q$	$2n \log q$ ($\overset{s}{\approx}$) $n(1 + \log q)$ ($\overset{c}{\approx}$)	$2.5 - \log q$

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Example parameters for (ring-based) GPV signatures:

	n	q	δ to break	pk size (bits)
Before (fast f^{-1})	436	2^{32}	1.007	$\approx 17 \times 10^{6}$
Now	284	2^{24}	1.007	$\approx 36 \times 10^{4}$

Bottom line: $\approx$ 45-fold improvement in key size.

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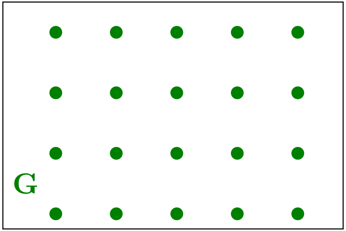

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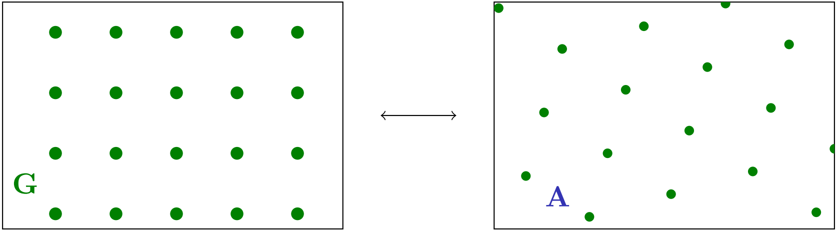

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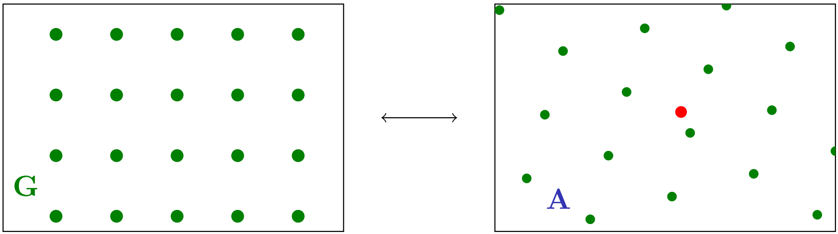

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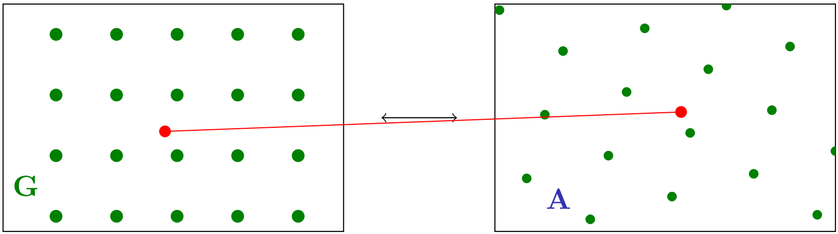

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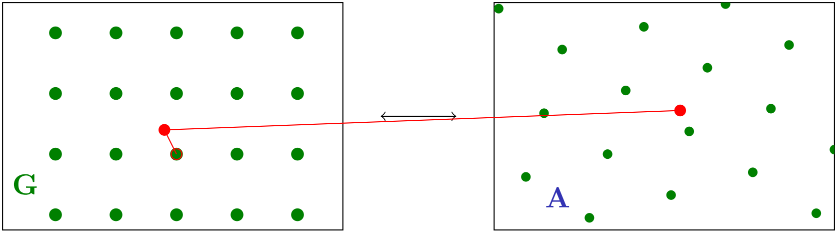

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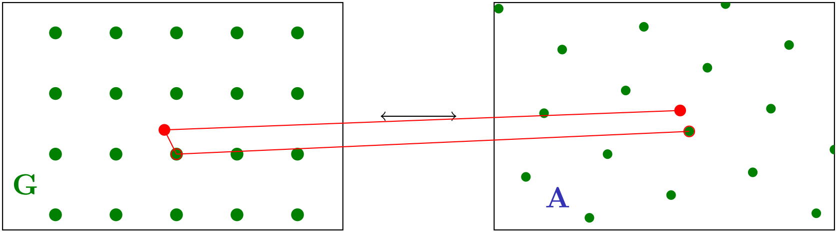

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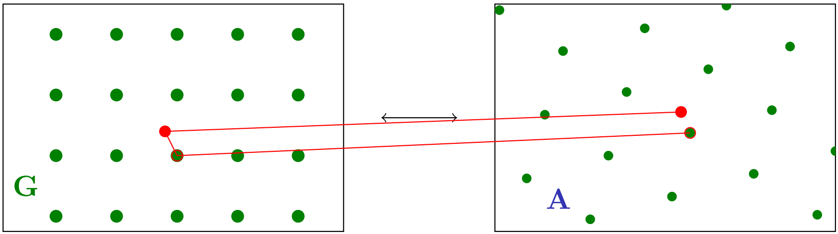

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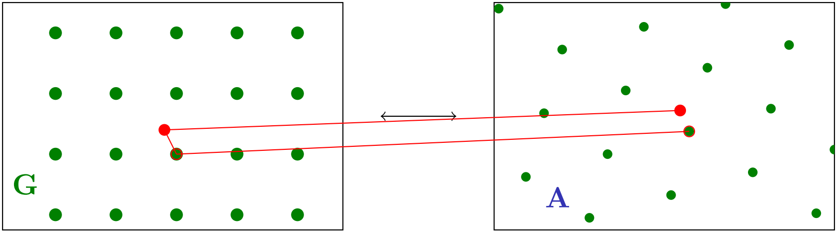

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Randomized Trasformation $\mathbf{G} \leftrightarrow \mathbf{A}$: Efficiency

1. Linear transformation is easily computable $(\bar{m} \cdot n \log q)$

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$$\mathbf{T} = \begin{bmatrix} \mathbf{I} & -\mathbf{R} \\ & \mathbf{I} \end{bmatrix} \in \mathbb{Z}_q^{(\bar{m}+n\log q)\times(\bar{m}+n\log q)}$$

2. Inverse transformation is just as simple

$$\mathbf{T}^{-1} = \begin{bmatrix} \mathbf{I} & +\mathbf{R} \\ & \mathbf{I} \end{bmatrix}$$

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5. A basis for $\Lambda^{\perp}(\mathbf{A})$ is easily computed using $\mathbf{T}$, but never needed: $\mathbf{R}$ serves as a new trapdoor

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Questions?