Efficient and Optimally Secure Key-Length Extension for Block Ciphers via Randomized Cascading

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Eurocrypt 2012

Outline

Block Ciphers and Key-Length Extension

Existing Approaches

Our Generic Attacks

Our Construction

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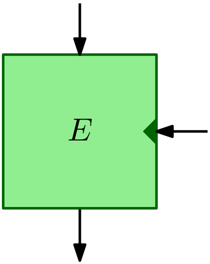
Block Ciphers



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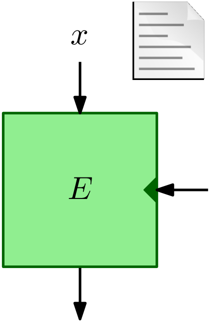

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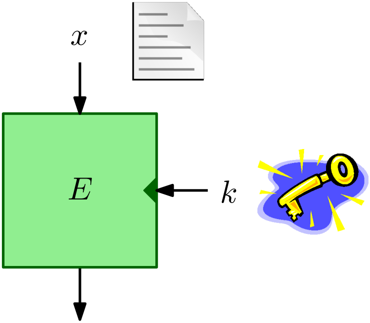

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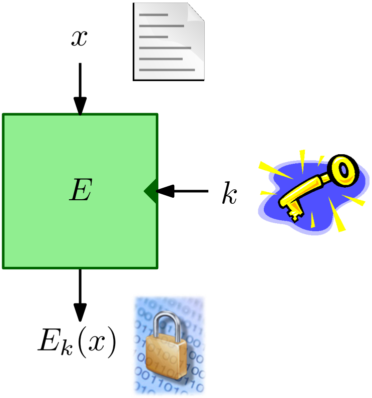
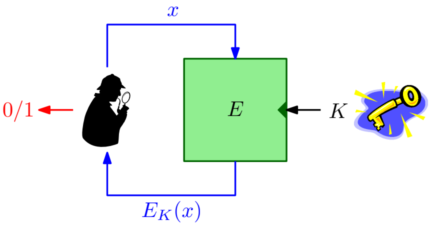


- e.g. DES, IDEA, AES
- $E: \{0,1\}^{\kappa} \times \{0,1\}^{n} \to \{0,1\}^{n}$

Block Cipher Security: Pseudo-Random Permutations



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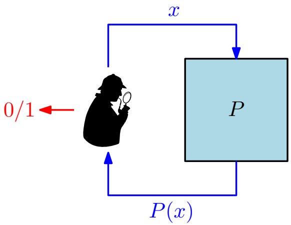

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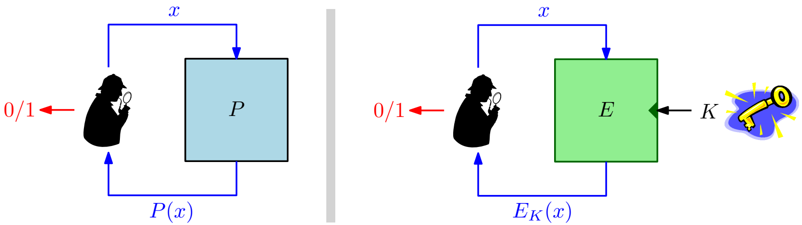

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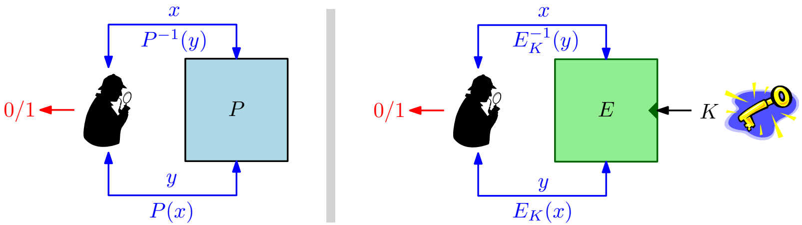

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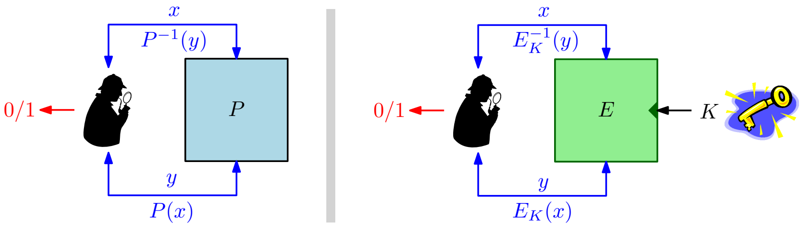

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$$\Delta^D(P, E_K) := |\Pr[D(P) = 1] - \Pr[D(E_K) = 1]|$$

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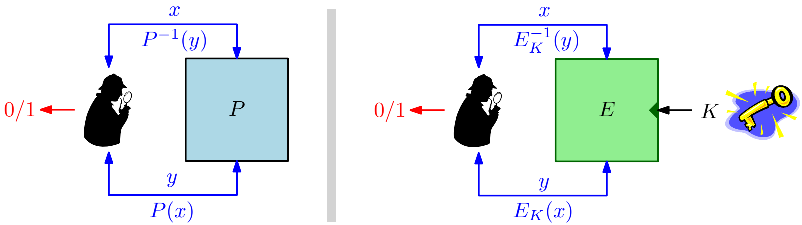

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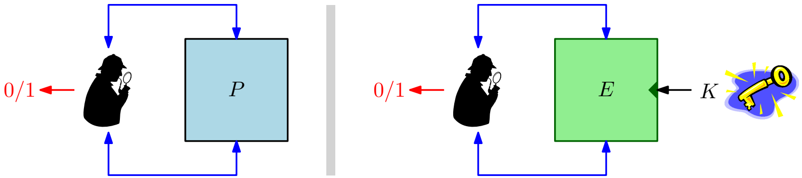

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Given $\mathcal{O} \in \{P, E_K\}$:

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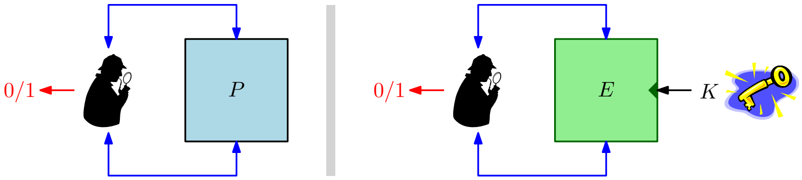

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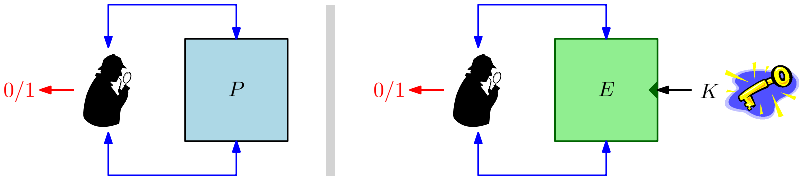

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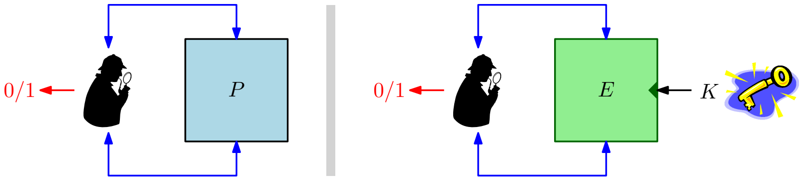

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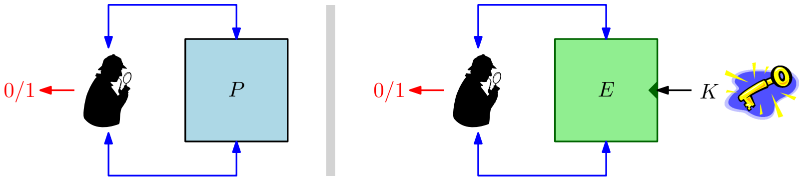

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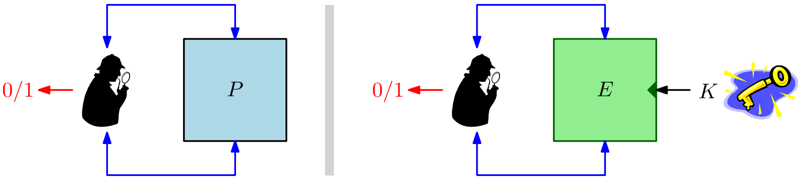

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  generic attack!

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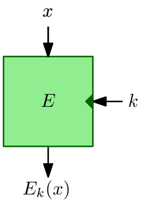

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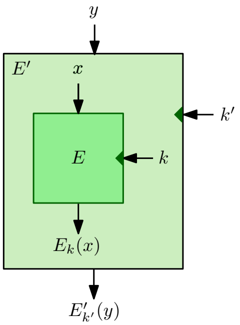

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$$E'[E]\colon \{0,1\}^{\kappa'} \times \{0,1\}^n \to \{0,1\}^n$$

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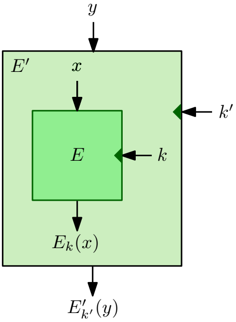

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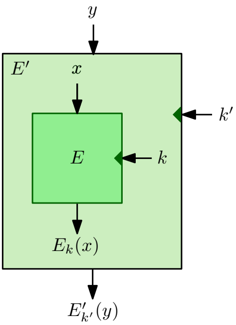

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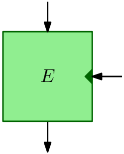
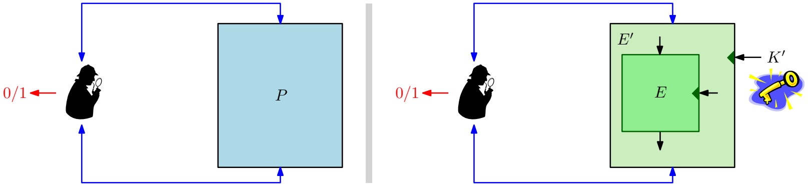
Generic Security: Ideal Block Cipher Model

- $\forall k$: independent uniformly random permutation



Key-Length Extension in ICM



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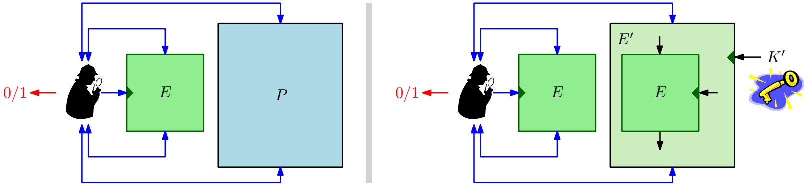

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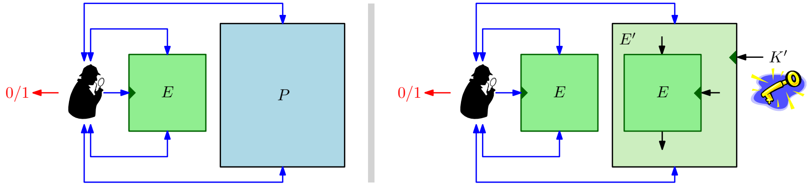

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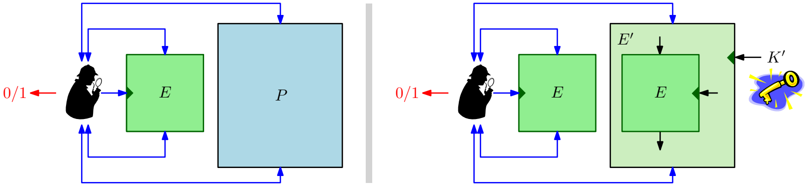

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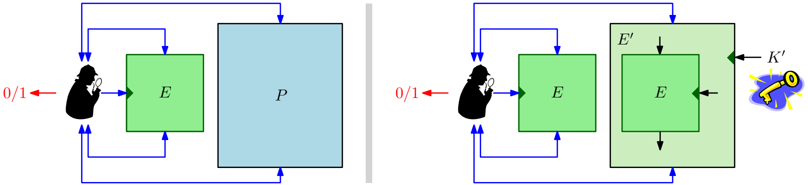

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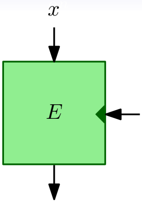

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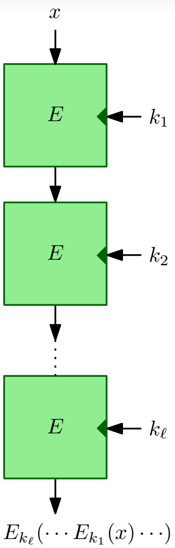

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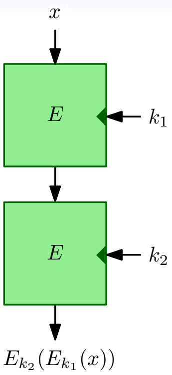

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    (2^{κ} evaluations of E and E^{-1})

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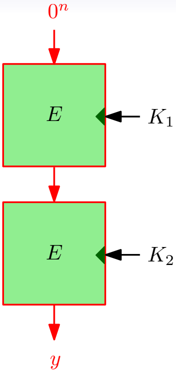

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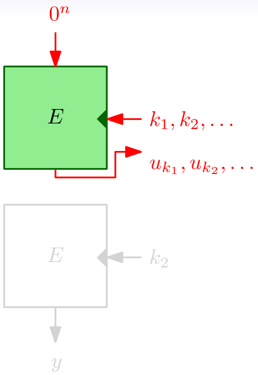

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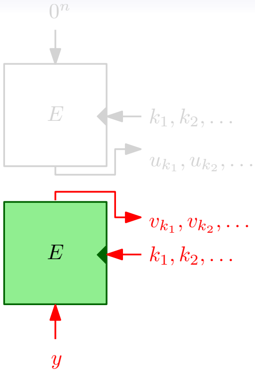

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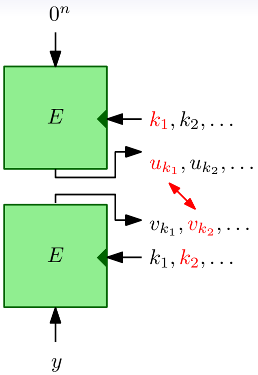

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    4. if $u_{k_i} = v_{k_j}$: verify (k_i, k_j)

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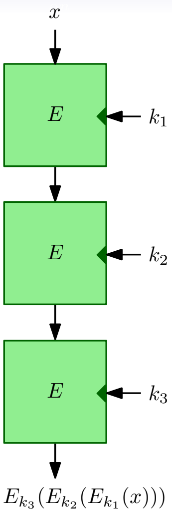

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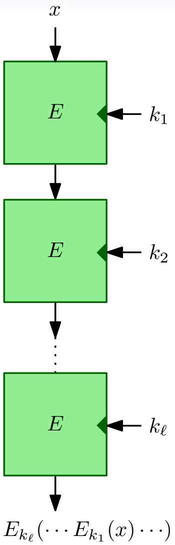

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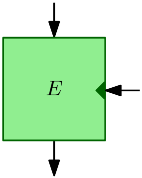
- Longer Cascades
  - Security improves for $\kappa < n$ in ICM (Gaži and Maurer, AC'09)

Approach II: Key Whitening



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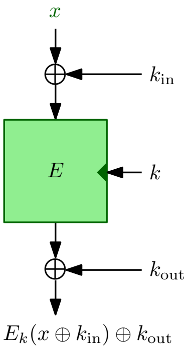

DESX [Rivest]

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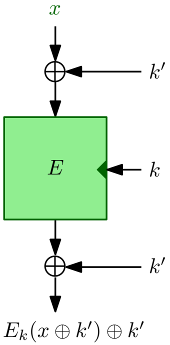

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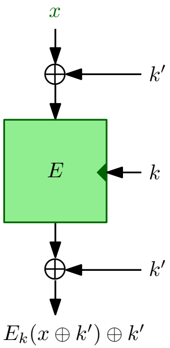

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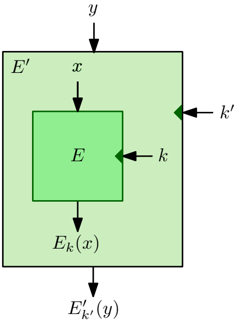
- Also secure if $k_{\text{in}} = k_{\text{out}}$

Can we do better?



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- security beyond $2^{\max\{\kappa,n\}}$ requires 3 BC queries

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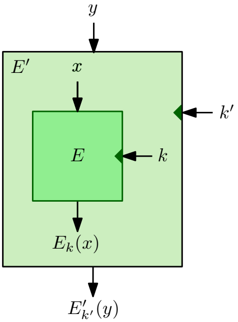

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What can be achieved with at most 2 queries to E?

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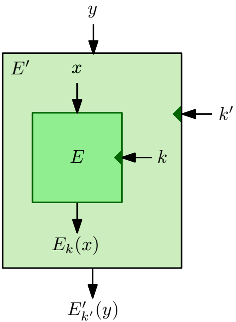
Any one-query construction can achieve at most $2^{\max\{\kappa, n\}}$ security!



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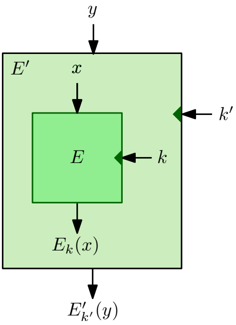

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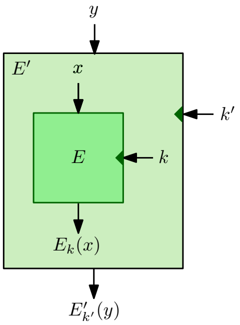

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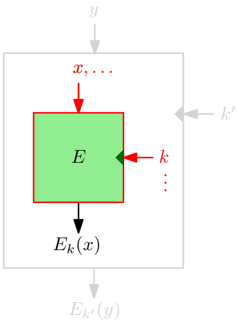

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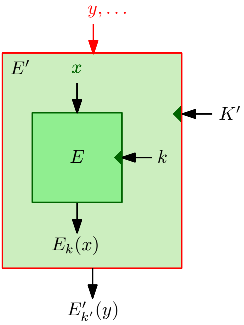

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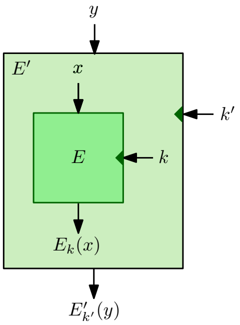

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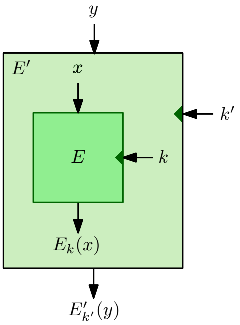
One Query Is Not Enough

Any one-query construction can achieve at most $2^{\max\{\kappa,n\}}$ security!



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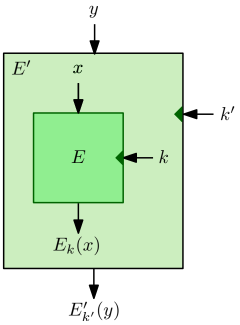

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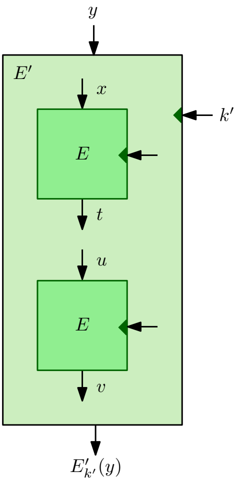
- Non-injective queries do no better

How About Two Queries?



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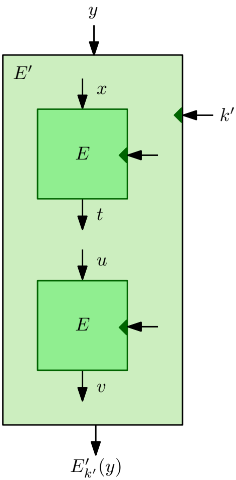

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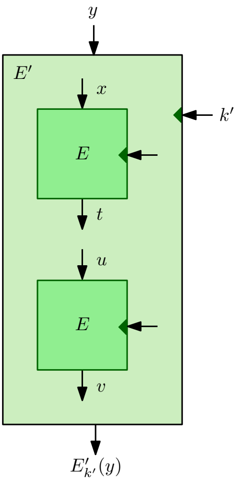

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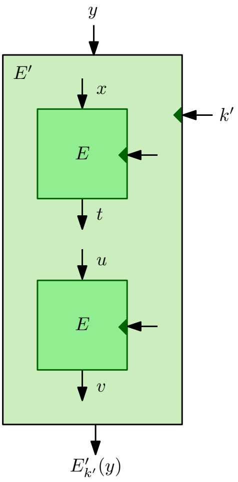

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There is room for security increase, we achieve it!

Outline

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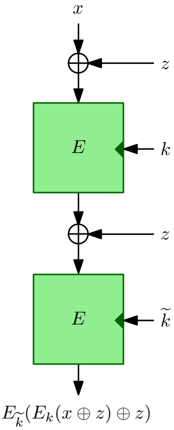
The Double XOR-Cascade



Definition

$$2XOR_{k,z}[E](x) := E_{\widetilde{k}}(E_k(x \oplus z) \oplus z)$$

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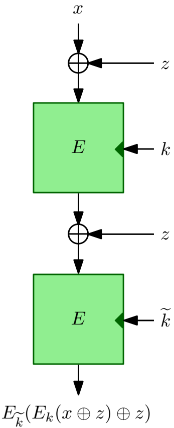

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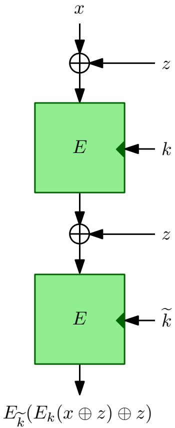

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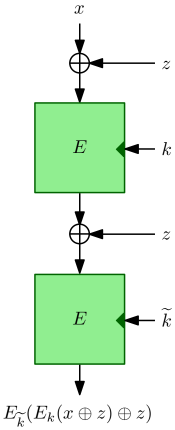

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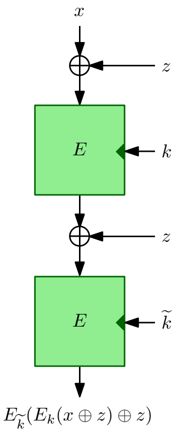

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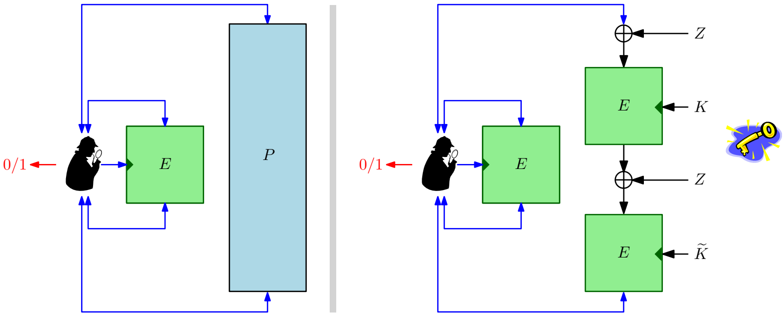
Main Result

Double XOR-Cascade is secure up to $2^{\kappa + n/2}$ queries.

A Glimpse at the Proof



The Initial Setting

$$\Delta^D\left((E, P), (E, 2XOR_{K,Z}[E])\right)$$

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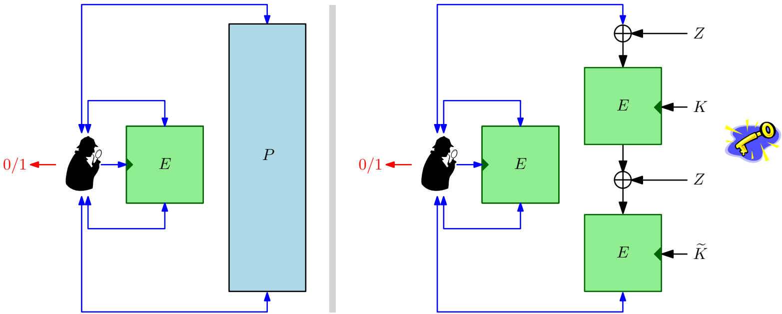

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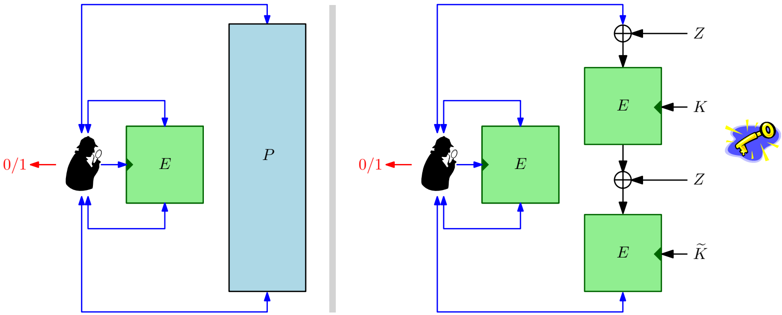

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Main Steps

- Reduce to a simpler combinatorial problem
- Show it is hard

A Glimpse at the Proof (2)

The New Problem: Distinguishing Permutations

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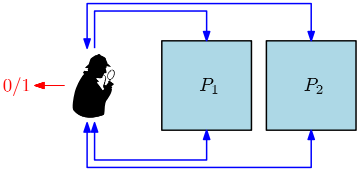

Independent

- P_1, P_2 independent uniformly random permutations

A Glimpse at the Proof (2)

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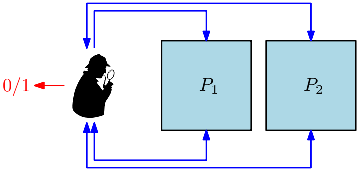

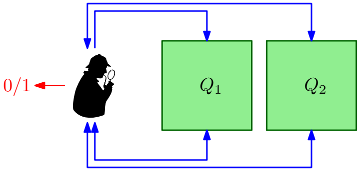

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- Q_1, Q_2 random perms s.t. for a random secret Z

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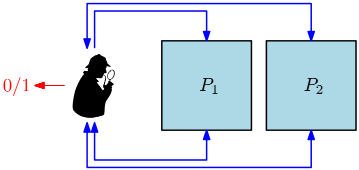

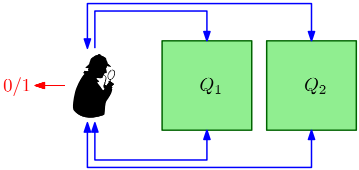

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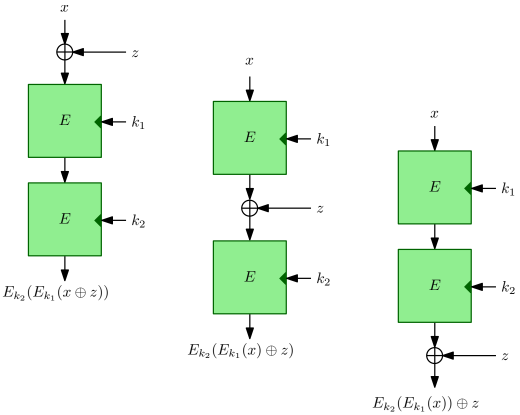
Hard for $< 2^{n/2}$ queries!

Details Are Important



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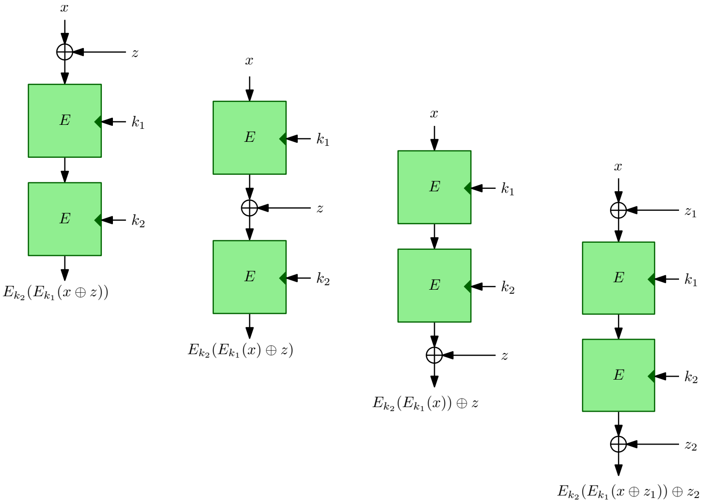

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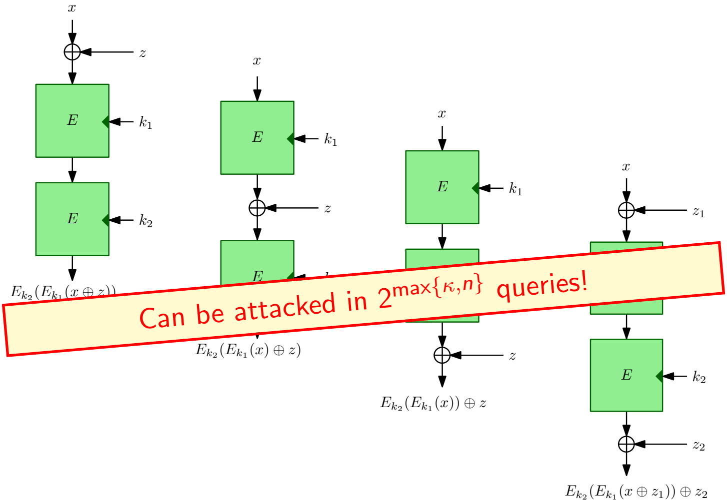

Summary

- New key-length extending construction for block ciphers
  - more efficient than triple encryption (2 BC queries per invocation)
  - more secure than triple encryption
    (Triple cascade: up to $2^{\kappa+\min\{\kappa/2,n/2\}}$)
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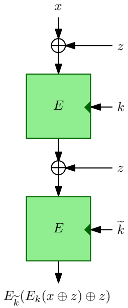

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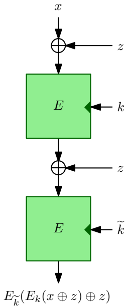

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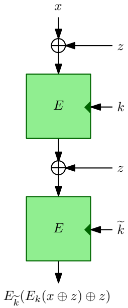

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