Efficient KDM-CCA Secure Public-Key Encryption for Polynomial Functions

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Key-Dependent Message

- KDM security: allow adversary to access encryptions of messages, which are closely dependent on the secret keys.

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- Applications:
  - Hard disk encryption
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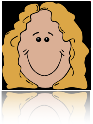
- Traditional security notion does not imply KDM security.
  [ABBC'10, CGH'12, MO'14, BHW'15, KRW'15, KW'16, AP'16] $\cdots$

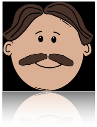
Public-Key Encryption

PKE = (Setup, Gen, Enc, Dec):

$(\mathsf{pk}, \mathsf{sk}) \leftarrow_{\$} \mathsf{Gen}(\mathsf{prm})$



Alice



Bob

Public-Key Encryption

PKE = (Setup, Gen, Enc, Dec):

$(\mathsf{pk}, \mathsf{sk}) \leftarrow_{\$} \mathsf{Gen}(\mathsf{prm})$

pke.ct

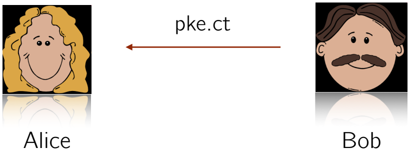
$\mathsf{pke.ct} \leftarrow_{\$} \mathsf{Enc}(\mathsf{pk}, m)$

Alice

Bob

Public-Key Encryption

$\mathsf{PKE} = (\mathsf{Setup}, \mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$:



$(\mathsf{pk}, \mathsf{sk}) \leftarrow_{\$} \mathsf{Gen}(\mathsf{prm})$

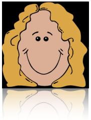
$\mathsf{m} \leftarrow \mathsf{Dec}(\mathsf{sk}, \mathsf{pke.ct})$

$\mathsf{pke.ct}$

$\mathsf{pke.ct} \leftarrow_{\$} \mathsf{Enc}(\mathsf{pk}, \mathsf{m})$

KDM Security

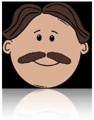
$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow_{\$} \mathsf{Gen}(\mathsf{prm})$



User 1

$\cdots$

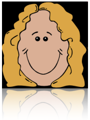
$(\mathsf{pk}_i, \mathsf{sk}_i) \leftarrow_{\$} \mathsf{Gen}(\mathsf{prm})$



User i

$\cdots$

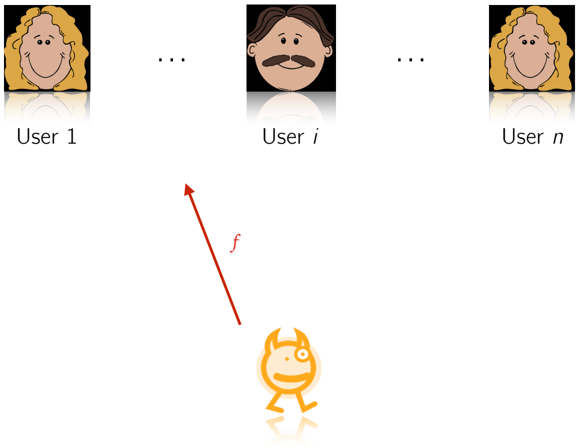
$(\mathsf{pk}_n, \mathsf{sk}_n) \leftarrow_{\$} \mathsf{Gen}(\mathsf{prm})$



User n

$\mathsf{pk}_1, \cdots, \mathsf{pk}_n$

KDM Security



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User 1 $\cdots$ User *i* $\cdots$ User *n*

f

$\mathsf{pk}_1, \cdots, \mathsf{pk}_n$

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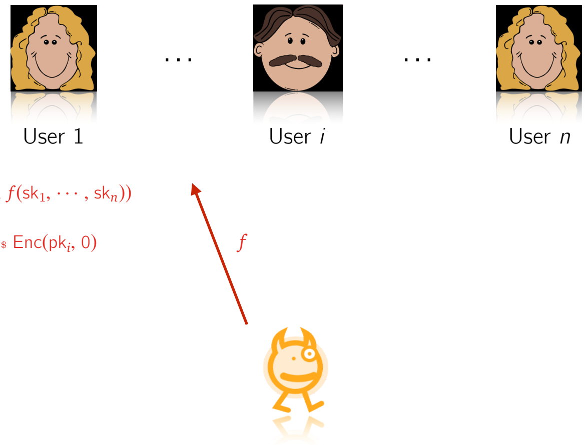

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$\mathsf{pke.ct}^* \leftarrow_\$ \mathsf{Enc}(\mathsf{pk}_i, f(\mathsf{sk}_1, \cdots, \mathsf{sk}_n))$

or $\mathsf{pke.ct}^* \leftarrow_\$ \mathsf{Enc}(\mathsf{pk}_i, 0)$

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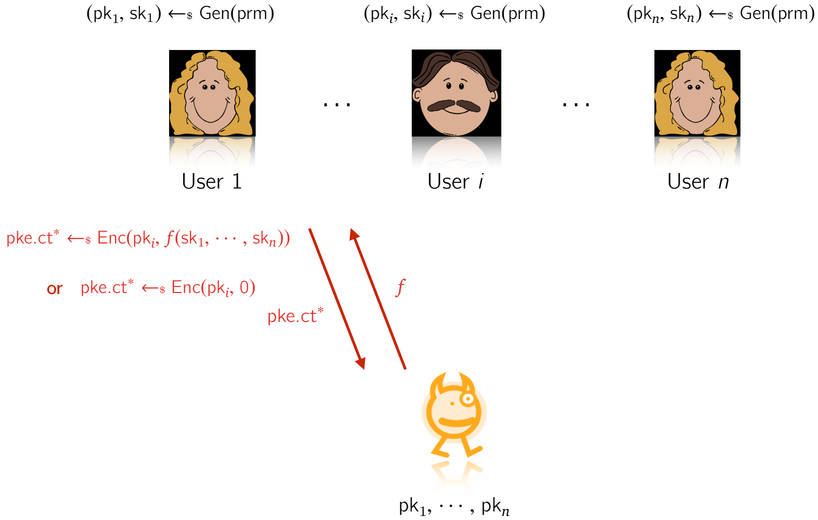
KDM Security



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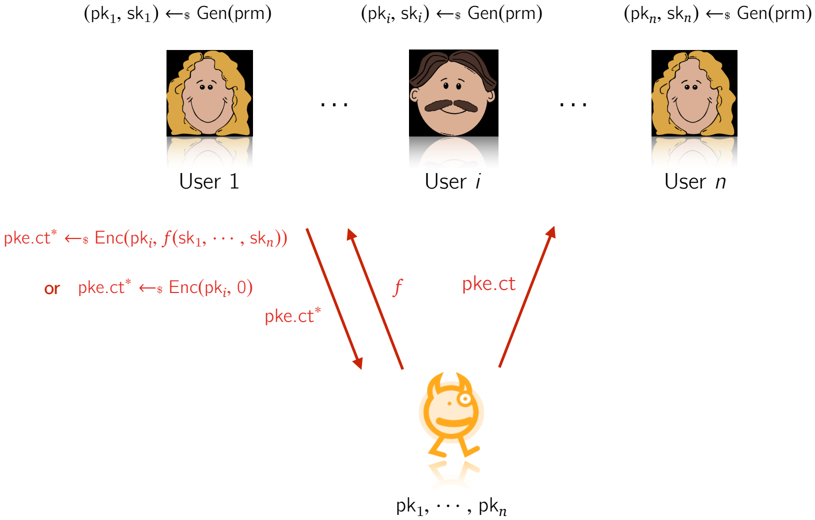

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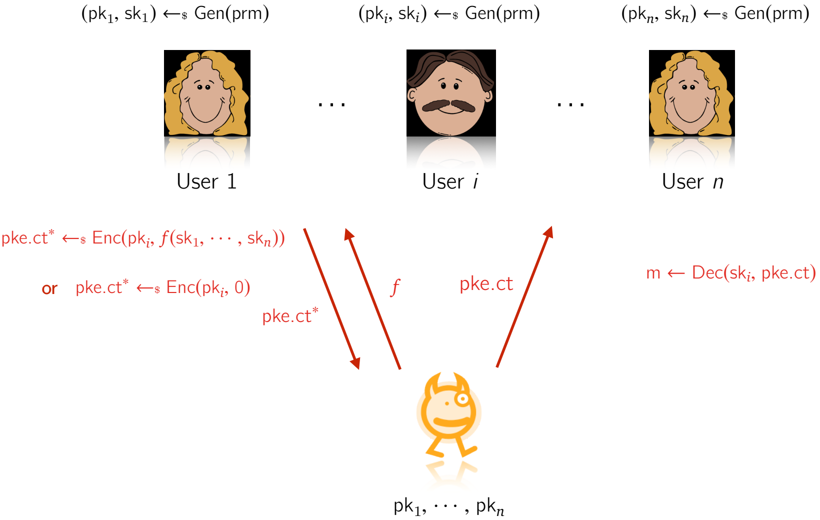

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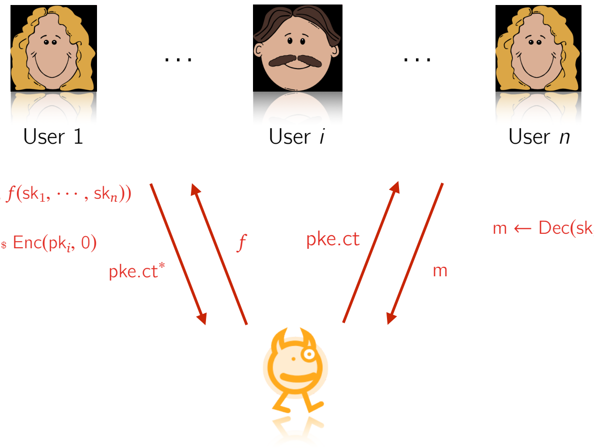

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pke.ct

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Function Set of KDM Security

KDM security is related to a set of functions $\mathcal{F}$ from $\mathcal{SK} \times \cdots \times \mathcal{SK}$ to $\mathcal{M}$.

- $\mathcal{F}_{\mathsf{circ}}$: the set of selection functions.

$$f : (\mathsf{sk}_1, \cdots, \mathsf{sk}_n) \longmapsto \mathsf{sk}_i$$

- $\mathcal{F}_{\mathsf{aff}}$: the set of affine functions.

$$f : (\mathsf{sk}_1, \cdots, \mathsf{sk}_n) \longmapsto \sum_{i=1}^{n} a_i \cdot \mathsf{sk}_i + b$$

- $\mathcal{F}_{\mathsf{poly}}^{d}$: the set of polynomial functions of bounded degree d.

$$f : (\mathsf{sk}_1, \cdots, \mathsf{sk}_n) \longmapsto \sum_{0 \le c_1 + \cdots + c_n \le d} a_{(c_1, \cdots, c_n)} \cdot \mathsf{sk}_1^{c_1} \cdots \mathsf{sk}_n^{c_n}$$

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The larger $\mathcal{F}$ is, the stronger the security is.

Related Works: KDM-CPA secure PKE

PKE Scheme	KDM-CPA Function Set	KDM-CCA?	|Ciphertext|	Assumption
[BHHO'08], [BG'10]	$\mathcal{F}_{\mathsf{aff}}$	–	$O(\ell)\ \lvert\mathbb{G}\rvert$	DDH/QR/DCR
[ACPS'09]	$\mathcal{F}_{\mathsf{aff}}$	–	$O(1)\ \lvert\mathbb{G}\rvert$	LWE
[BGK'11]	$\mathcal{F}^{d}_{\mathsf{poly}}$	–	$O(\ell^{d+1})\ \lvert\mathbb{G}\rvert$	DDH/LWE
[MTY'11]	$\mathcal{F}^{d}_{\mathsf{poly}}$	–	$O(d)\ \lvert\mathbb{G}\rvert$	DCR

- ℓ: security parameter.
- d: bounded degree of polynomial functions.

Related Works: KDM-CCA secure PKE

PKE Scheme	KDM-CCA Function Set	KDM-CCA?	|Ciphertext|	Assumption
[BHHO'08] + [CCS'09]	$\mathcal{F}_{\text{aff}}$	√	$O(\ell)$ $\lvert\mathbb{G}\rvert$	DDH
[Hofheinz'13]	$\mathcal{F}_{\text{circ}}$	√	$O(1)$ $\lvert\mathbb{G}\rvert$	DDH & DCR
[LLJ'15]	$\mathcal{F}_{\text{aff}}$	?	$O(1)$ $\lvert\mathbb{G}\rvert$	DDH & DCR

- ℓ: security parameter.
- d: bounded degree of polynomial functions.

Our Contribution

PKE Scheme	KDM-CCA Function Set	KDM-CCA?	|Ciphertext|	Assumption
Our first scheme	$\mathcal{F}_{\mathsf{aff}}$	$\surd$	$O(1)\ \lvert\mathbb{G}\rvert$	DDH & DCR
Our second scheme	$\mathcal{F}_{\mathsf{poly}}^{d}$	$\surd$	$O(d^9)\ \lvert\mathbb{G}\rvert$	DDH & DCR

- We give the first efficient KDM[$\mathcal{F}_{\mathsf{aff}}$]-CCA secure PKE with compact ciphertexts.
  - Compact: the ciphertexts consist only a constant number of group elements.
  - Efficient: our scheme is free of NIZK and free of pairing.

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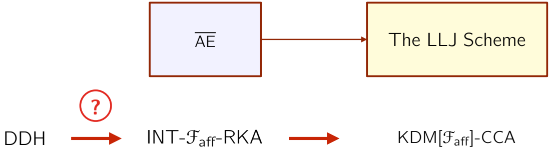
  - Compact: the ciphertexts consist only a constant number of group elements.
  - Efficient: our scheme is free of NIZK and free of pairing.
- We extend our technique, and construct the first efficient KDM[$\mathcal{F}^{d}_{\mathsf{poly}}$]-CCA secure PKE with almost compact ciphertexts.

Synopsis

1. The LLJ Scheme [Lu, Li and Jia, 2015]
2. Introducing: Authenticated Encryption with Auxiliary-Input
3. KDM-CCA secure PKE for Affine Functions
4. KDM-CCA secure PKE for Polynomial Functions

The LLJ Scheme from Related-Key Attack secure “$\overline{\text{AE}}$”



$\overline{\text{AE}}$

The LLJ Scheme

?

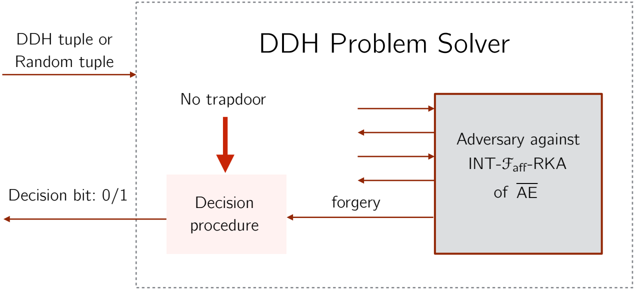
DDH

INT-$\mathcal{F}_{\text{aff}}$-RKA

KDM[$\mathcal{F}_{\text{aff}}$]-CCA

- One essential building block called “Authenticated Encryption” ($\overline{\text{AE}}$) is employed.
- The “INT-$\mathcal{F}_{\text{aff}}$-RKA” (ciphertext-integrity against related-key attacks) security proof of the LLJ’s $\overline{\text{AE}}$ does not go through to the DDH assumption.

INT-$\mathcal{F}_{\text{aff}}$-RKA security of LLJ's $\overline{\text{AE}}$



- LLJ's $\overline{\text{AE}}$: (ElGamal)-type.

$$(g^r, g^{kr}).$$

- The DDH adversary does not have any trapdoor to convert the forgery from the adversary of $\overline{\text{AE}}$ to a decision bit in an efficient way.

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A Plausible Solution

- Our new AIAE: (Kurosawa-Desmedt [KD'04])-type.

$$\left(g_1^r, g_2^r, g_1^{r(k_1+k_3t)}, g_2^{r(k_2+k_4t)}\right).$$

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New Problem!

The secret key of our AIAE consists of several elements $\mathsf{k} = (k_1, k_2, k_3, k_4)$.

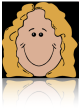
The affine function of k is too complicated to prove the INT-$\mathcal{F}_{\mathsf{aff}}$-RKA security.

$$f : (k_1, k_2, k_3, k_4) \longmapsto (\sum_{i=1}^{4} a_{i,1} \cdot k_i + b_1, \sum_{i=1}^{4} a_{i,2} \cdot k_i + b_2, \sum_{i=1}^{4} a_{i,3} \cdot k_i + b_3, \sum_{i=1}^{4} a_{i,4} \cdot k_i + b_4)$$

Our Solution: Authenticated Encryption with Auxiliary-Input

AIAE = (AIAE.Setup, AIAE.Enc, AIAE.Dec):

k



Alice

k

Bob

- We introduce "Authenticated Encryption with Auxiliary-Input" (AIAE).

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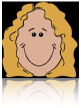

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  - AIAE must have auxiliary input “aux”.

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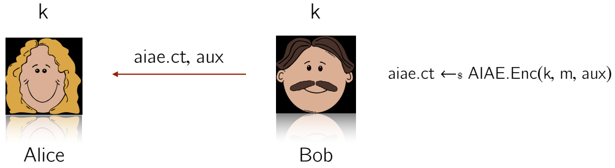

k

k

aiae.ct, aux

aiae.ct $\leftarrow_{\$}$ AIAE.Enc(k, m, aux)

Alice

Bob

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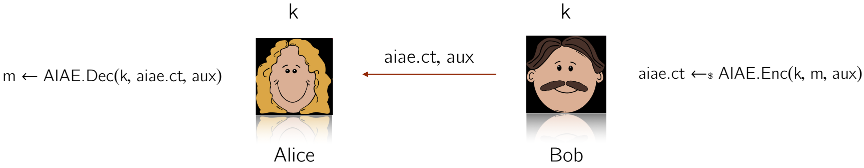

k

k

m ← AIAE.Dec(k, aiae.ct, aux)

aiae.ct, aux

aiae.ct $\leftarrow_{\$}$ AIAE.Enc(k, m, aux)

Alice

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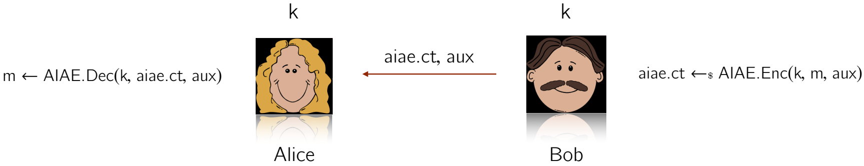


m $\leftarrow$ AIAE.Dec(k, aiae.ct, aux)

k

Alice

aiae.ct, aux

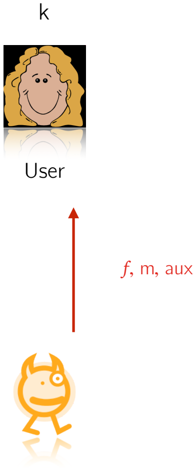
k

Bob

aiae.ct $\leftarrow_{\$}$ AIAE.Enc(k, m, aux)

- We introduce "Authenticated Encryption with Auxiliary-Input" (AIAE).
  - AIAE must have auxiliary input "aux".
  - Weak INT-$\mathcal{F}$-RKA security: an additional "special rule" for the forgery.

Weak INT-$\mathcal{F}$-RKA security for AIAE

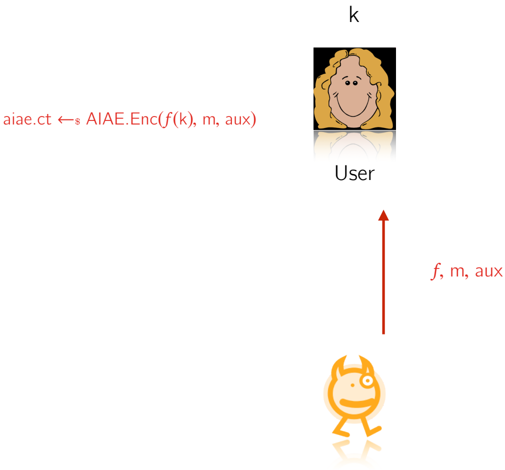


k

User

f, m, aux

Weak INT-$\mathcal{F}$-RKA security for AIAE



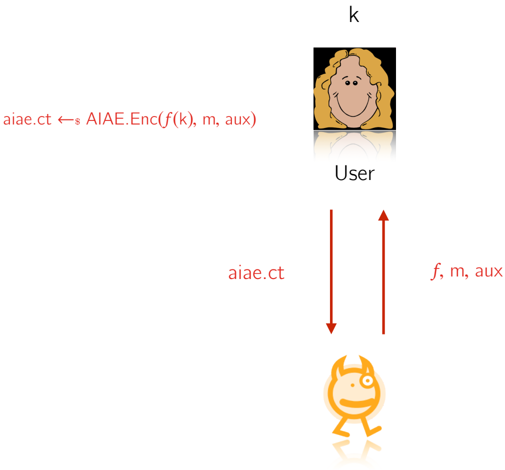
k

$\mathsf{aiae.ct} \leftarrow_{\$} \mathsf{AIAE.Enc}(f(\mathsf{k}), \mathsf{m}, \mathsf{aux})$

User

f, m, aux

Weak INT-$\mathcal{F}$-RKA security for AIAE



k

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aiae.ct

f, m, aux

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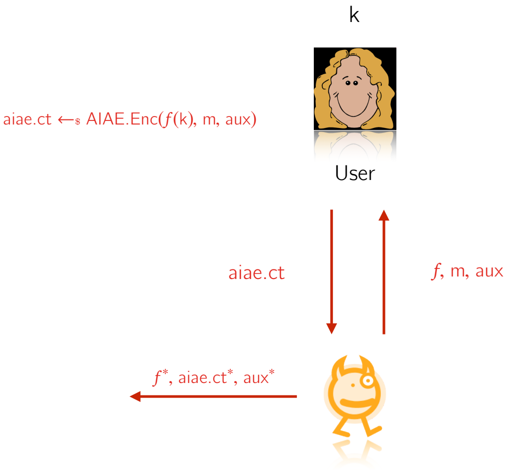

k

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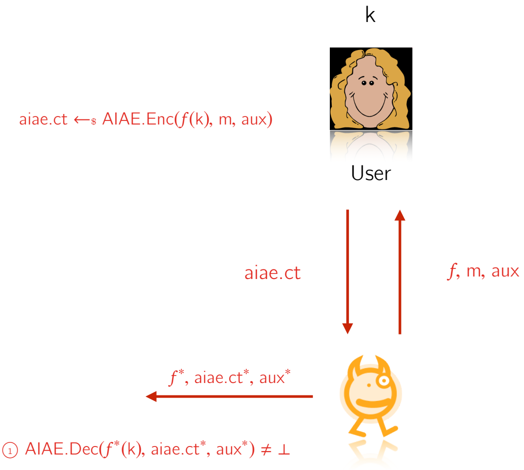
aiae.ct

f, m, aux

f^*, aiae.ct*, aux*

Weak INT-$\mathcal{F}$-RKA security for AIAE



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aiae.ct

f, m, aux

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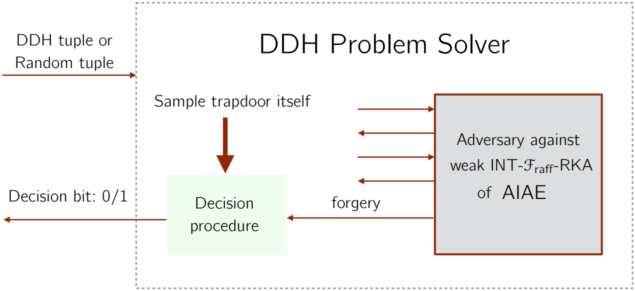
① $\text{AIAE.Dec}(f^*(\text{k}), \text{aiae.ct}^*, \text{aux}^*) \neq \perp$

② Special rule

Our AIAE



- We prove the weak INT-$\mathcal{F}_{\text{raff}}$-RKA security of our AIAE w.r.t. a smaller restricted affine function set $\mathcal{F}_{\text{raff}}$.

$$f : (k_1, k_2, k_3, k_4) \longmapsto (a \cdot k_1 + b_1, a \cdot k_2 + b_2, a \cdot k_3 + b_3, a \cdot k_4 + b_4)$$

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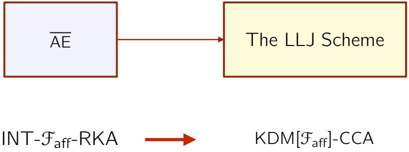
The LLJ's Method does not work for Our AIAE



$\overline{\mathsf{AE}}$ → The LLJ Scheme

INT-$\mathcal{F}_{\mathsf{aff}}$-RKA → KDM[$\mathcal{F}_{\mathsf{aff}}$]-CCA

- Our AIAE only achieves a very weak INT-$\mathcal{F}_{\mathsf{raff}}$-RKA security w.r.t. a small $\mathcal{F}_{\mathsf{raff}}$.

We cannot apply the LLJ's method to construct KDM[$\mathcal{F}_{\mathsf{aff}}$]-CCA secure PKE.

Our Approach

- Build KDM-CCA secure PKE from three building blocks: KEM, $\mathcal{E}$ and AIAE.
  - KEM: a key encapsulation mechanism.

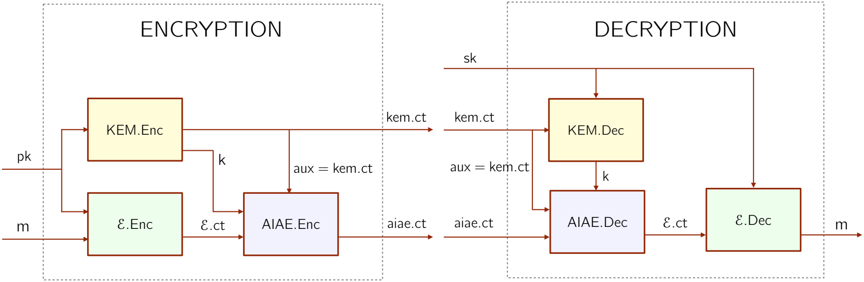
    $$(\mathsf{k}, \mathsf{kem.ct}) \leftarrow_{\$} \mathsf{KEM.Enc}(\mathsf{pk}), \qquad \mathsf{k} \leftarrow \mathsf{KEM.Dec}(\mathsf{sk}, \mathsf{kem.ct}).$$

  - $\mathcal{E}$: a public-key encryption scheme.

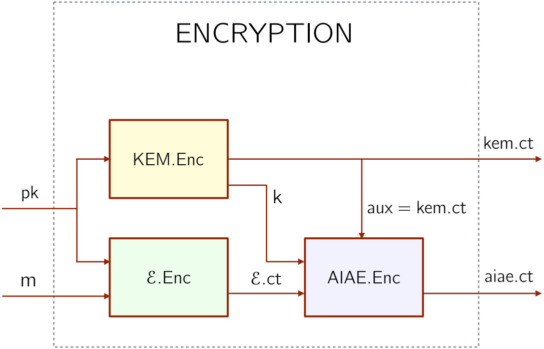
    $$\mathcal{E}.\mathsf{ct} \leftarrow_{\$} \mathcal{E}.\mathsf{Enc}(\mathsf{pk}, \mathsf{m}), \qquad \mathsf{m} \leftarrow \mathcal{E}.\mathsf{Dec}(\mathsf{sk}, \mathcal{E}.\mathsf{ct}).$$

  - AIAE: an authenticated encryption with auxiliary-input.

    $$\mathsf{AIAE.ct} \leftarrow_{\$} \mathsf{AIAE.Enc}(\mathsf{k}, \mathsf{m}, \mathsf{aux}), \qquad \mathsf{m} \leftarrow \mathsf{AIAE.Dec}(\mathsf{k}, \mathsf{AIAE.ct}, \mathsf{aux}).$$

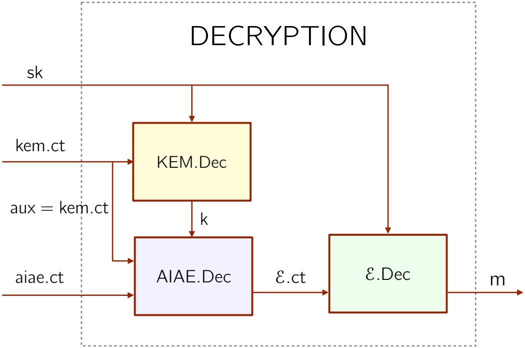


Our Construction



- KEM and $\mathcal{E}$ share the same key pair (pk, sk).
- AIAE.Enc uses k encapsulated by KEM to encrypt $\mathcal{E}$.ct with aux = kem.ct.

Our Construction



- KEM and $\mathcal{E}$ share the same key pair (pk, sk).

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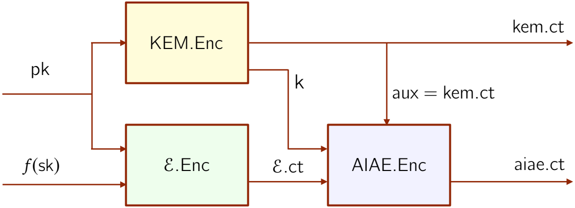
Proof Idea of KDM[$\mathcal{F}_{\mathsf{aff}}$]-CCA Security

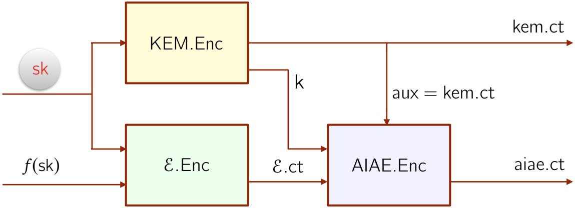
The Encryption Oracle:



- Divide the secret key sk to two independent parts, 🟦 sk mod N ▨ sk mod $\phi(N)$

Proof Idea of KDM[$\mathcal{F}_{\mathsf{aff}}$]-CCA Security

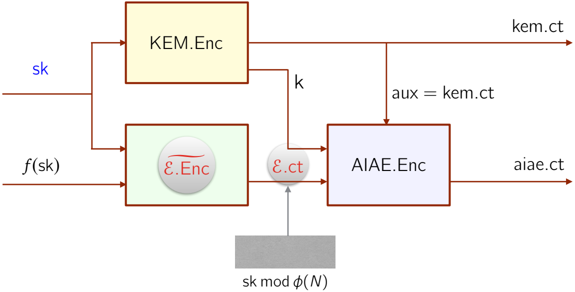
The Encryption Oracle:



- Use sk to answer the encryption queries.

Proof Idea of KDM[$\mathcal{F}_{\mathsf{aff}}$]-CCA Security

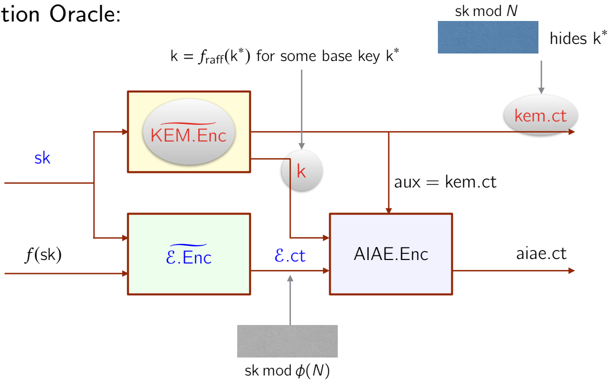
The Encryption Oracle:



- Under the DCR assumption, $\mathcal{E}.\mathsf{Enc}$ is changed to $\widetilde{\mathcal{E}.\mathsf{Enc}}$.
  - $\widetilde{\mathcal{E}.\mathsf{Enc}}$ behaves like an entropy filter for $\mathcal{F}_{\mathsf{aff}}$, such that $\underset{\text{sk mod } N}{▨}$ is reserved.

Proof Idea of KDM[$\mathcal{F}_{\mathsf{aff}}$]-CCA Security

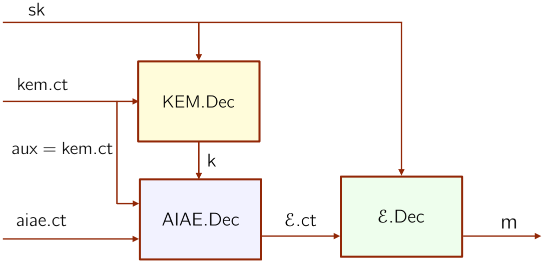
The Encryption Oracle:


sk mod N
hides k^*
$k = f_{\mathsf{raff}}(k^*)$ for some base key k^*
kem.ct
$\widetilde{\mathsf{KEM.Enc}}$
sk
k
aux = kem.ct
$f(\mathsf{sk})$
$\widetilde{\mathcal{E}.\mathsf{Enc}}$
$\mathcal{E}$.ct
AIAE.Enc
aiae.ct
sk mod $\phi(N)$

- Under the DCR assumption, KEM.Enc is changed to $\widetilde{\mathsf{KEM.Enc}}$.
  - k is expressed as an $\mathcal{F}_{\mathsf{raff}}$-function of a fixed base key k^*.
  - In kem.ct, ▨ protects the base key k^*.
    sk mod N

Proof Idea of KDM[$\mathcal{F}_{\mathsf{aff}}$]-CCA Security

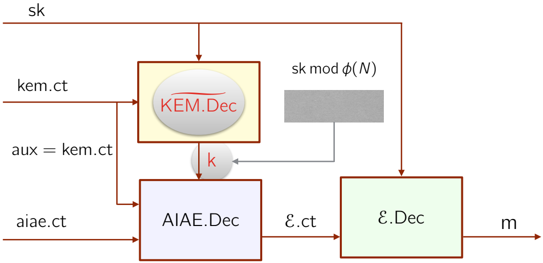
The Decryption Oracle:



- Divide the secret key sk to two independent parts, 🟦 sk mod N ▨ sk mod $\phi(N)$

Proof Idea of KDM[$\mathcal{F}_{\text{aff}}$]-CCA Security

The Decryption Oracle:



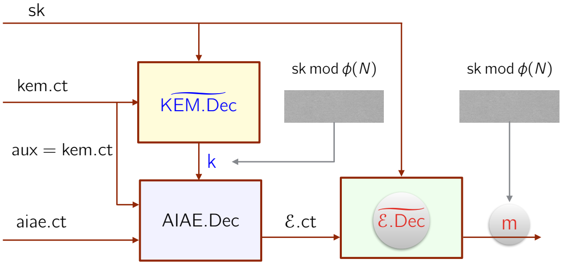
- $\widetilde{\text{KEM.Dec}}$ rejects the query, if the computation of k involves ▨ .

  sk mod N

  – By the weak INT-$\mathcal{F}_{\text{raff}}$-RKA security of AIAE, this change is computationally indistinguishable.

Proof Idea of KDM[$\mathcal{F}_{\mathsf{aff}}$]-CCA Security

The Decryption Oracle:



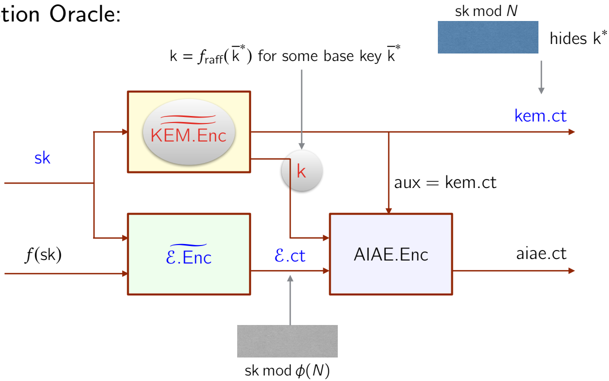
- $\widetilde{\mathcal{E}\text{.Dec}}$ rejects the query, if the computation of m involves ▨ .

  sk mod N

  - Since $\mathcal{E}$ has an authentication functionality, this change is computationally indistinguishable.

Proof Idea of KDM[$\mathcal{F}_{\text{aff}}$]-CCA Security

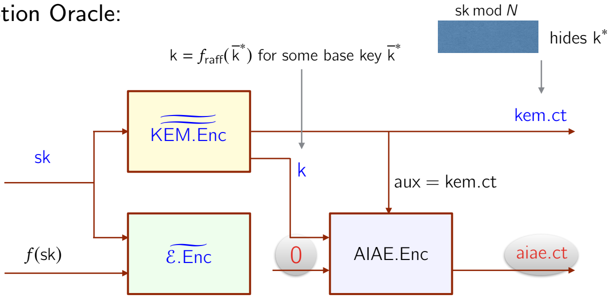
The Encryption Oracle:


sk mod N
hides k^*
$\mathsf{k} = f_{\text{raff}}(\overline{\mathsf{k}}^*)$ for some base key $\overline{\mathsf{k}}^*$
kem.ct
$\widetilde{\text{KEM.Enc}}$
k
sk
aux = kem.ct
f(sk)
$\widetilde{\mathcal{E}.\text{Enc}}$
$\mathcal{E}$.ct
AIAE.Enc
aiae.ct
sk mod $\phi(N)$

- We compute k as $\mathcal{F}_{\text{raff}}$-functions of an independent base key $\overline{\mathsf{k}}^*$.
  - In $\widetilde{\mathcal{E}.\text{Enc}}$ and the Decryption Oracle, ▨ is not involved.
    sk mod N
  - In kem.ct, the base key k^* is protected by ▨ perfectly.
    sk mod N

Proof Idea of KDM[$\mathcal{F}_{\mathsf{aff}}$]-CCA Security

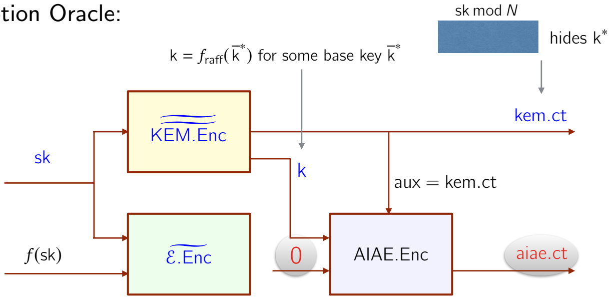
The Encryption Oracle:


sk mod N
hides k*
k = $f_{\mathsf{raff}}(\overline{\mathsf{k}}^*)$ for some base key $\overline{\mathsf{k}}^*$
kem.ct
$\widetilde{\mathsf{KEM.Enc}}$
sk
k
aux = kem.ct
f(sk)
$\widetilde{\mathcal{E}}$.Enc
0
AIAE.Enc
aiae.ct

- By the IND-$\mathcal{F}_{\mathsf{raff}}$-RKA security of AIAE, we change aiae.ct as encryptions of 0.
  - k is an $\mathcal{F}_{\mathsf{raff}}$-function of $\overline{\mathsf{k}}^*$, which is independent of other parts of the game.

Proof Idea of KDM[$\mathcal{F}_{\text{aff}}$]-CCA Security

The Encryption Oracle:

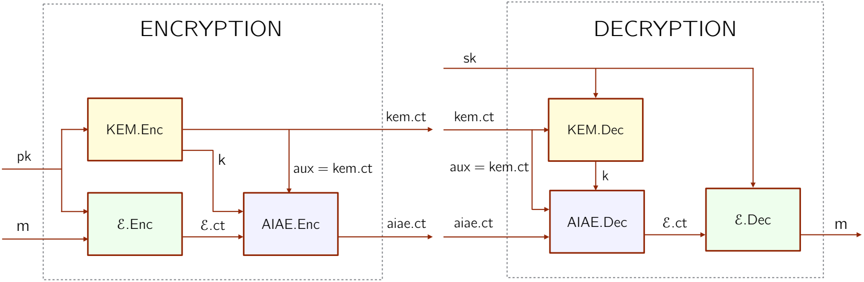

sk mod N
hides k^*
$\mathsf{k} = f_{\text{raff}}(\bar{\mathsf{k}}^*)$ for some base key $\bar{\mathsf{k}}^*$
kem.ct
$\widetilde{\widetilde{\text{KEM}}}$.Enc
sk
k
aux = kem.ct
$\widetilde{\mathcal{E}}$.Enc
$f(\mathsf{sk})$
0
AIAE.Enc
aiae.ct

- By the IND-$\mathcal{F}_{\text{raff}}$-RKA security of AIAE, we change aiae.ct as encryptions of 0.
  - k is an $\mathcal{F}_{\text{raff}}$-function of $\bar{\mathsf{k}}^*$, which is independent of other parts of the game.
- The advantage of the adversary is zero.

Synopsis

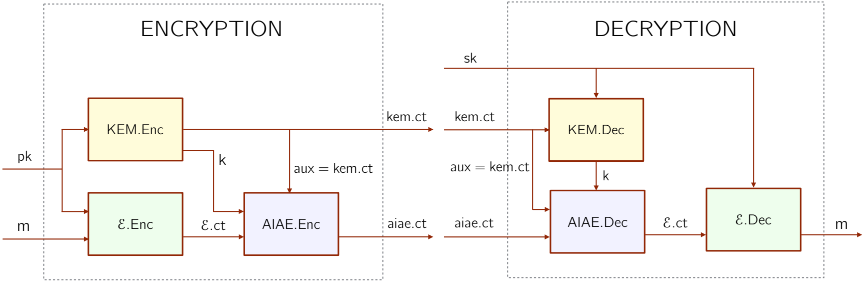
1. The LLJ Scheme [Lu, Li and Jia, 2015]
2. Introducing: Authenticated Encryption with Auxiliary-Input
3. KDM-CCA secure PKE for Affine Functions
4. KDM-CCA secure PKE for Polynomial Functions

Our Approach



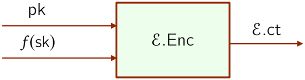
- We design a new $\mathcal{E}$: an entropy filter for the set of polynomial functions $\mathcal{F}^d_{\text{poly}}$.
    - Entropy Filter ([LLJ'15]): through some computationally indistinguishable change, $\underset{\text{sk mod } N}{\blacksquare}$ can be reserved by $\mathcal{E}$.Enc(pk, f(sk)), for $f \in \mathcal{F}^d_{\text{poly}}$.

Our Approach



- We design a new $\mathcal{E}$: an entropy filter for the set of polynomial functions $\mathcal{F}^d_{\text{poly}}$.
  - Entropy Filter ([LLJ'15]): through some computationally indistinguishable change, sk mod N can be reserved by $\mathcal{E}$.Enc(pk, f(sk)), for $f \in \mathcal{F}^d_{\text{poly}}$.
- The other two building blocks KEM and AIAE are the same.

$\mathcal{E}$ designed for monomial $f(\mathsf{sk}) = a \cdot x_1 y_1 x_2 y_2 x_3 y_3 x_4 y_4$


pk

$f(\mathsf{sk})$

$\mathcal{E}$.Enc

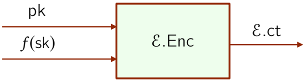
$\mathcal{E}$.ct

$\mathcal{E}$.ct = (table, e, t)

- prm = $(g_1, \cdots, g_5)$. sk = $(x_1, \cdots, x_4, y_1, \cdots, y_4)$.

  pk = $(h_1, \cdots, h_4) = (g_1^{-x_1} g_2^{-y_1}, g_2^{-x_2} g_3^{-y_2}, g_3^{-x_3} g_4^{-y_3}, g_4^{-x_4} g_5^{-y_4})$.

$\mathcal{E}$ designed for monomial $f(\mathsf{sk}) = a \cdot x_1 y_1 x_2 y_2 x_3 y_3 x_4 y_4$



$\mathcal{E}$.ct = (table, e, t)

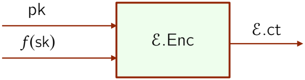
- prm = $(g_1, \cdots, g_5)$. sk = $(x_1, \cdots, x_4, y_1, \cdots, y_4)$.

  pk = $(h_1, \cdots, h_4) = (g_1^{-x_1} g_2^{-y_1}, g_2^{-x_2} g_3^{-y_2}, g_3^{-x_3} g_4^{-y_3}, g_4^{-x_4} g_5^{-y_4})$.

- For $j \in [0, 8]$,

$$\begin{array}{|c|c|c|c|}\hline u_{j,1} & u_{j,2} & \cdots & u_{j,8} \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|c|}\hline g_1^{r_{j,1}} & g_2^{r_{j,1}} & g_2^{r_{j,2}} & g_3^{r_{j,2}} & g_3^{r_{j,3}} & g_4^{r_{j,3}} & g_4^{r_{j,4}} & g_5^{r_{j,4}} \\ \hline \end{array}\,. \quad v_j = h_1^{r_{j,1}} h_2^{r_{j,2}} h_3^{r_{j,3}} h_4^{r_{j,4}}\,.$$

$\mathcal{E}$ designed for monomial $f(\mathsf{sk}) = a \cdot x_1 y_1 x_2 y_2 x_3 y_3 x_4 y_4$



$\mathcal{E}.\mathsf{ct} = (\mathsf{table}, e, t)$

- prm $= (g_1, \cdots, g_5)$. sk $= (x_1, \cdots, x_4, y_1, \cdots, y_4)$.

  pk $= (h_1, \cdots, h_4) = (g_1^{-x_1} g_2^{-y_1}, g_2^{-x_2} g_3^{-y_2}, g_3^{-x_3} g_4^{-y_3}, g_4^{-x_4} g_5^{-y_4})$.

- For $j \in [0, 8]$,

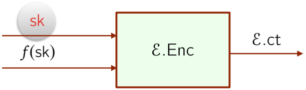
$$\begin{array}{|c|c|c|c|}\hline u_{j,1} & u_{j,2} & \cdots & u_{j,8} \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|c|}\hline g_1^{r_{j,1}} & g_2^{r_{j,1}} & g_2^{r_{j,2}} & g_3^{r_{j,2}} & g_3^{r_{j,3}} & g_4^{r_{j,3}} & g_4^{r_{j,4}} & g_5^{r_{j,4}} \\ \hline \end{array}. \quad v_j = h_1^{r_{j,1}} h_2^{r_{j,2}} h_3^{r_{j,3}} h_4^{r_{j,4}}.$$

- table $=$

$$\begin{array}{|c|c|c|c|}\hline u_{0,1} & u_{0,2} & \cdots & u_{0,8} \\ \hline u_{1,1} \cdot v_0 & u_{1,2} & \cdots & u_{1,8} \\ \hline u_{2,1} & u_{2,2} \cdot v_1 & \cdots & u_{2,8} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline u_{8,1} & u_{8,2} & \cdots & u_{8,8} \cdot v_7 \\ \hline \end{array}.$$

- $e = v_8 \cdot T^{f(\mathsf{sk})}$. $\quad t = g_1^{f(\mathsf{sk}) \bmod \phi(N)}$.

$\mathcal{E}$ designed for monomial $f(\mathsf{sk}) = a \cdot x_1 y_1 x_2 y_2 x_3 y_3 x_4 y_4$



$\mathcal{E}$.ct = (table, e, t)

- For $j \in [0, 8]$,

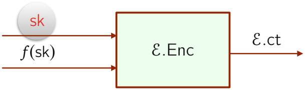
$$\begin{array}{|c|c|c|c|}\hline u_{j,1} & u_{j,2} & \cdots & u_{j,8} \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|c|}\hline g_1^{r_{j,1}} & g_2^{r_{j,1}} & g_2^{r_{j,2}} & g_3^{r_{j,2}} & g_3^{r_{j,3}} & g_4^{r_{j,3}} & g_4^{r_{j,4}} & g_5^{r_{j,4}} \\ \hline \end{array} . \quad v_j = h_1^{r_{j,1}} h_2^{r_{j,2}} h_3^{r_{j,3}} h_4^{r_{j,4}} .$$

$$\Rightarrow \hat{v}_j = u_{j,1}^{-x_1} u_{j,2}^{-y_1} u_{j,3}^{-x_2} u_{j,4}^{-y_2} u_{j,5}^{-x_3} u_{j,6}^{-y_3} u_{j,7}^{-x_4} u_{j,8}^{-y_4}$$

- table =

$u_{0,1}$	$u_{0,2}$	$\cdots$	$u_{0,8}$	$\Rightarrow \hat{v}_0 = v_0$
$u_{1,1} \cdot v_0$	$u_{1,2}$	$\cdots$	$u_{1,8}$	$\Rightarrow \hat{v}_1 = v_1$
$u_{2,1}$	$u_{2,2} \cdot v_1$	$\cdots$	$u_{2,8}$	$\Rightarrow \hat{v}_2 = v_2$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	
$u_{8,1}$	$u_{8,2}$	$\cdots$	$u_{8,8} \cdot v_7$	$\Rightarrow \hat{v}_8 = v_8$

$\mathcal{E}$ designed for monomial $f(\mathsf{sk}) = a \cdot x_1 y_1 x_2 y_2 x_3 y_3 x_4 y_4$



$\mathcal{E}$.ct = (table, e, t)

- For $j \in [0, 8]$,

$$\begin{array}{|c|c|c|c|}\hline u_{j,1} & u_{j,2} & \cdots & u_{j,8} \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|c|}\hline g_1^{r_{j,1}} & g_2^{r_{j,1}} & g_2^{r_{j,2}} & g_3^{r_{j,2}} & g_3^{r_{j,3}} & g_4^{r_{j,3}} & g_4^{r_{j,4}} & g_5^{r_{j,4}} \\ \hline \end{array} . \quad v_j = h_1^{r_{j,1}} h_2^{r_{j,2}} h_3^{r_{j,3}} h_4^{r_{j,4}} .$$

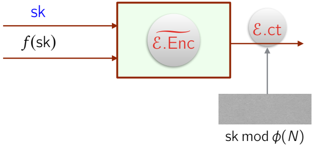
$$\Rightarrow \hat{v}_j = u_{j,1}^{-x_1} u_{j,2}^{-y_1} u_{j,3}^{-x_2} u_{j,4}^{-y_2} u_{j,5}^{-x_3} u_{j,6}^{-y_3} u_{j,7}^{-x_4} u_{j,8}^{-y_4}$$

- table =

$$\begin{array}{|c|c|c|c|l}\cline{1-4} u_{0,1} & u_{0,2} & \cdots & u_{0,8} & \Rightarrow \hat{v}_0 = v_0 \\ \cline{1-4} u_{1,1} \cdot v_0 & u_{1,2} & \cdots & u_{1,8} & \Rightarrow \hat{v}_1 = v_1 \\ \cline{1-4} u_{2,1} & u_{2,2} \cdot v_1 & \cdots & u_{2,8} & \Rightarrow \hat{v}_2 = v_2 \\ \cline{1-4} \vdots & \vdots & \ddots & \vdots & \\ \cline{1-4} u_{8,1} & u_{8,2} & \cdots & u_{8,8} \cdot v_7 & \Rightarrow \hat{v}_8 = v_8 \\ \cline{1-4} \end{array}$$

- $e = v_8 \cdot T^{f(\mathsf{sk})} \Rightarrow e = \hat{v}_8 \cdot T^{f(\mathsf{sk})}$. $\quad t = g_1^{f(\mathsf{sk}) \bmod \phi(N)}$.

$\mathcal{E}$ designed for monomial $f(\mathsf{sk}) = a \cdot x_1 y_1 x_2 y_2 x_3 y_3 x_4 y_4$



$\mathcal{E}$.ct = (table, e, t)

- For $j \in [0, 8]$,

$$\begin{array}{|c|c|c|c|}\hline u_{j,1} & u_{j,2} & \cdots & u_{j,8} \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|c|}\hline g_1^{r_{j,1}} & g_2^{r_{j,1}} & g_2^{r_{j,2}} & g_3^{r_{j,2}} & g_3^{r_{j,3}} & g_4^{r_{j,3}} & g_4^{r_{j,4}} & g_5^{r_{j,4}} \\ \hline \end{array} . \quad v_j = h_1^{r_{j,1}} h_2^{r_{j,2}} h_3^{r_{j,3}} h_4^{r_{j,4}} .$$

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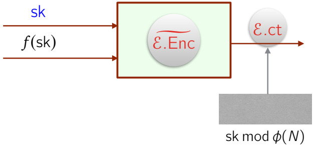
- table =

$u_{0,1}$	$u_{0,2}$	$\cdots$	$u_{0,8}$	$\Rightarrow \hat{v}_0 = v_0$
$u_{1,1} \cdot v_0 \cdot T^a$	$u_{1,2}$	$\cdots$	$u_{1,8}$	$\Rightarrow \hat{v}_1 = v_1 \cdot T^{-ax_1}$
$u_{2,1}$	$u_{2,2} \cdot v_1$	$\cdots$	$u_{2,8}$	$\Rightarrow \hat{v}_2 = v_2 \cdot T^{-ax_1 y_1}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	
$u_{8,1}$	$u_{8,2}$	$\cdots$	$u_{8,8} \cdot v_7$	$\Rightarrow \hat{v}_8 = v_8 \cdot T^{-ax_1 y_1 \cdots x_4 y_4} = v_8 \cdot T^{-f(\mathsf{sk})}$

$\mathcal{E}$ designed for monomial $f(\mathsf{sk}) = a \cdot x_1y_1x_2y_2x_3y_3x_4y_4$



$\mathcal{E}.\mathsf{ct} = (\mathsf{table}, e, t)$

- For $j \in [0, 8]$,

$$\begin{array}{|c|c|c|c|}\hline u_{j,1} & u_{j,2} & \cdots & u_{j,8} \\ \hline\end{array} = \begin{array}{|c|c|c|c|c|c|c|c|}\hline g_1^{r_{j,1}} & g_2^{r_{j,1}} & g_2^{r_{j,2}} & g_3^{r_{j,2}} & g_3^{r_{j,3}} & g_4^{r_{j,3}} & g_4^{r_{j,4}} & g_5^{r_{j,4}} \\ \hline\end{array}. \quad v_j = h_1^{r_{j,1}} h_2^{r_{j,2}} h_3^{r_{j,3}} h_4^{r_{j,4}}.$$

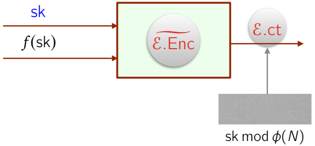
$$\Rightarrow \hat{v}_j = u_{j,1}^{-x_1} u_{j,2}^{-y_1} u_{j,3}^{-x_2} u_{j,4}^{-y_2} u_{j,5}^{-x_3} u_{j,6}^{-y_3} u_{j,7}^{-x_4} u_{j,8}^{-y_4}$$

- table =

$$\begin{array}{|c|c|c|c|l}\hline u_{0,1} & u_{0,2} & \cdots & u_{0,8} & \Rightarrow \hat{v}_0 = v_0 \\ \hline u_{1,1} \cdot v_0 \cdot \textcircled{T^a} & u_{1,2} & \cdots & u_{1,8} & \Rightarrow \hat{v}_1 = v_1 \cdot T^{-ax_1} \\ \hline u_{2,1} & u_{2,2} \cdot v_1 & \cdots & u_{2,8} & \Rightarrow \hat{v}_2 = v_2 \cdot T^{-ax_1y_1} \\ \hline \vdots & \vdots & \ddots & \vdots & \\ \hline u_{8,1} & u_{8,2} & \cdots & u_{8,8} \cdot v_7 & \Rightarrow \hat{v}_8 = v_8 \cdot T^{-ax_1y_1\cdots x_4y_4} = v_8 \cdot T^{-f(\mathsf{sk})} \\ \hline\end{array}$$

- $e = v_8 \cdot T^{f(\mathsf{sk})} \Rightarrow e = \hat{v}_8 \cdot T^{f(\mathsf{sk})} \Rightarrow e = v_8.$ $\qquad t = g_1^{f(\mathsf{sk}) \bmod \phi(N)}.$

$\mathcal{E}$ designed for monomial $f(\mathsf{sk}) = a \cdot x_1y_1x_2y_2x_3y_3x_4y_4$


sk
$f(\mathsf{sk})$
$\widetilde{\mathcal{E}.\mathsf{Enc}}$
$\mathcal{E}.\mathsf{ct}$
sk mod $\phi(N)$

$\mathcal{E}.\mathsf{ct} = (\mathsf{table}, e, t)$

- For $j \in [0, 8]$,

$$\begin{array}{|c|c|c|c|}\hline u_{j,1} & u_{j,2} & \cdots & u_{j,8} \\ \hline\end{array} = \begin{array}{|c|c|c|c|c|c|c|c|}\hline g_1^{r_{j,1}} & g_2^{r_{j,1}} & g_2^{r_{j,2}} & g_3^{r_{j,2}} & g_3^{r_{j,3}} & g_4^{r_{j,3}} & g_4^{r_{j,4}} & g_5^{r_{j,4}} \\ \hline\end{array}. \quad v_j = h_1^{r_{j,1}} h_2^{r_{j,2}} h_3^{r_{j,3}} h_4^{r_{j,4}}.$$

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$u_{0,1}$	$u_{0,2}$	$\cdots$	$u_{0,8}$
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$u_{2,1}$	$u_{2,2} \cdot v_1$	$\cdots$	$u_{2,8}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$u_{8,1}$	$u_{8,2}$	$\cdots$	$u_{8,8} \cdot v_7$

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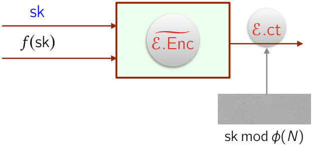

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$\widetilde{\mathcal{E}}$.Enc behaves like an entropy filter for the monomial.

General $\mathcal{E}$ designed for Polynomial Functions

- A polynomial function f in $\mathsf{sk} = (x_1, \cdots, x_4, y_1, \cdots, y_4)$ of degree d is

$$f(\mathsf{sk}) = \sum_{0 \leq c_1 + \cdots + c_8 \leq d} a_{(c_1, \cdots, c_8)} \cdot x_1^{c_1} y_1^{c_2} \cdots x_4^{c_7} y_4^{c_8}.$$

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- For each monomial $\mathsf{c} = (c_1, \cdots, c_8)$, $\mathcal{E}$.Enc creates a pair of $\mathsf{table}^{(\mathsf{c})}$ and $v^{(\mathsf{c})}$.

  The products of these $v^{(\mathsf{c})}$ are used to hide the message: $e = \prod_{\mathsf{c}} v^{(\mathsf{c})} \cdot T^{f(\mathsf{sk})}$.

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- Under the DCR assumption, $\mathcal{E}$.Enc is changed to $\widetilde{\mathcal{E}.\mathsf{Enc}}$, such that each $v^{(\mathsf{c})}$ is multiplied with an additional term:

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$\widetilde{\mathcal{E}.\mathsf{Enc}}$ behaves like an entropy filter for polynomial functions.

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- A new approach for constructing KDM-CCA secure PKE scheme, from KEM, $\mathcal{E}$, and a new primitive called "AIAE".
- Efficient KDM[$\mathcal{F}_{\mathsf{aff}}$]-CCA secure PKE with compact ciphertexts.
- Efficient KDM[$\mathcal{F}^{d}_{\mathsf{poly}}$]-CCA secure PKE with almost compact ciphertexts.

Thank You