A SHUFFLE ARGUMENT SECURE IN THE GENERIC MODEL

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Panoramix

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- A new efficient CRS-based NIZK shuffle argument

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  - Use computer algebra to solve systems of polyn. eq.
    - Esp. to find **Gröbner bases**

A BIT OF MOTIVATION: E-VOTING

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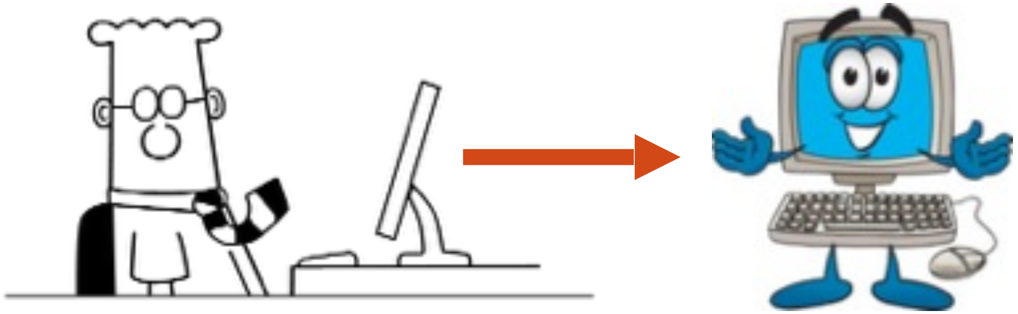

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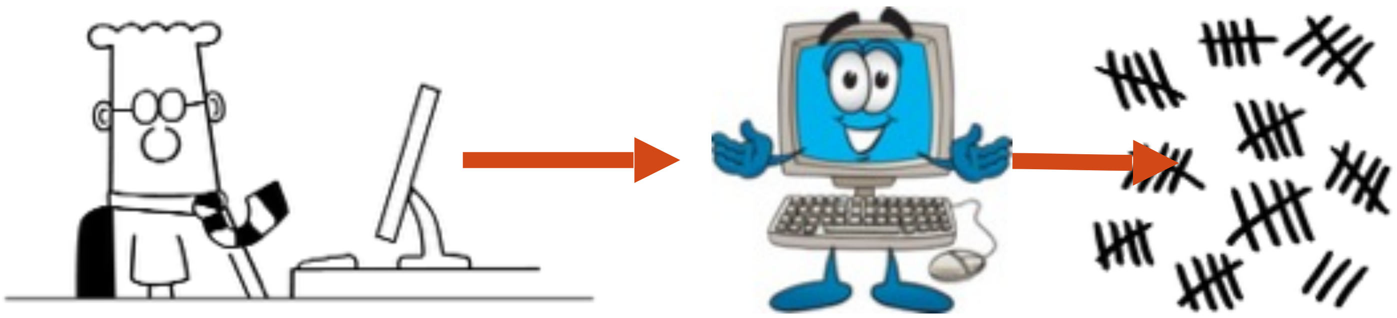

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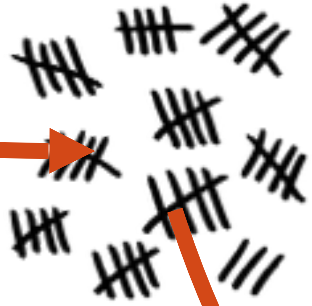

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It is not voters who counts,
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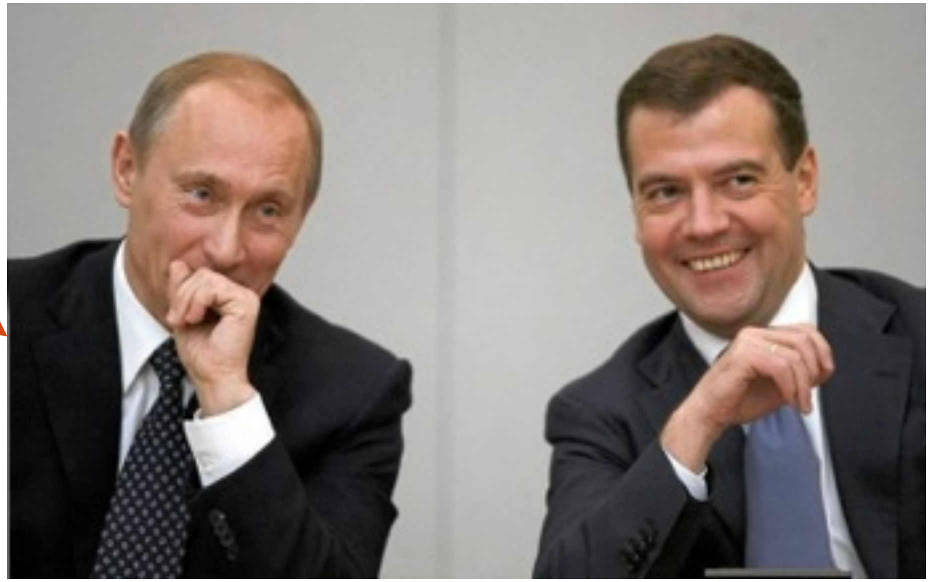

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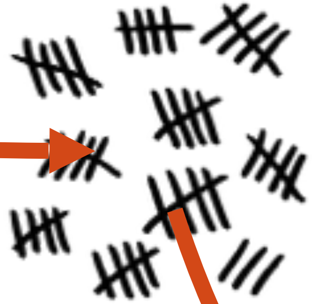

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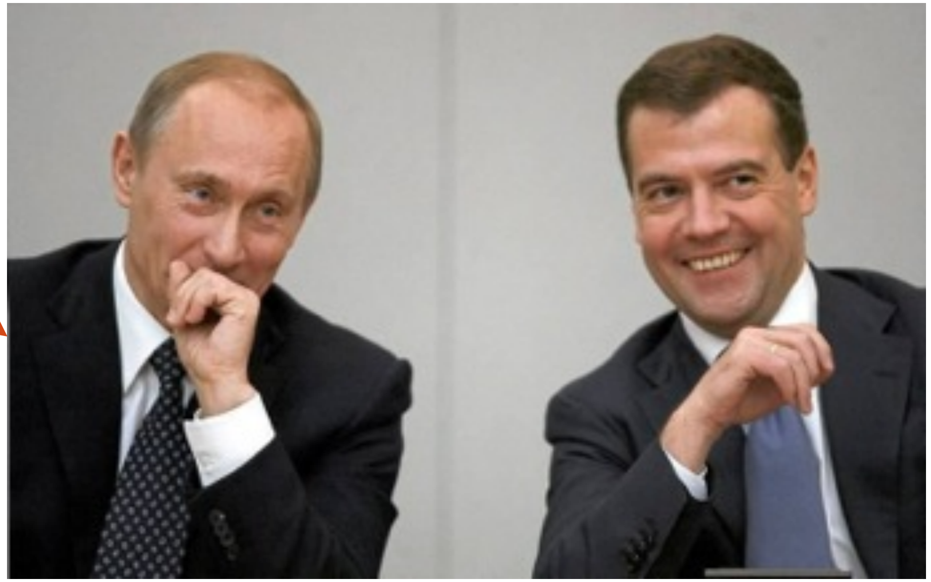

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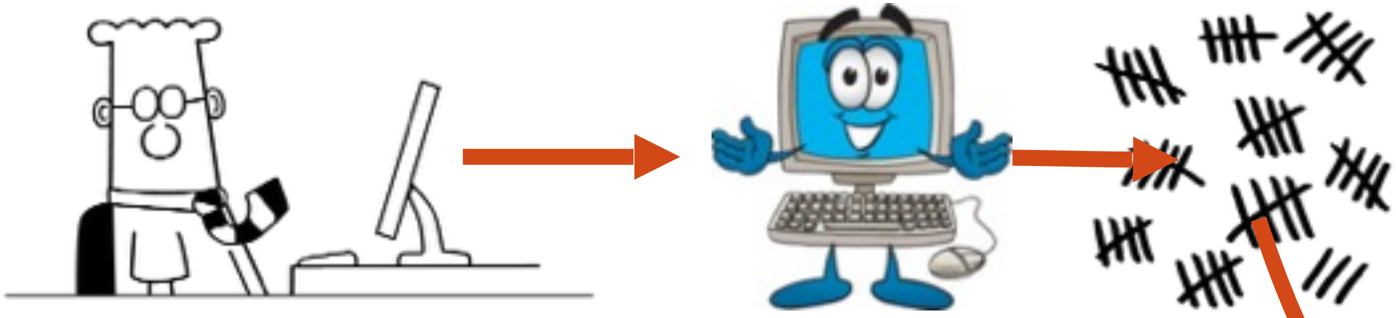

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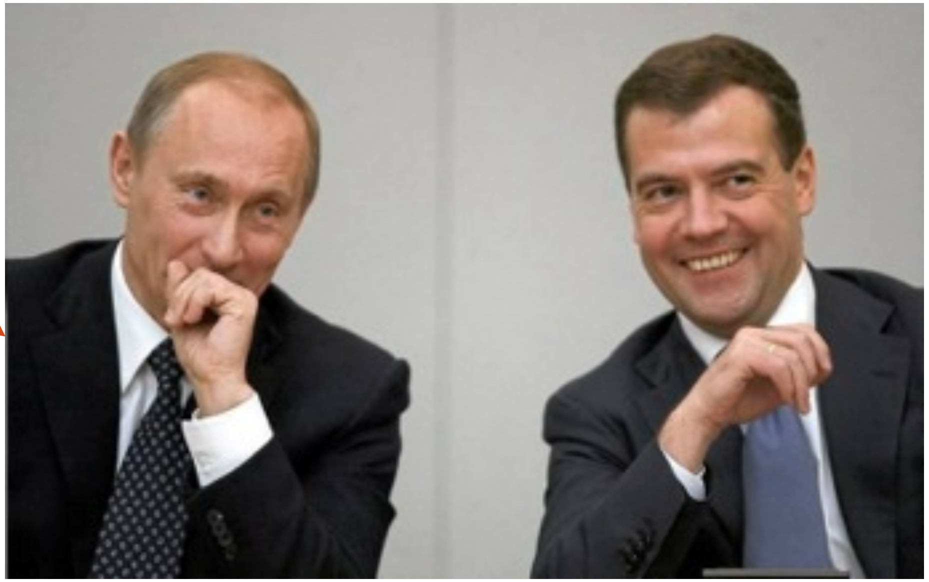

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Data is public
(Data, source) is private

SIMPLE MIX-NETS

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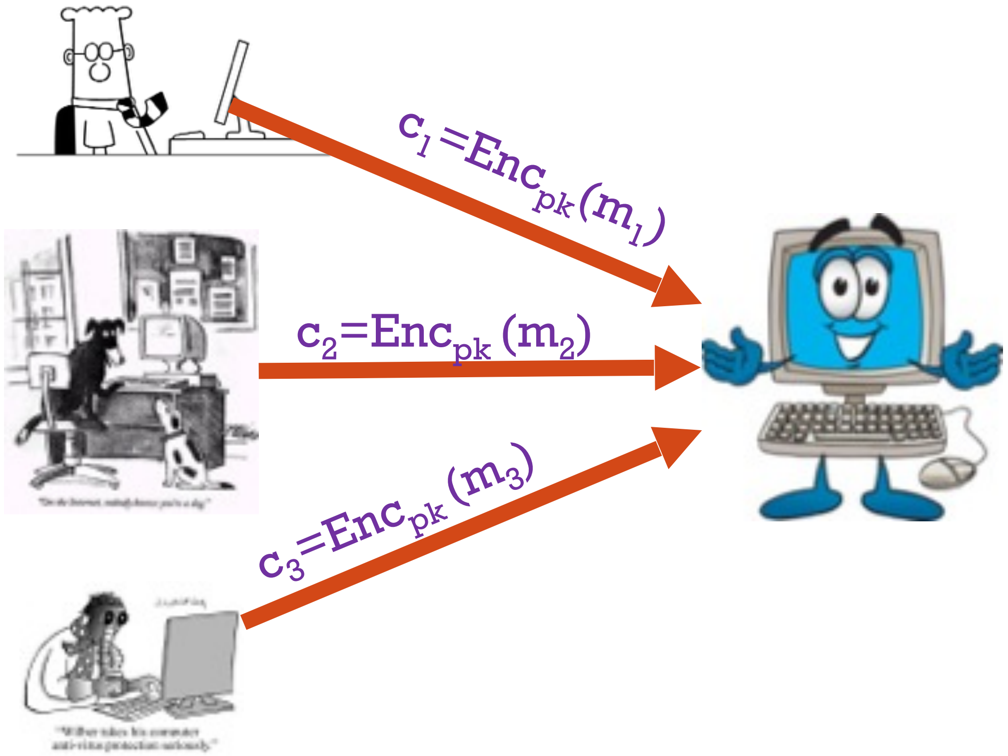

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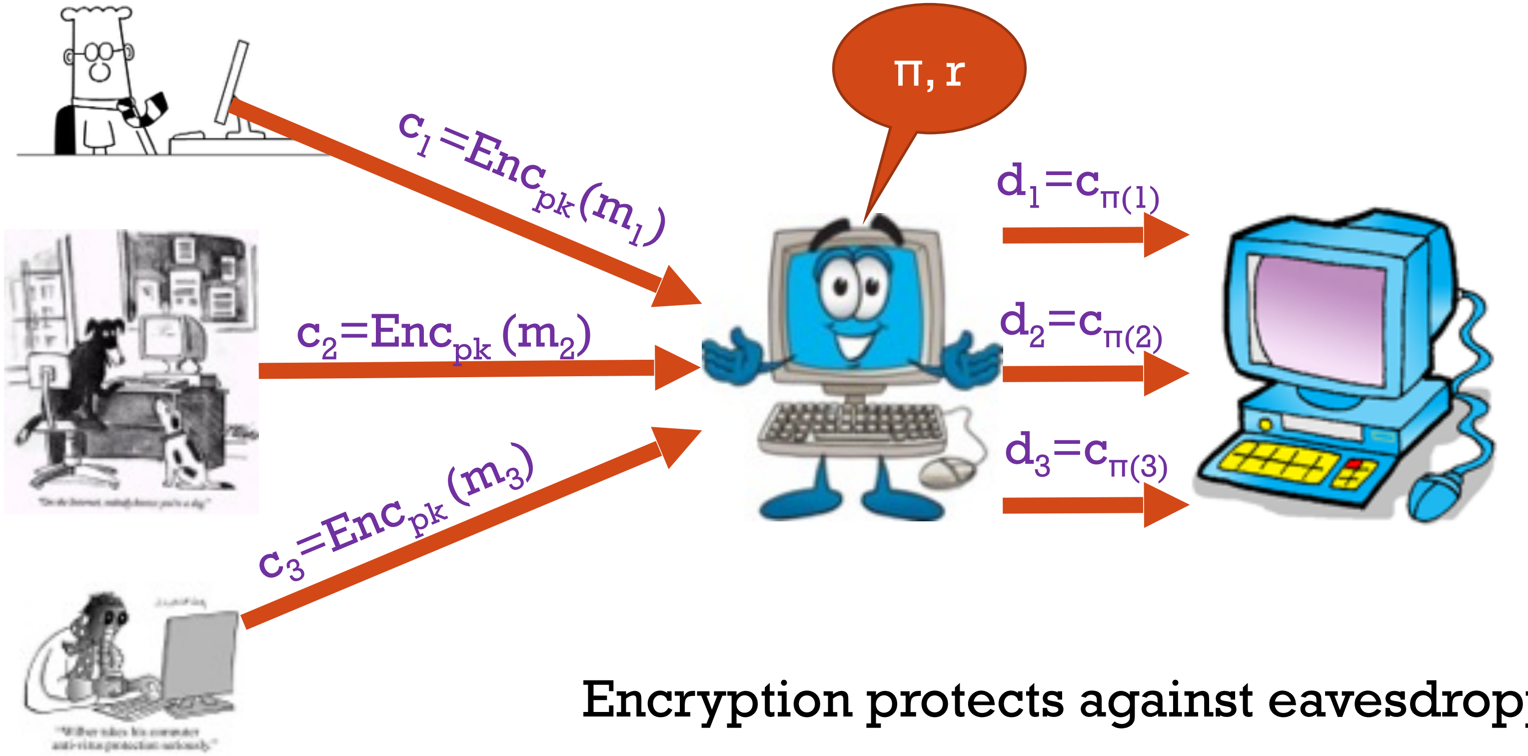

Encryption protects against eavesdropping on the Internet

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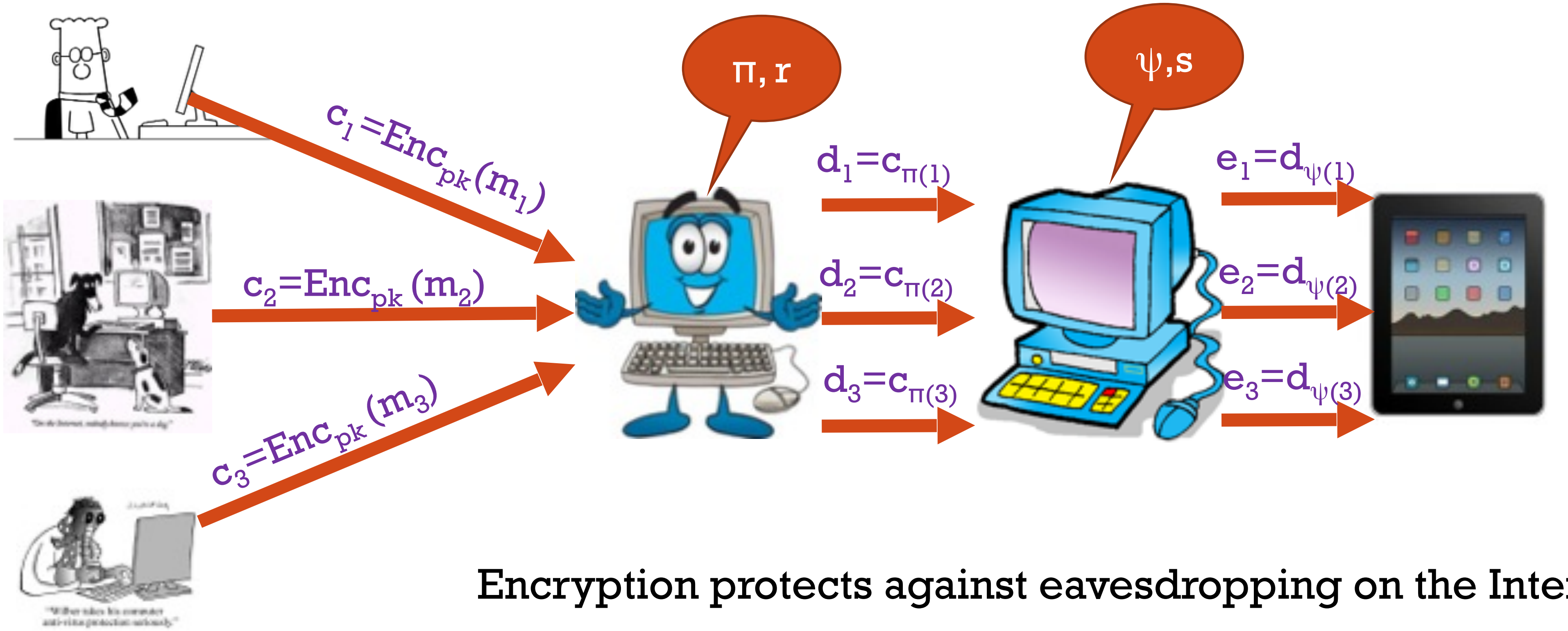

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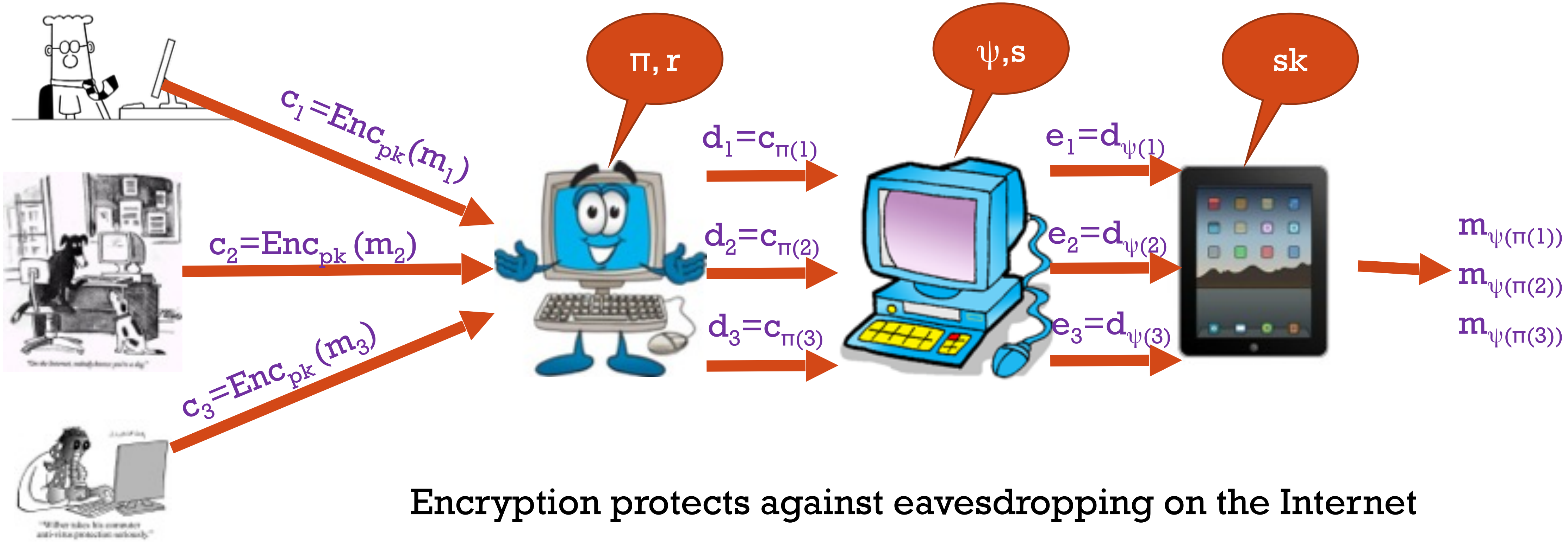

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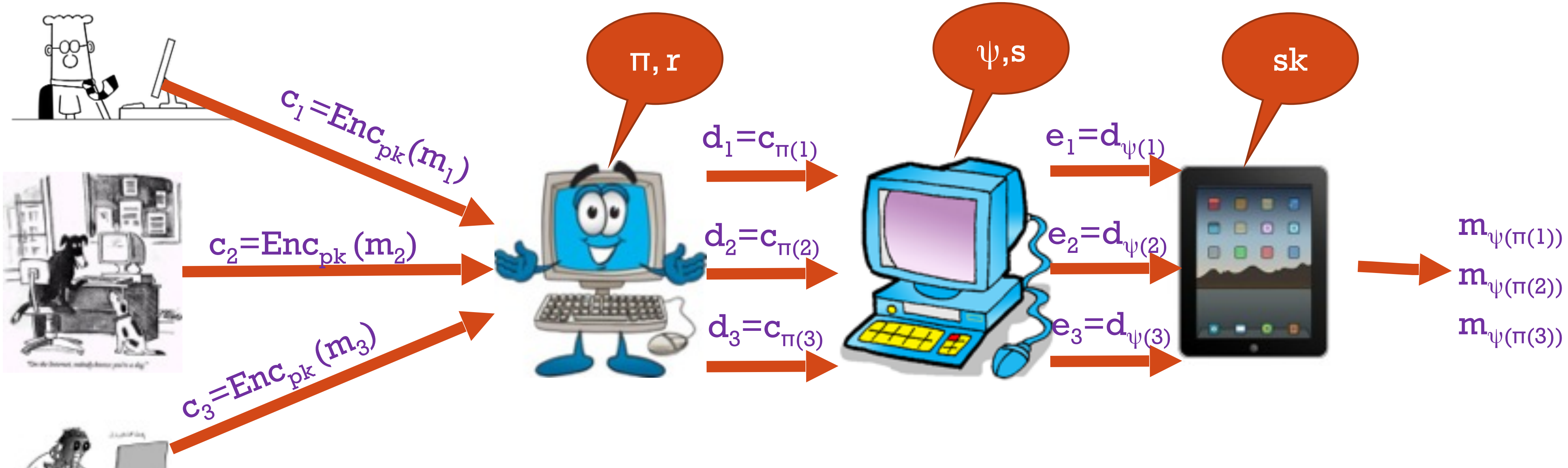

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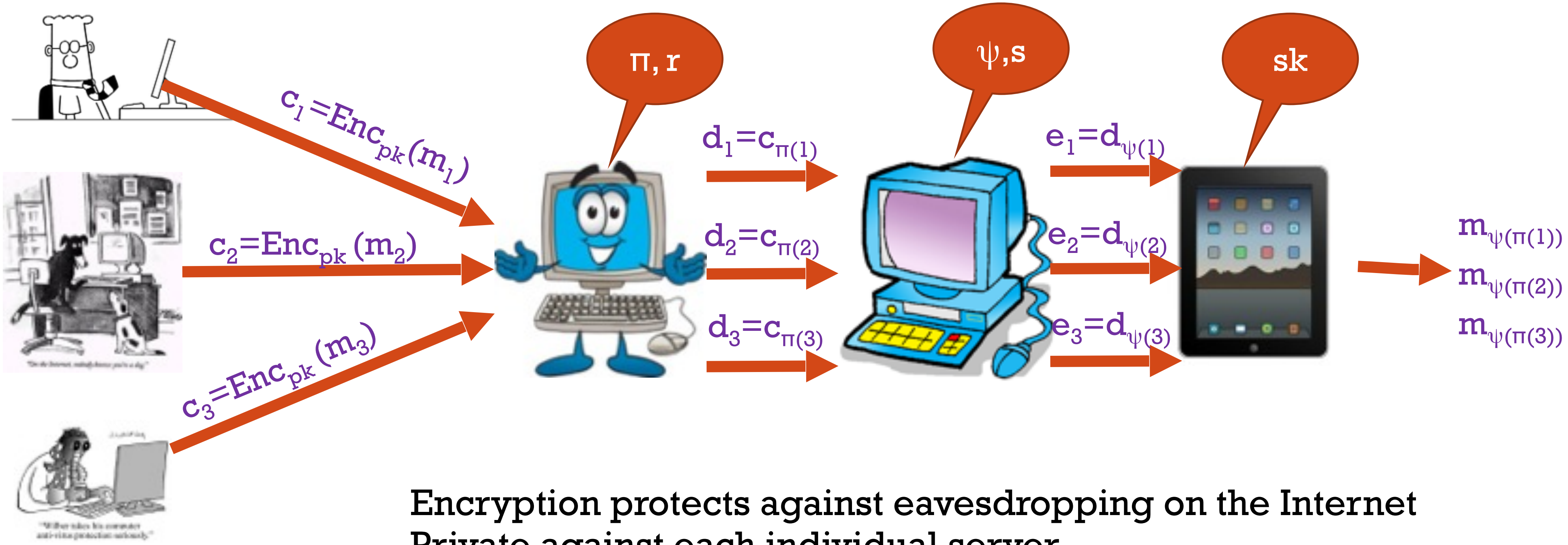

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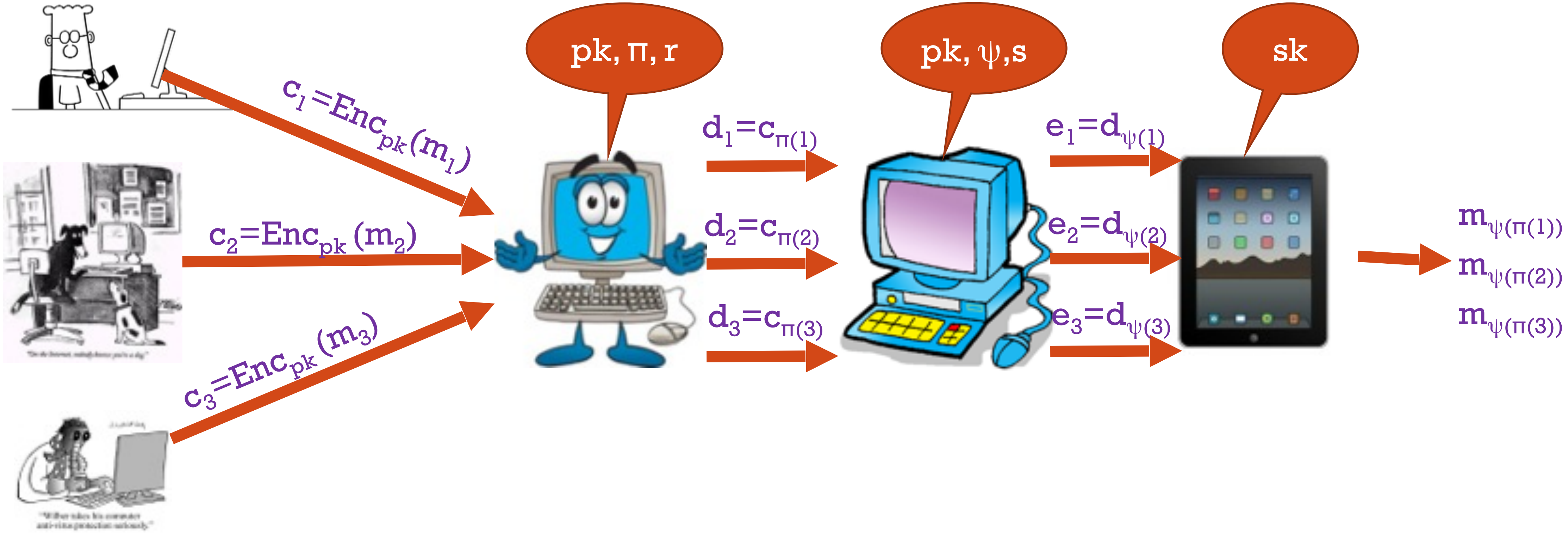
Not enough: what if a server cheats?

ACCOUNTABLE MIX-NETS



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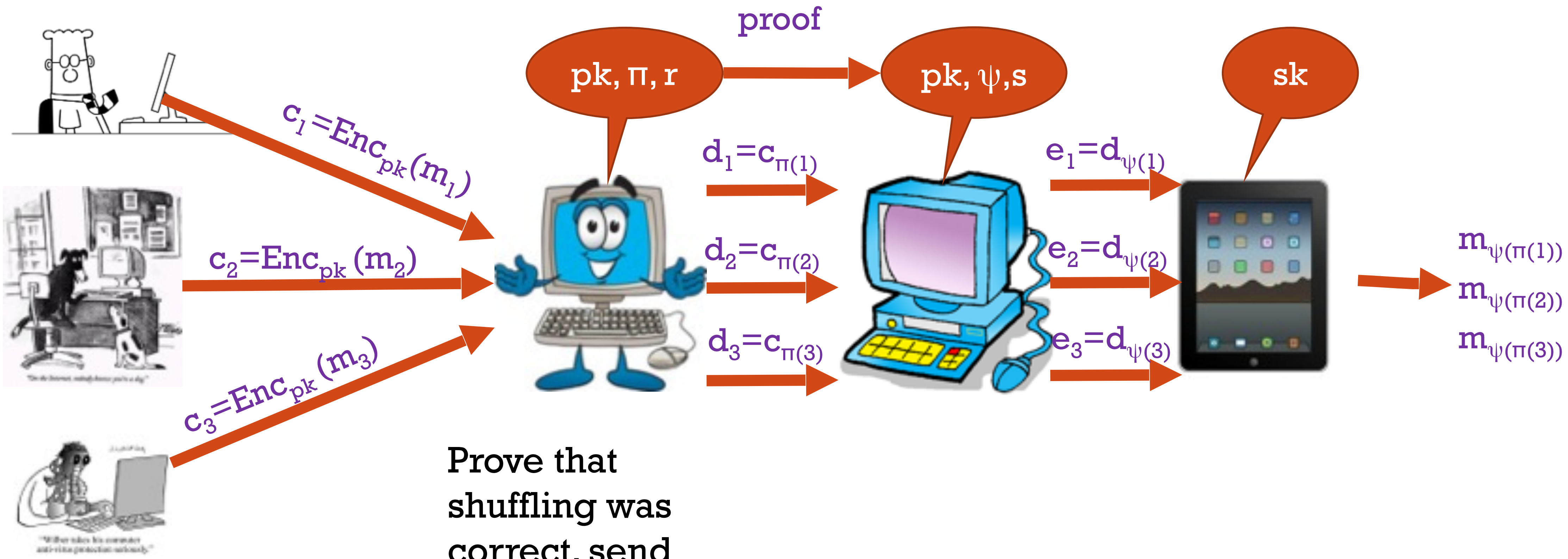

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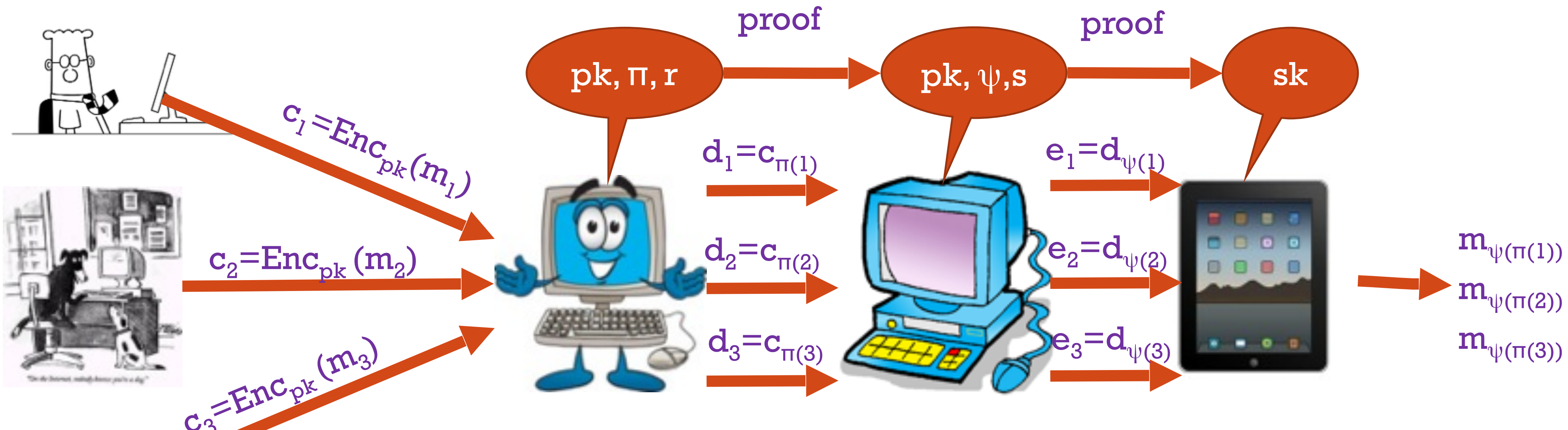

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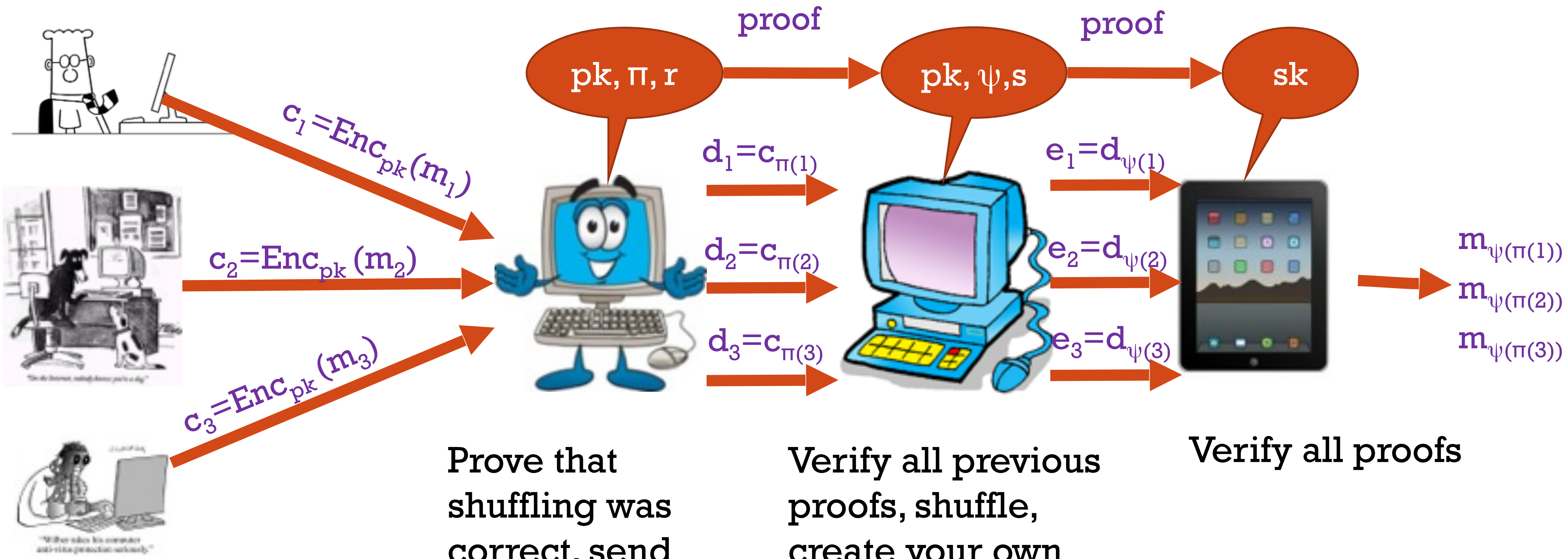

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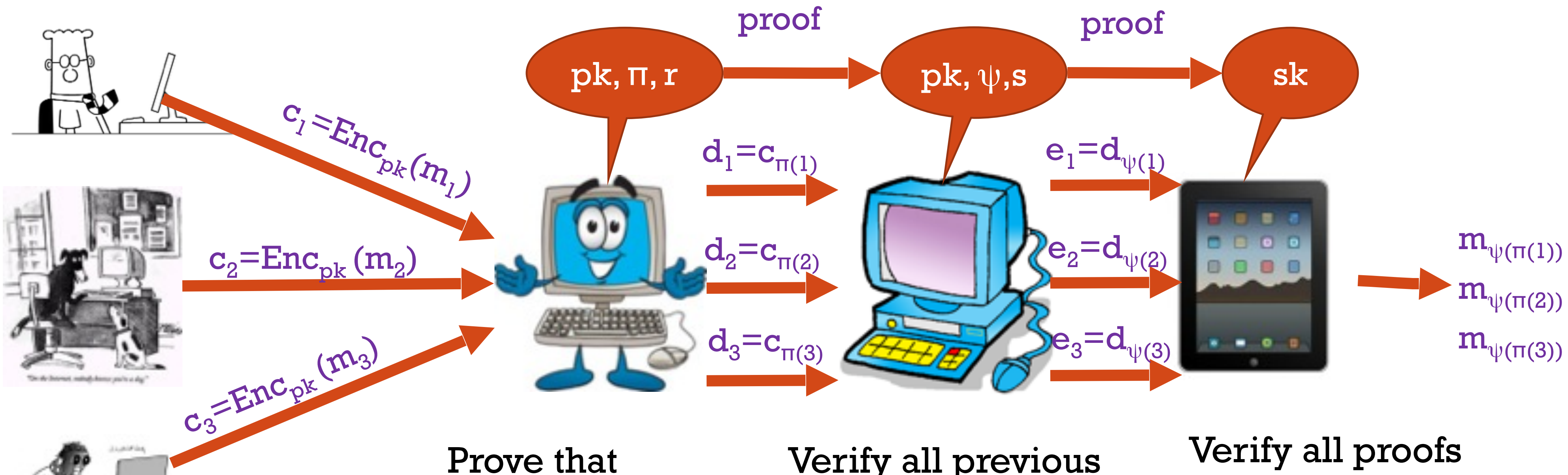

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    Mix-server permutes ciphertexts, re-encrypt them and provides a proof that he has done it correctly.
- Existing CRS model arguments not very efficient

CRS-BASED SHUFFLE ARGUMENTS

Assumption proposed in that paper, proof in GBGM

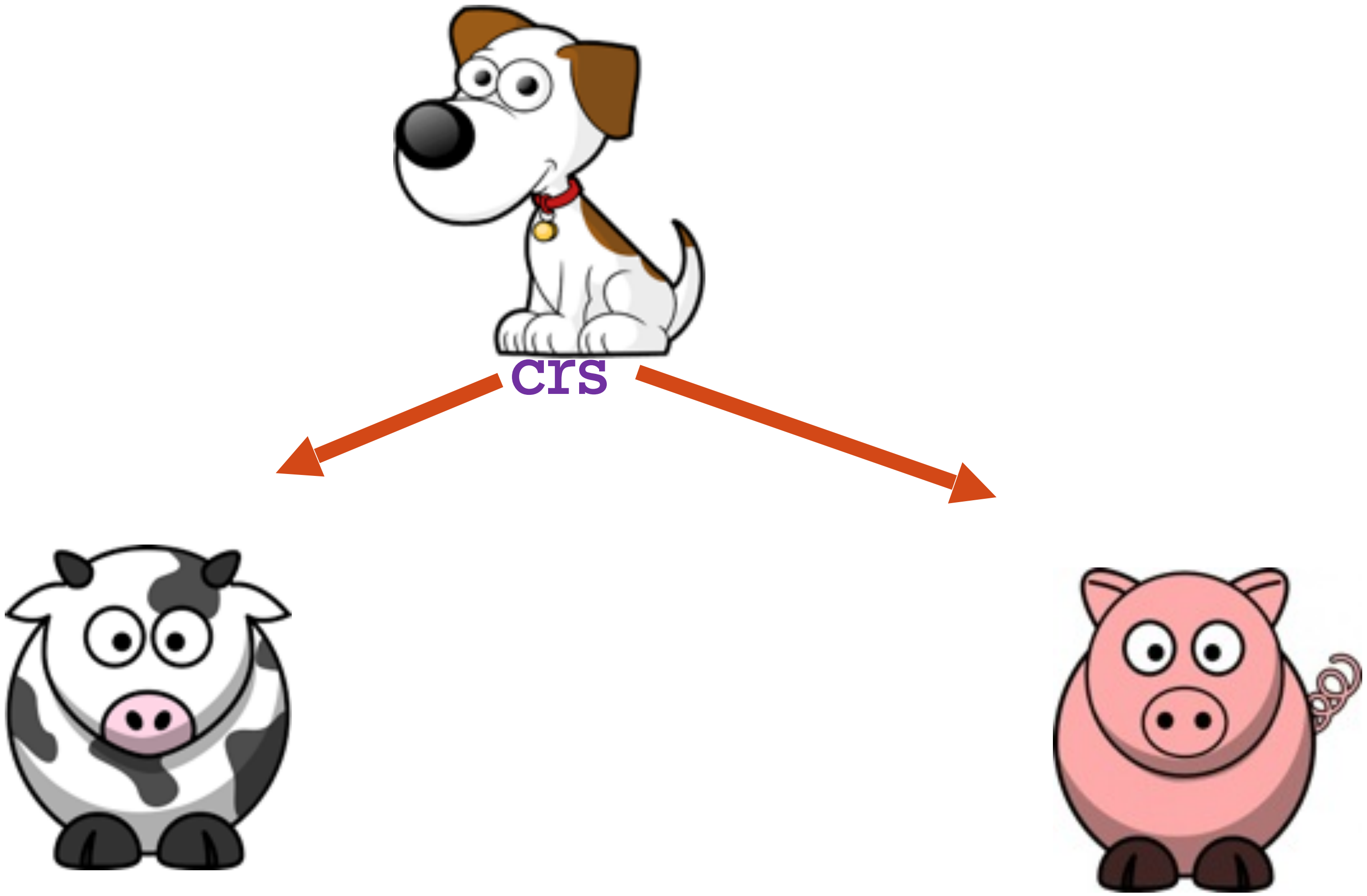
Assmpt. proposed 2010+, but not in that paper, proof in GBGM

	Lipmaa-Zhang (2012)	Fauzi-Lipmaa (2016)	This paper
CRS length	*7n + 6*	*8n + 17*	*3n + 14*
Communic.	*12n + 11*	*9n + 2*	*7n + 3*
P comp. (units)	*36*	*19.8*	*24.3*
V comp. (units)	*196*	*126*	*36.3*
GBGM?	*PSDL, DLIN (comp.)*	*TSDH, PCDH, PSP (comp.)*	*Pure GBGM*
Soundness	*Full*	*Culpable*	*Full*

n: number of ciphertexts (say 100,000)

1 unit = *n* million machine cycles
According to speed records on BN curves

ZERO KNOWLEDGE: CRS MODEL



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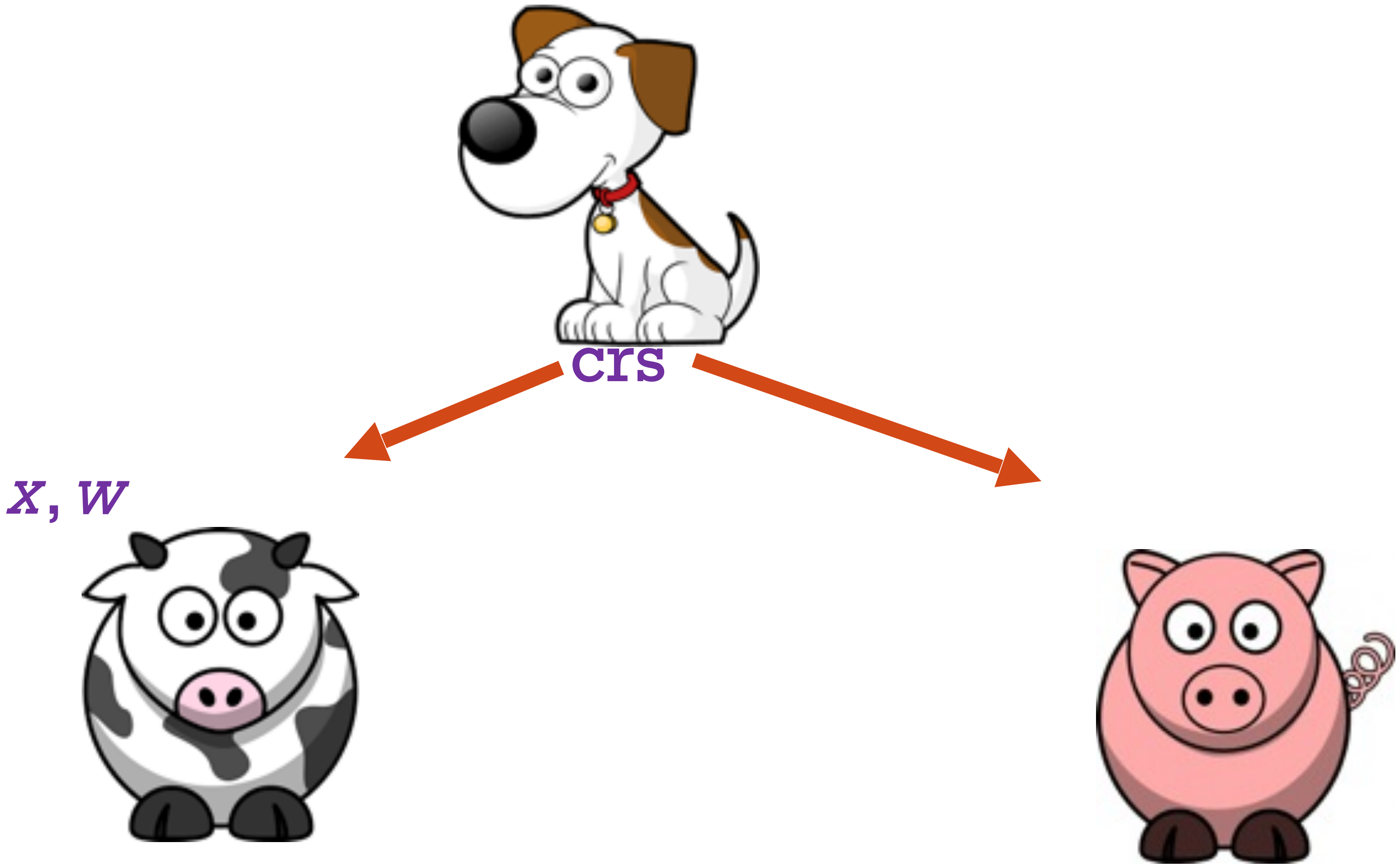

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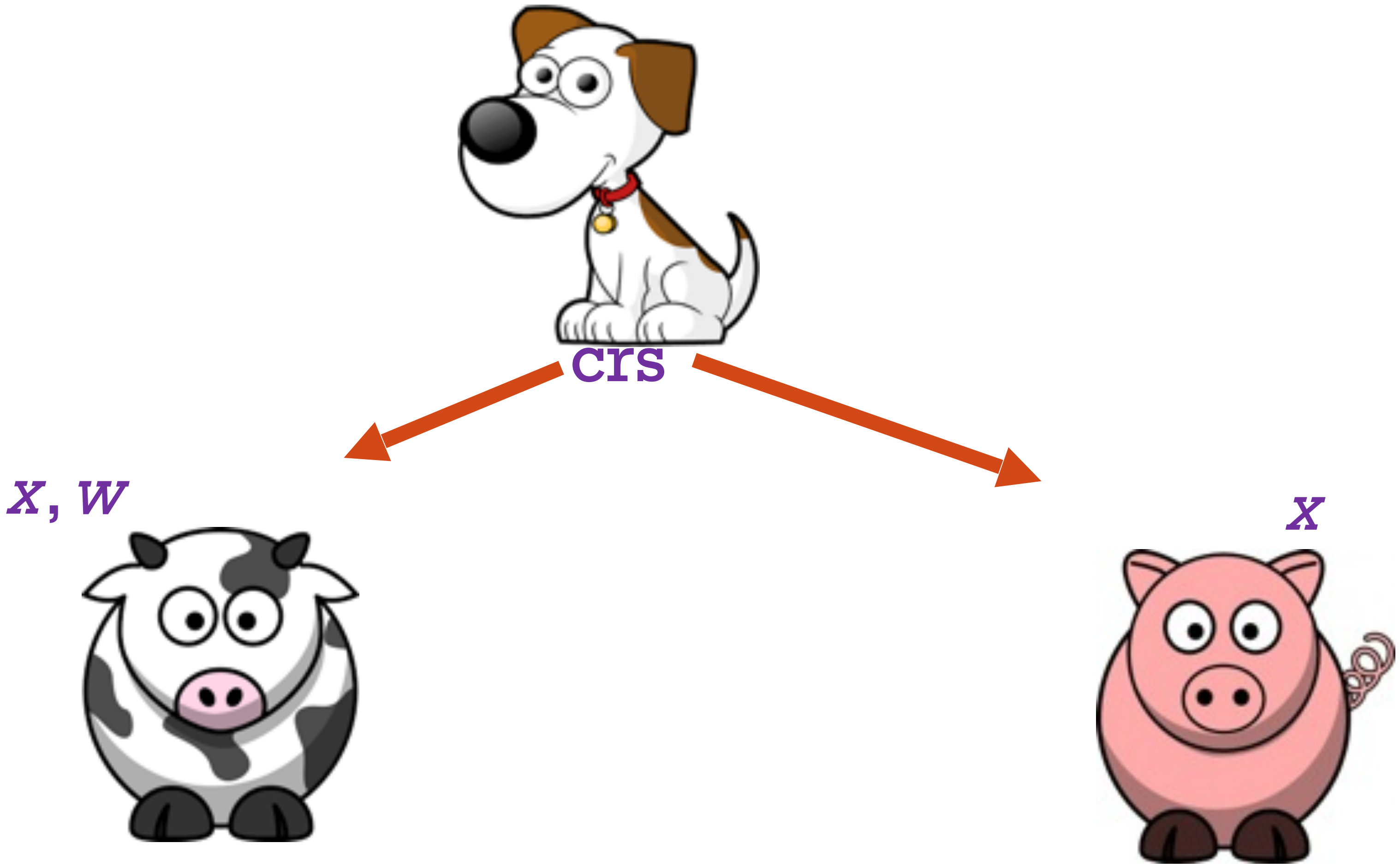

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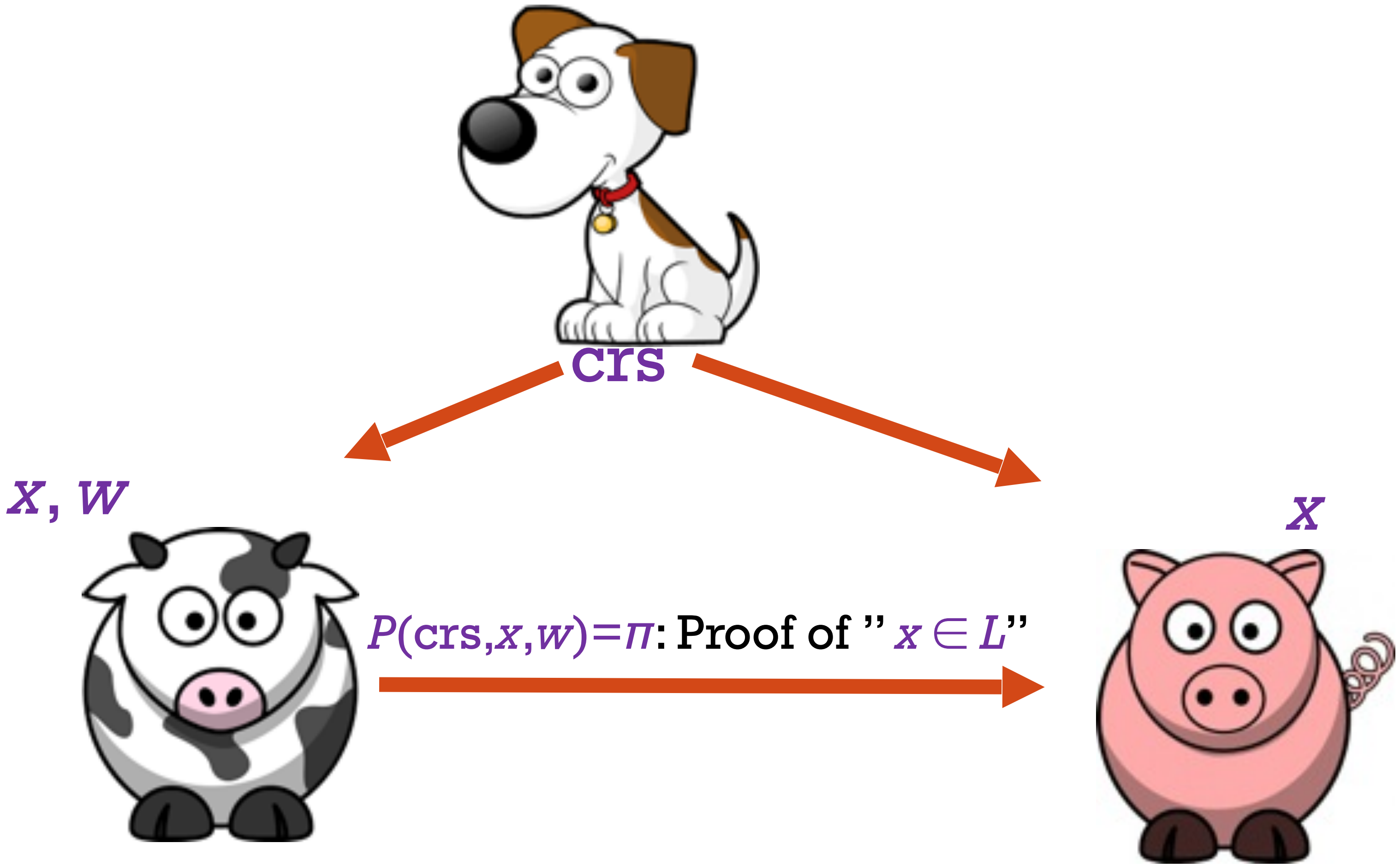

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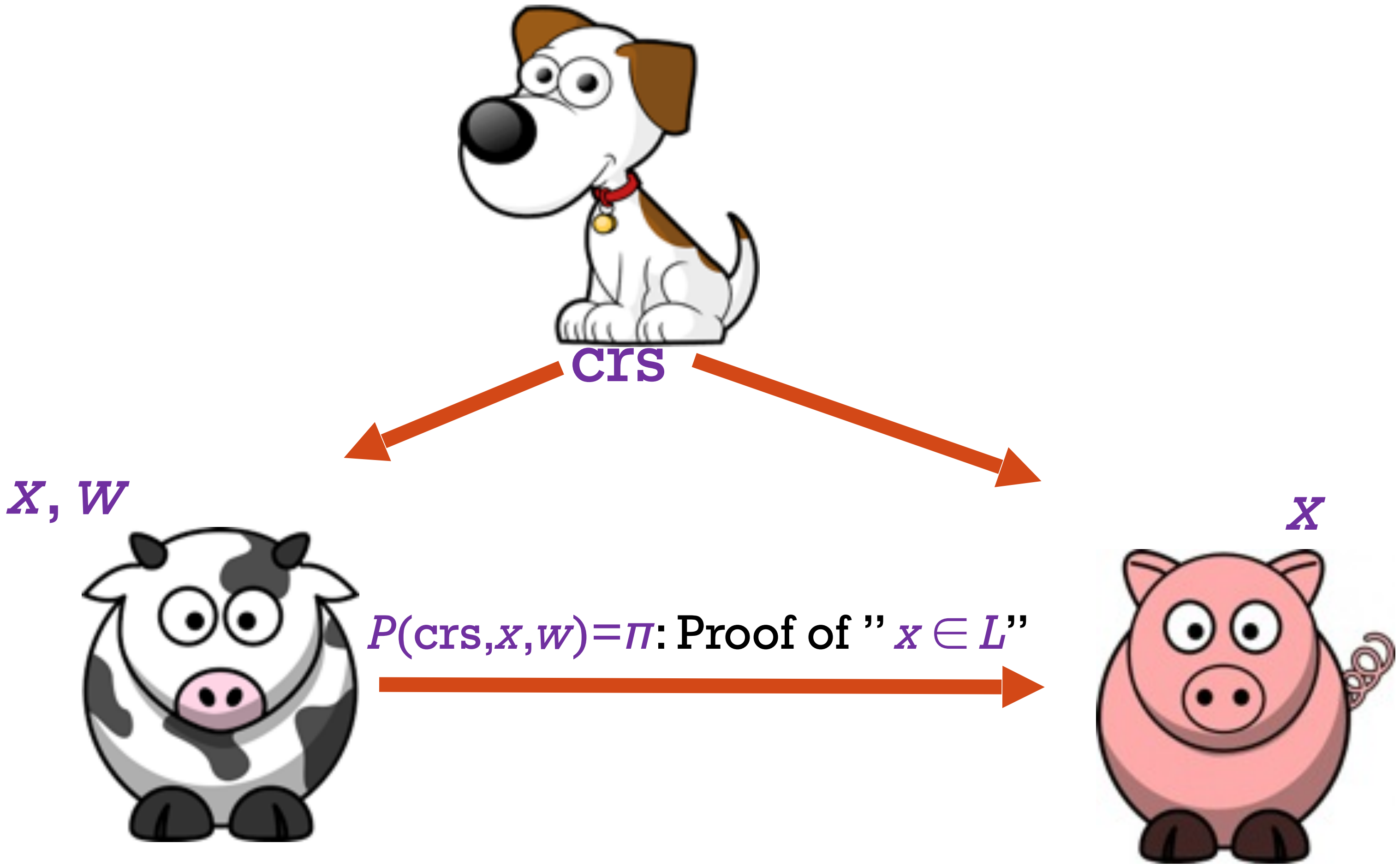

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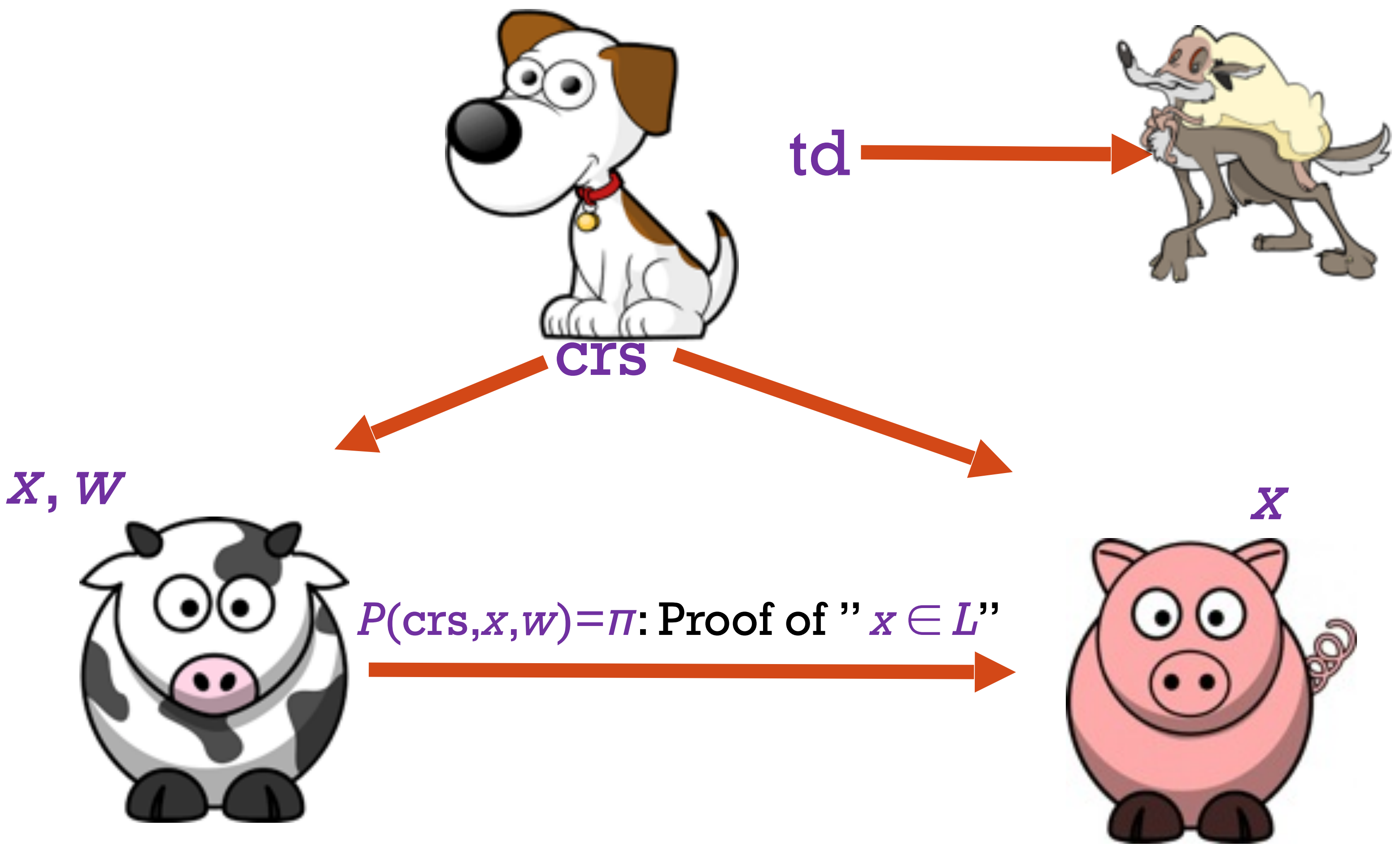

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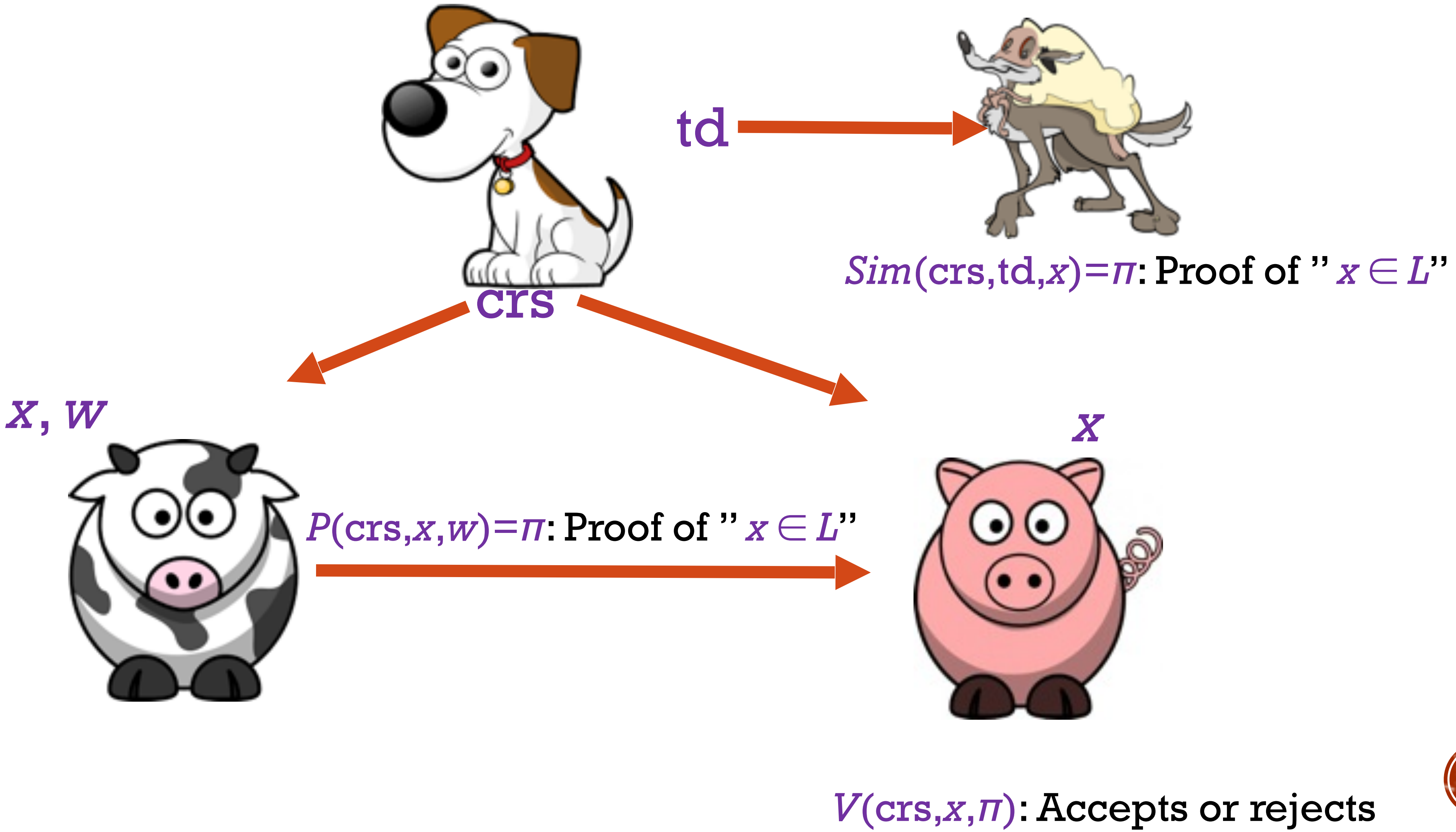

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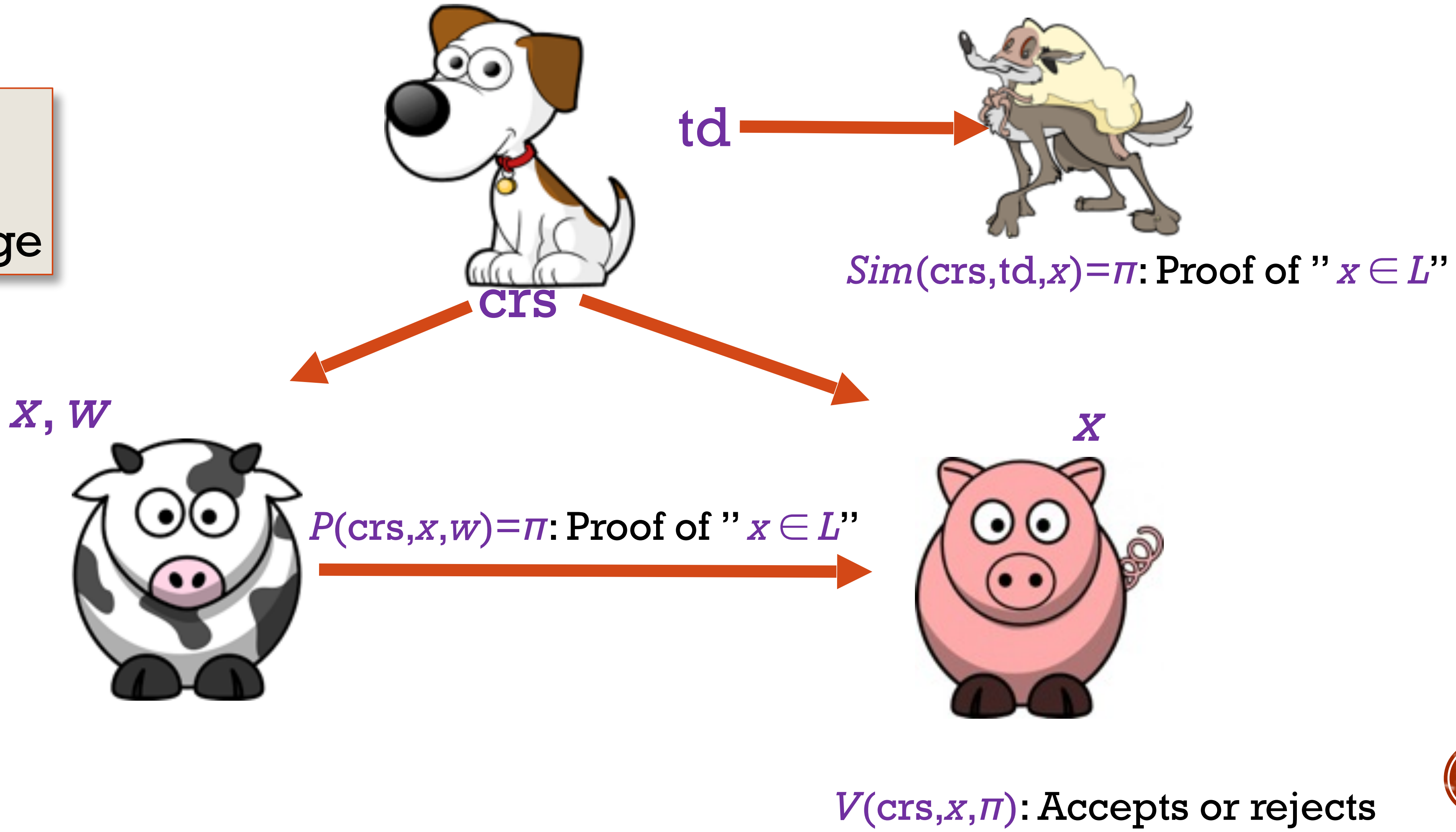
Correctness
Soundness
Zero knowledge



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- **Bilinear map:** $e: \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$
- Requirements:
  - Efficiently computable
  - Non-degeneracy: $e\,(g_1, g_2) \neq 1$
  - Bilinearity: $e\,(g_1{}^a, g_2{}^b) = e\,(g_1, g_2)^{ab}$

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  - Analogous to DL: given g^a, compute a
- What else should be hard?

NON-GENERIC APPROACH

Protocol

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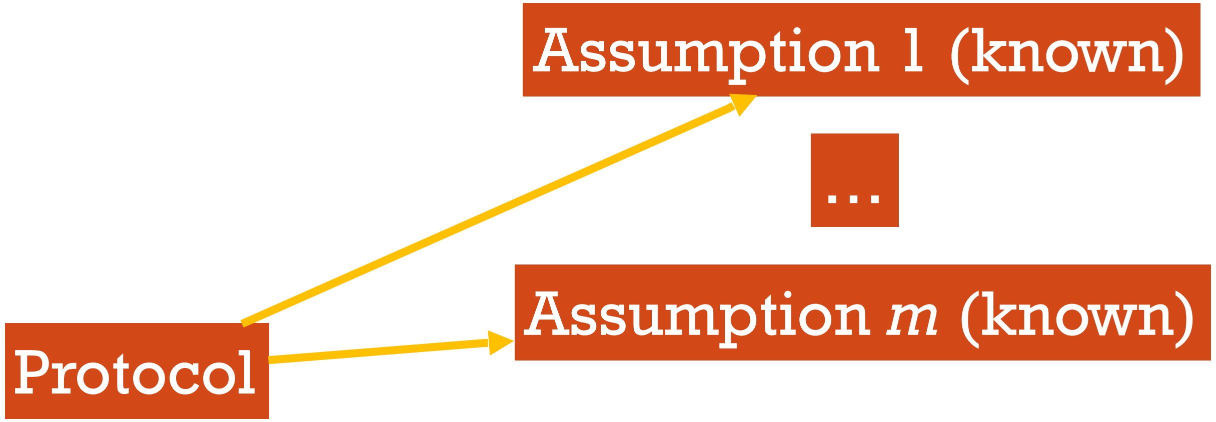

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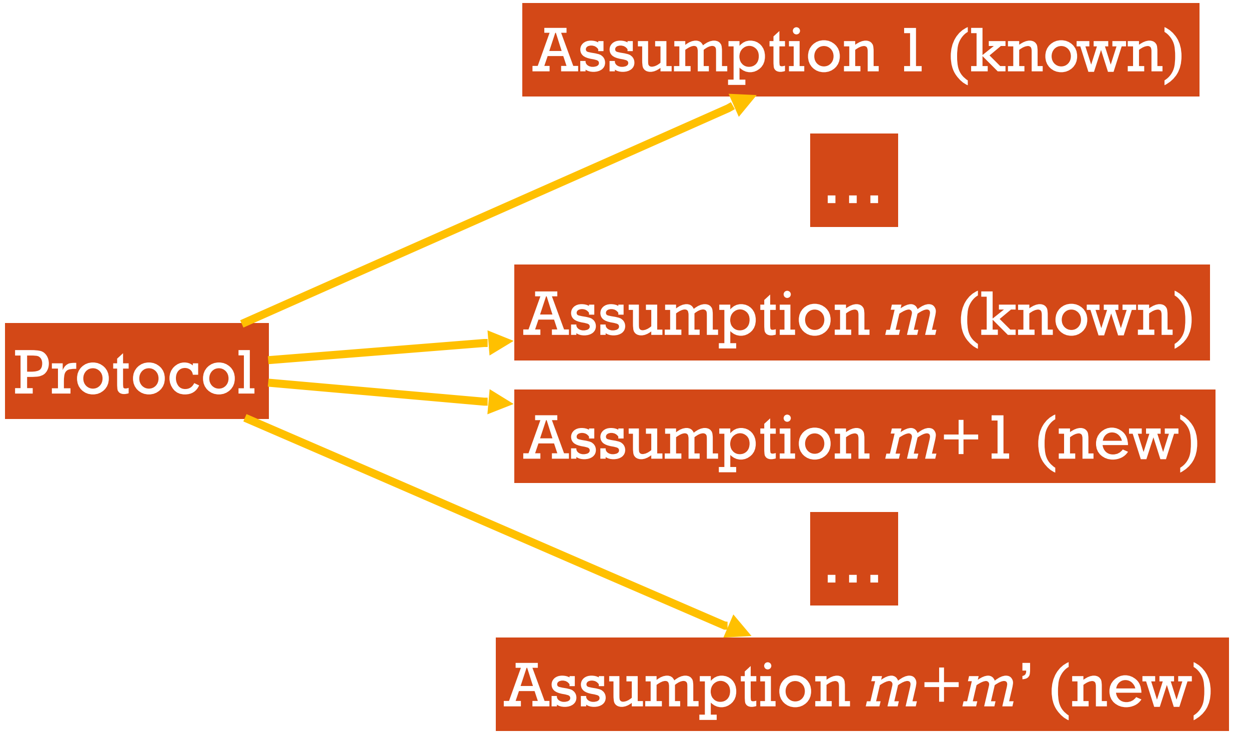

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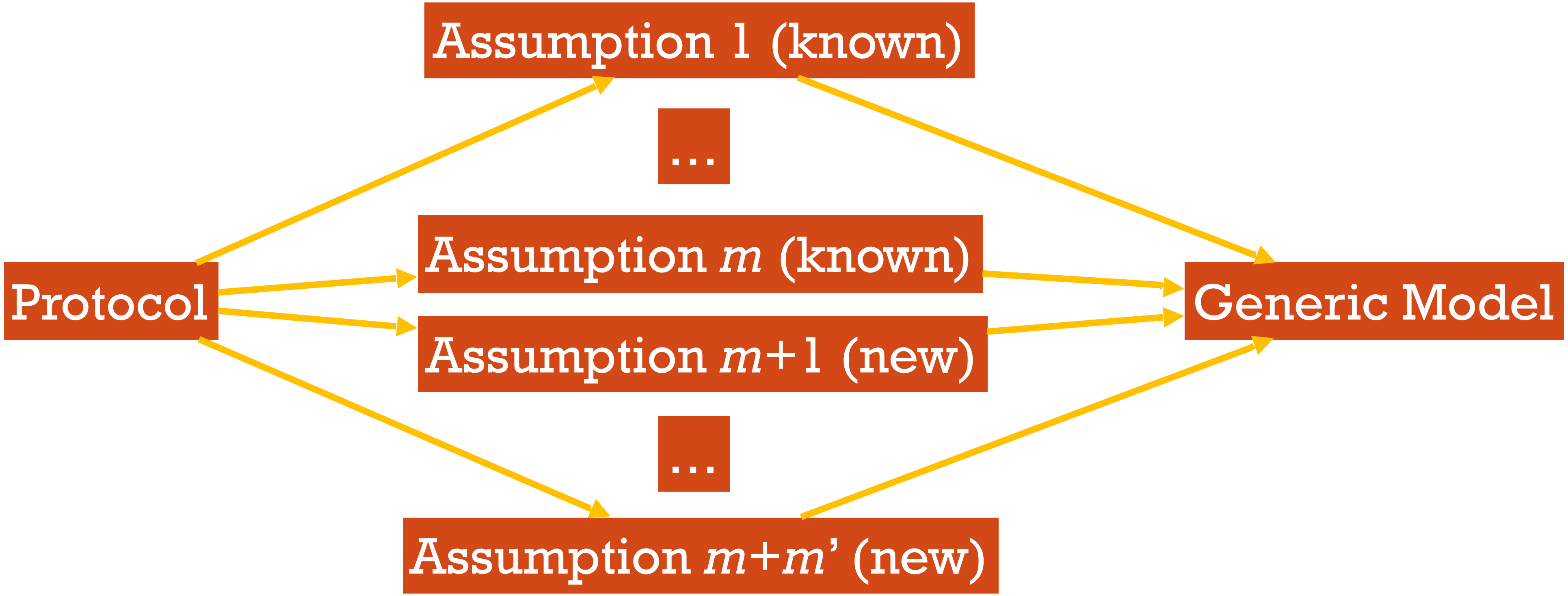

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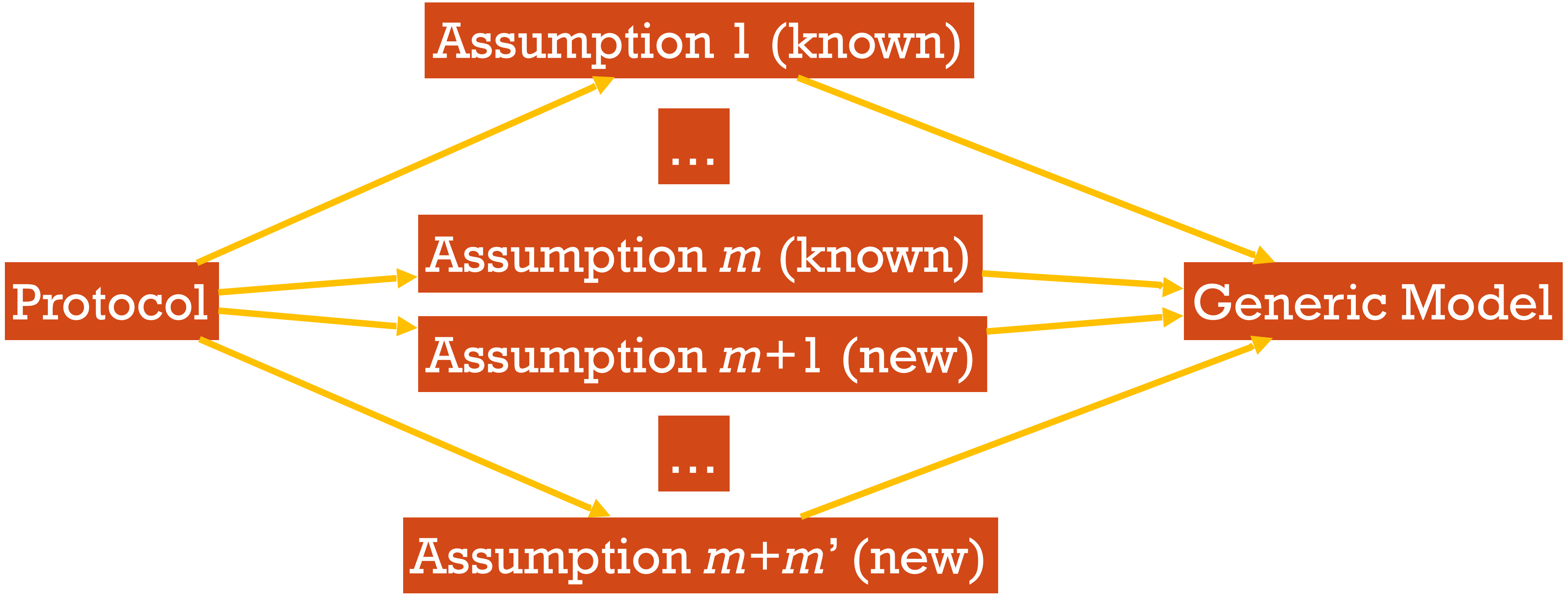

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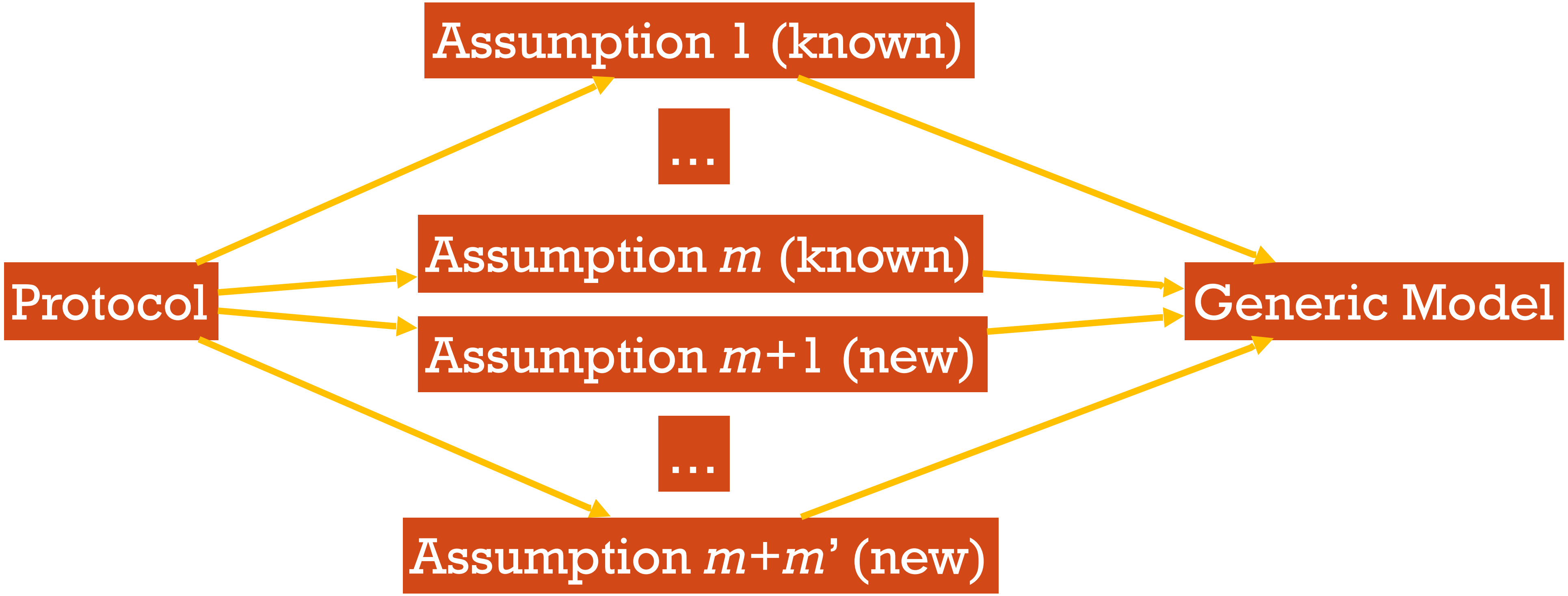

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Con: each arrow might mean a loss in efficiency

GENERIC MODEL APPROACH

Protocol $\longrightarrow$ Generic Model

Con: proof in GGM is only for restricted adversaries

Pro: only one arrow, thus smaller loss in efficiency

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- Note: we do not handle G_T as a generic group

SOUNDNESS IN GBGM

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X_1

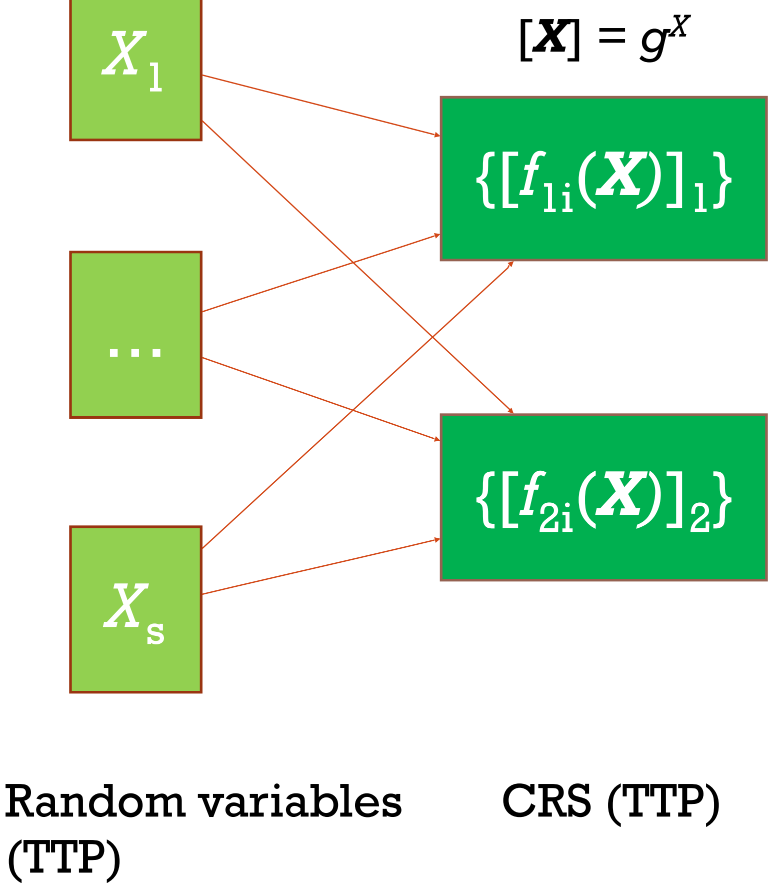
…

X_s

Random variables
(TTP)

SOUNDNESS IN GBGM

Polynomials
(TTP knows $\boldsymbol{X}$)



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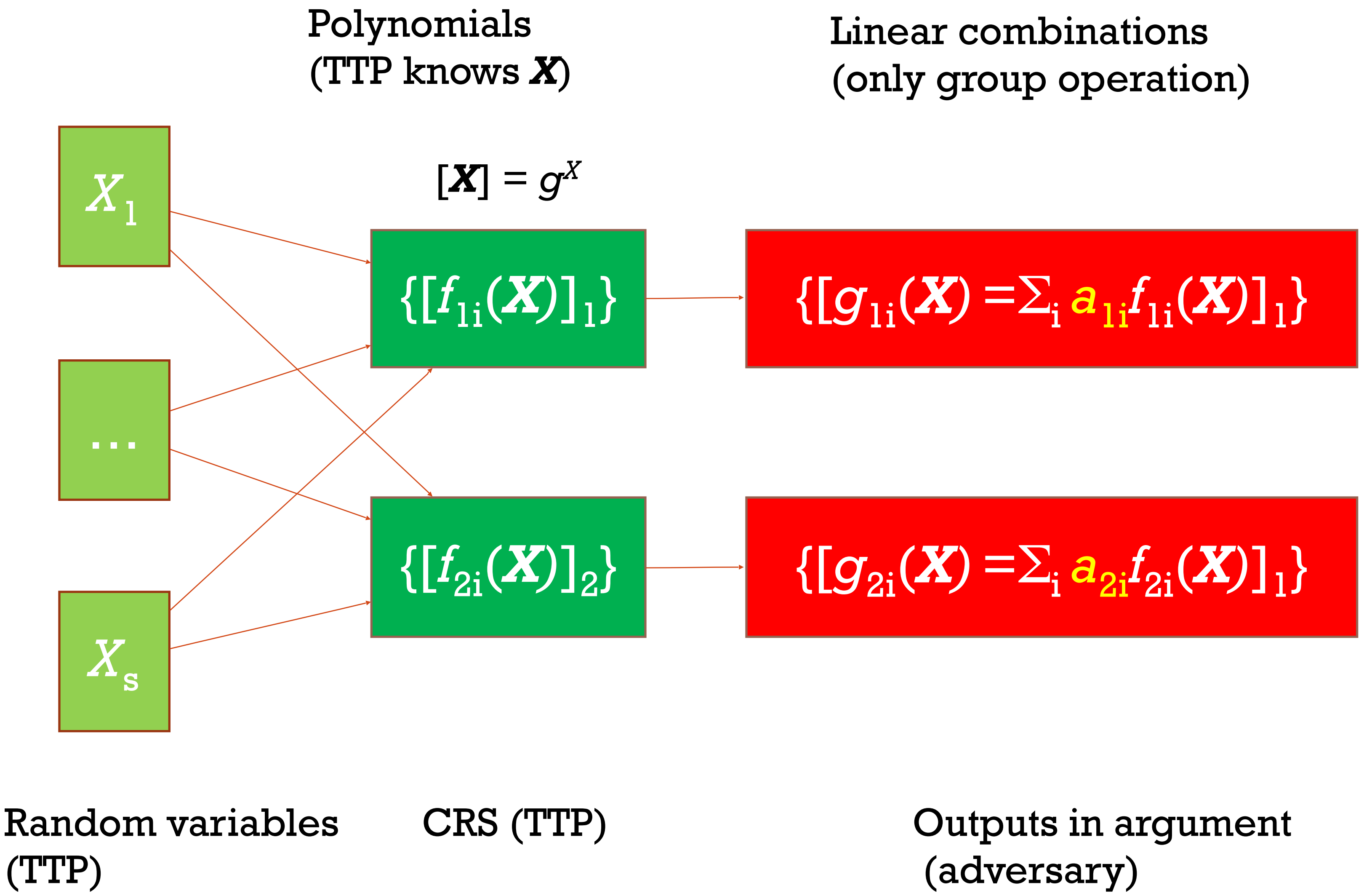

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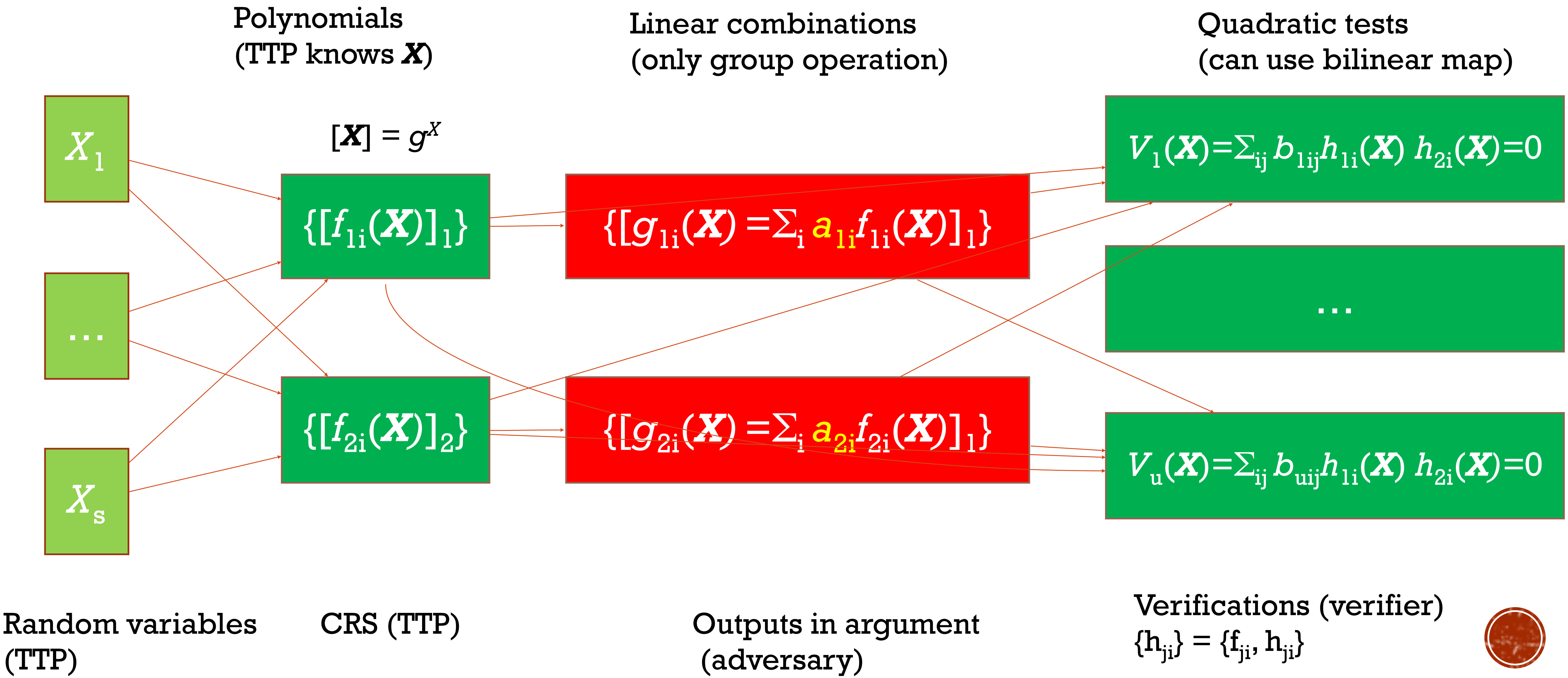

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- Show that solution's coefficients are "nice"
  - = restricted to be as in the honest case

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  - Compose CRS-s of individual subarguments together, getting one big CRS

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- **Soundness check:**
  - Is the composed protocol sound?
    - Subarguments get extra inputs in CRS
  - If not: introduce new random variables that guarantee CRS elements are used in only correct subarguments, reiterate

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- **Argument:** // "square span programs"

$$[\pi(\boldsymbol{X})]_1 = [((a_I P_I\,(X) + P_0\,(X) + rX_\varrho)^2 - 1) \,/\, X_\varrho]_1$$

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- **Argument:** // ”square span programs”

$$[\pi(\boldsymbol{X})]_1 = [((a_I P_I\,(X) + P_0\,(X) + rX_\varrho)^2 - 1) \,/\, X_\varrho]_1$$

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- Goal: find coefficients s.t. verification equation is satisfied

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- This is a system of polynomial equations
  - … and a nasty one
  - of more than 20 polynomial equations

$\{i_1,\ldots,i_4\}$	$\mathrm{coeff}_{\mu(\boldsymbol{i})}(\mathcal{V}_{1sp}(\boldsymbol{X})\cdot\mathcal{V}^*_{1sp}(\boldsymbol{X}))$
$\{1,2,1,0\}$	$-A_\varrho(B_\alpha+1)+(A_\alpha+1)B_\varrho-C_\alpha$
$\{1,2,0,1\}$	$-A_\gamma(B_\alpha+1)$
$\{1,2,0,0\}$	$-A_{\varrho\beta}(B_\alpha+1)$
$\{1,1,2,0\}$	$(A_\alpha+1)B_\beta$
$\{1,1,1,1\}$	$(A_\alpha+1)B_\gamma$
$\{1,1,1,0\}$	$-a(X)(B_\alpha+1)+(A_\alpha+1)\,(b(X)+B_1)-A_0(B_\alpha+1)P_0(X)$
$\{1,0,1,0\}$	$-(B_\alpha+1)Z(X)a^\dagger(X)$
$\{0,3,1,0\}$	$A_\varrho B_\varrho-C_\varrho$
$\{0,3,0,1\}$	$A_\gamma B_\varrho-C_\gamma$
$\{0,3,0,0\}$	$A_{\varrho\beta}B_\varrho-C_{\varrho\beta}$
$\{0,2,2,0\}$	$A_\varrho B_\beta$
$\{0,2,1,1\}$	$A_\gamma B_\beta+A_\varrho B_\gamma$
$\{0,2,1,0\}$	$a(X)B_\varrho+A_\varrho\,(b(X)+B_1)+A_{\varrho\beta}B_\beta+$ $P_0(X)\,(A_\varrho(B_\alpha+1)+(A_\alpha+A_0+1)\,B_\varrho-C_\alpha-C_0)-c(X)$
$\{0,2,0,2\}$	$A_\gamma B_\gamma$
$\{0,2,0,1\}$	$A_\gamma\,(b(X)+B_1)+A_{\varrho\beta}B_\gamma+A_\gamma(B_\alpha+1)P_0(X)$
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$\{0,1,2,0\}$	$a(X)B_\beta+(A_\alpha+A_0+1)\,B_\beta P_0(X)$
$\{0,1,1,1\}$	$a(X)B_\gamma+(A_\alpha+A_0+1)\,B_\gamma P_0(X)$
$\{0,1,1,0\}$	$-Z(X)c^\dagger(X)+P_0(X)\,(a(X)(B_\alpha+1)+(A_\alpha+A_0+1)\,(b(X)+B_1))+$ $a(X)\,(b(X)+B_1)+(A_\alpha+A_0+1)\,(B_\alpha+1)P_0(X)^2-1+$ $B_\varrho Z(X)a^\dagger(X)$
$\{0,0,2,0\}$	$B_\beta Z(X)a^\dagger(X)$
$\{0,0,1,1\}$	$B_\gamma Z(X)a^\dagger(X)$
$\{0,0,1,0\}$	$Z(X)\,(b(X)+B_1)\,a^\dagger(X)+(B_\alpha+1)P_0(X)Z(X)a^\dagger(X)$

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- Obtain that $A_i(X) = a_I P_I(X)$ => Sound

THANK YOU!

Panoramix