Adaptive Oblivious Transfer And Generalization

Olivier Blazy, Céline Chevalier, **Paul Germouty**

December 5, 2016

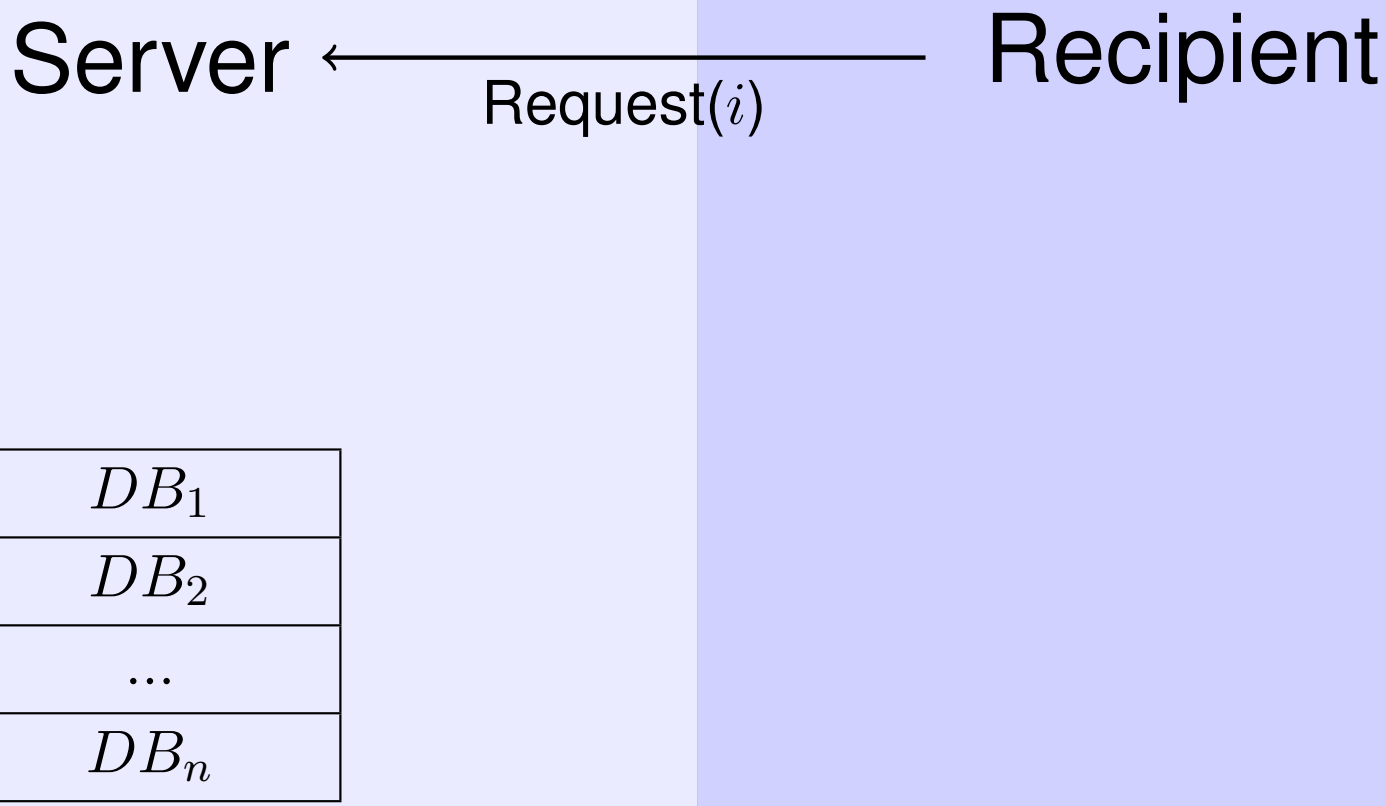
Oblivious Transfer

Oblivious Transfer

Server

DB_1
DB_2
...
DB_n

Oblivious Transfer



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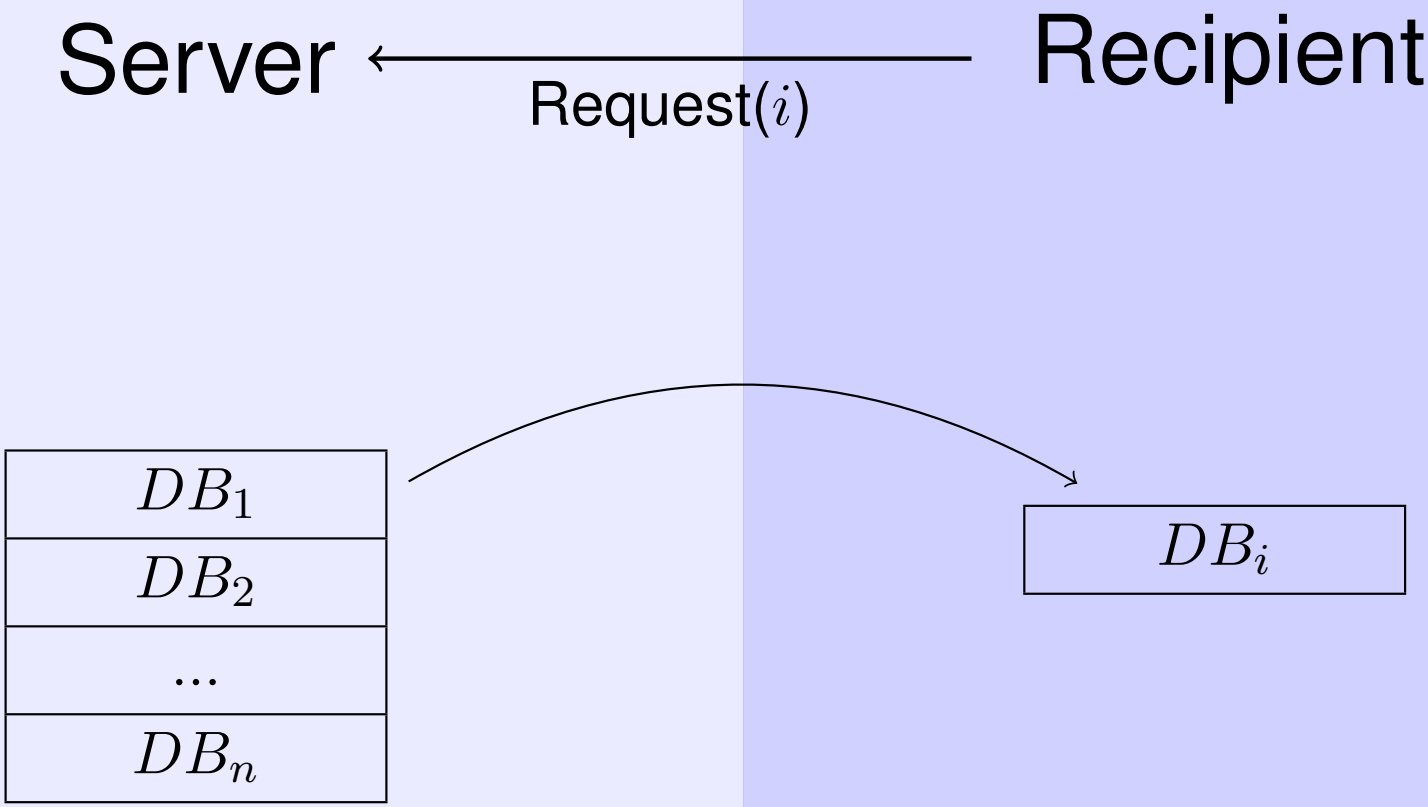

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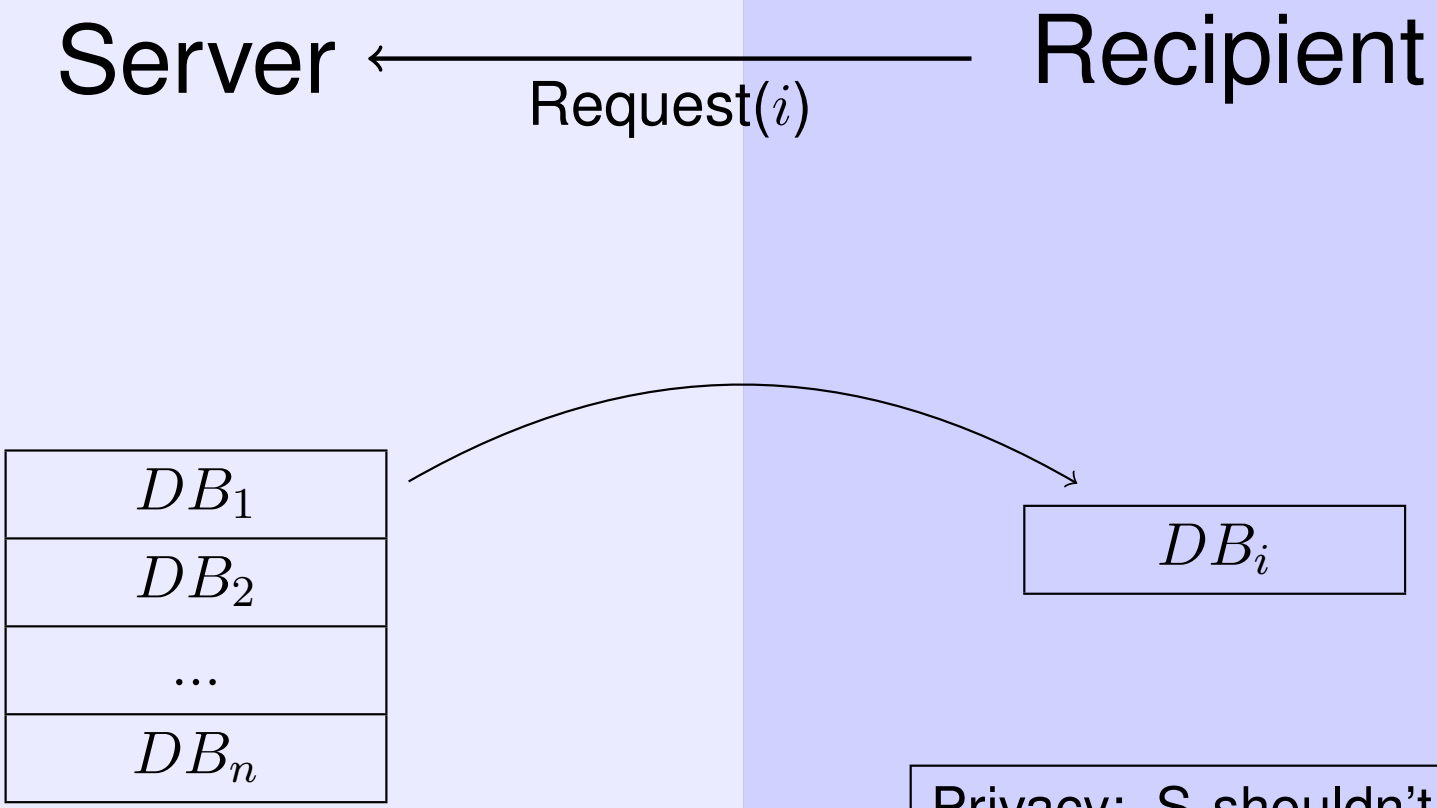

Privacy: S shouldn't know i and R shouldn't have any information about other lines.

Identity Based Encryption

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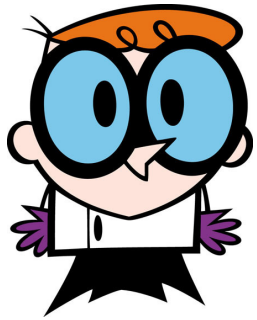
Alice

Identity Based Encryption

Bob



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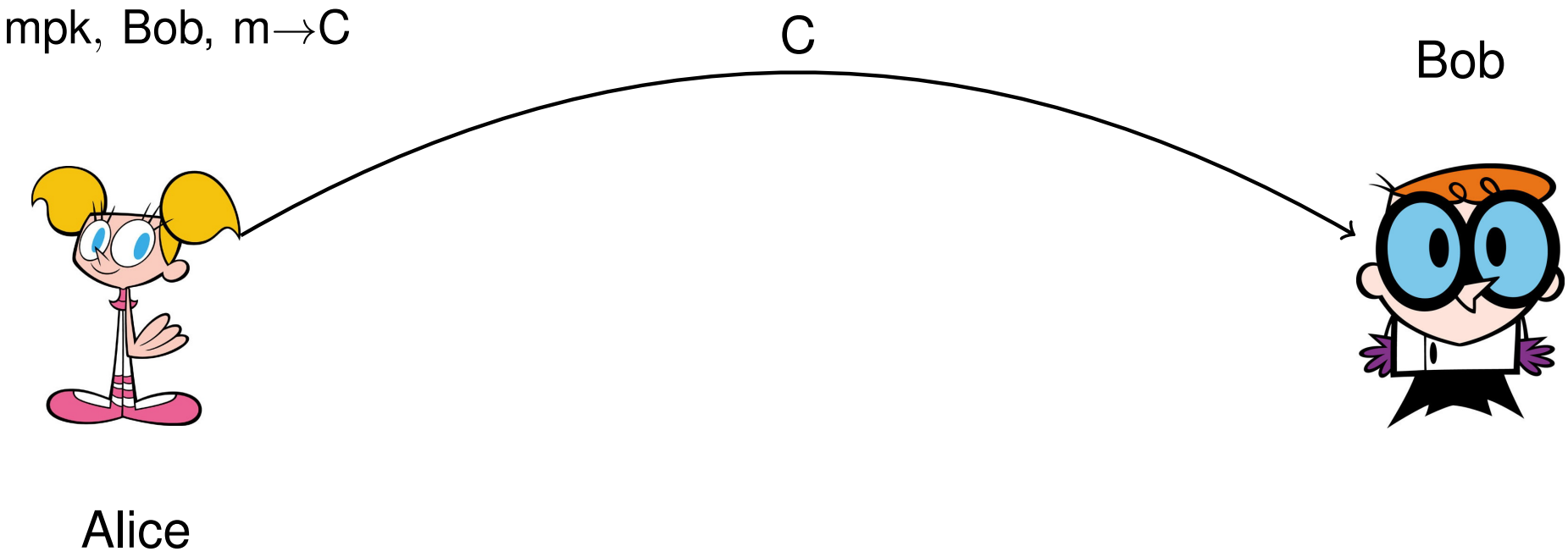

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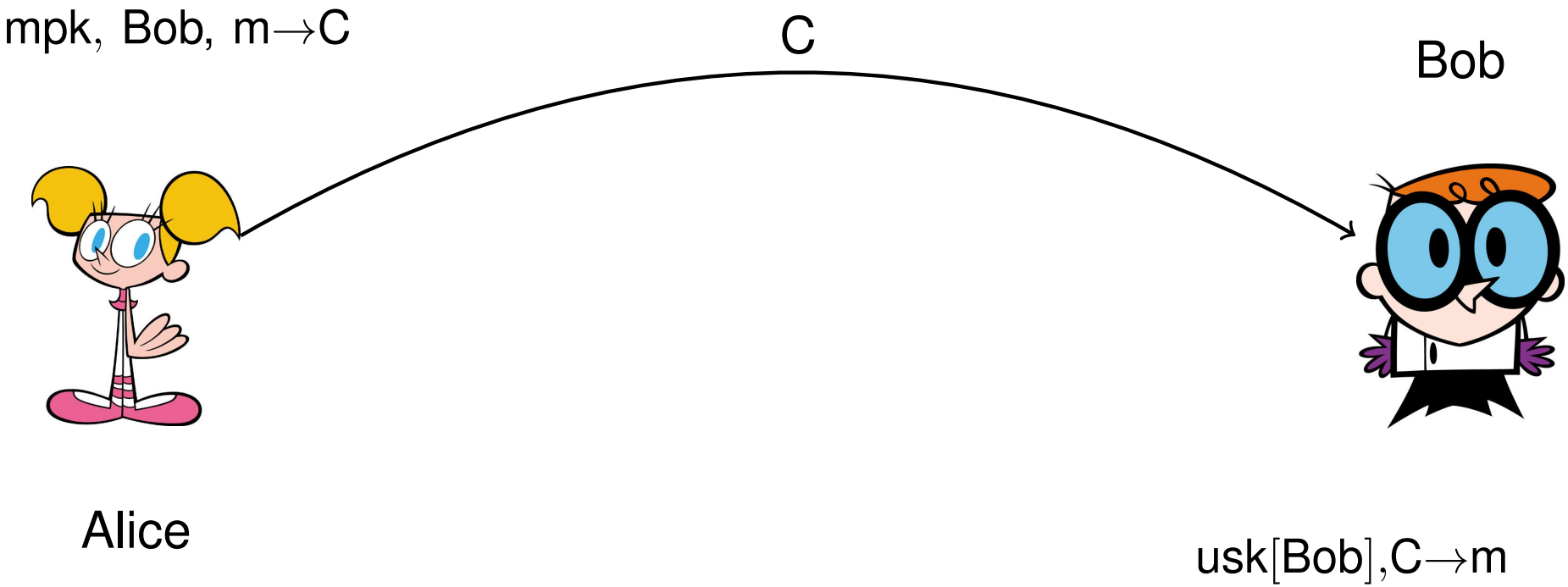

Identity Based Key Encapsulation Mechanism

- **Gen**(param): generates $(\mathsf{mpk}, \mathsf{msk})$
- **USKGen**(**msk**, **id**): computes usk[id]
- **Enc**(**mpk**, **id**): encrypts a key K into C
- **Dec**(**usk**[**id**], C): decrypts C into K

UC-framework and Security Model

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- Ideal functionality vs real world

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- Ideal functionality vs real world
- Adaptive corruptions: the adversary can ask for internal state of the recipient at any moment and then play his role.

The Oblivious Signature Based Envelope Protocol

Server

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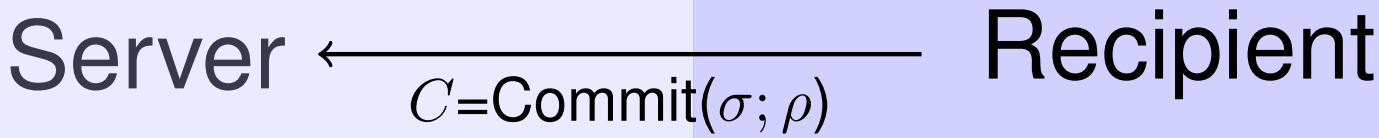

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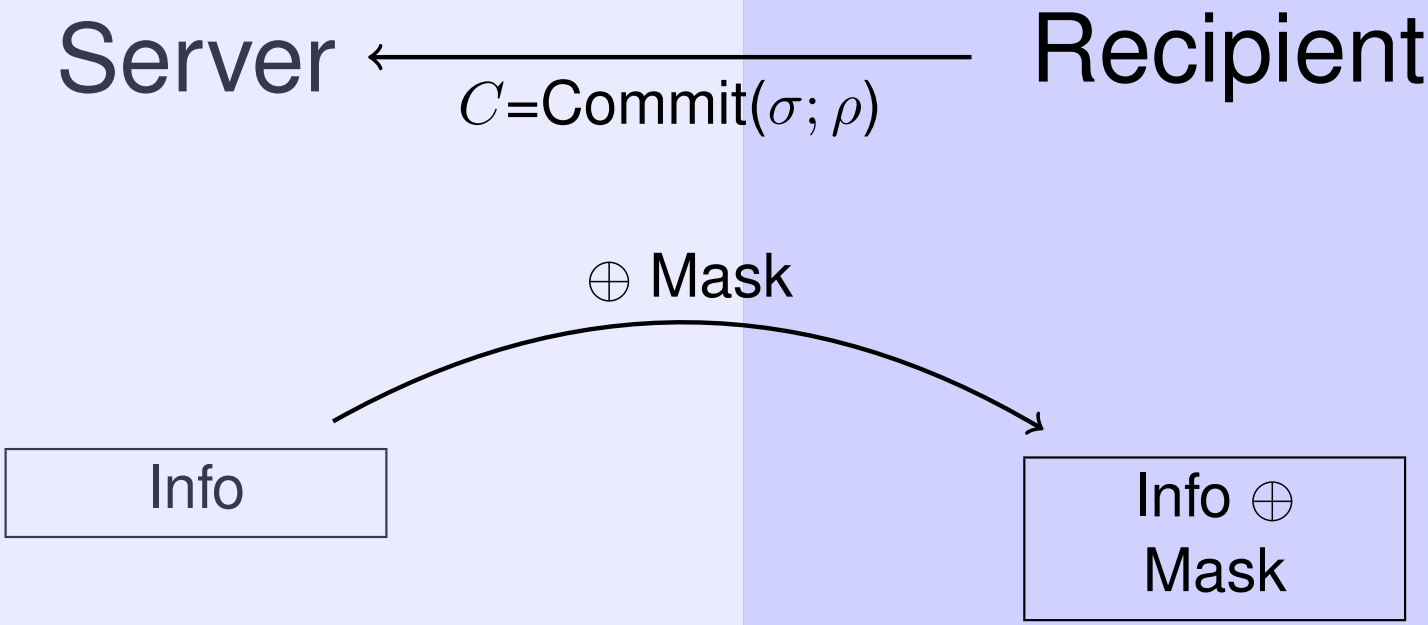

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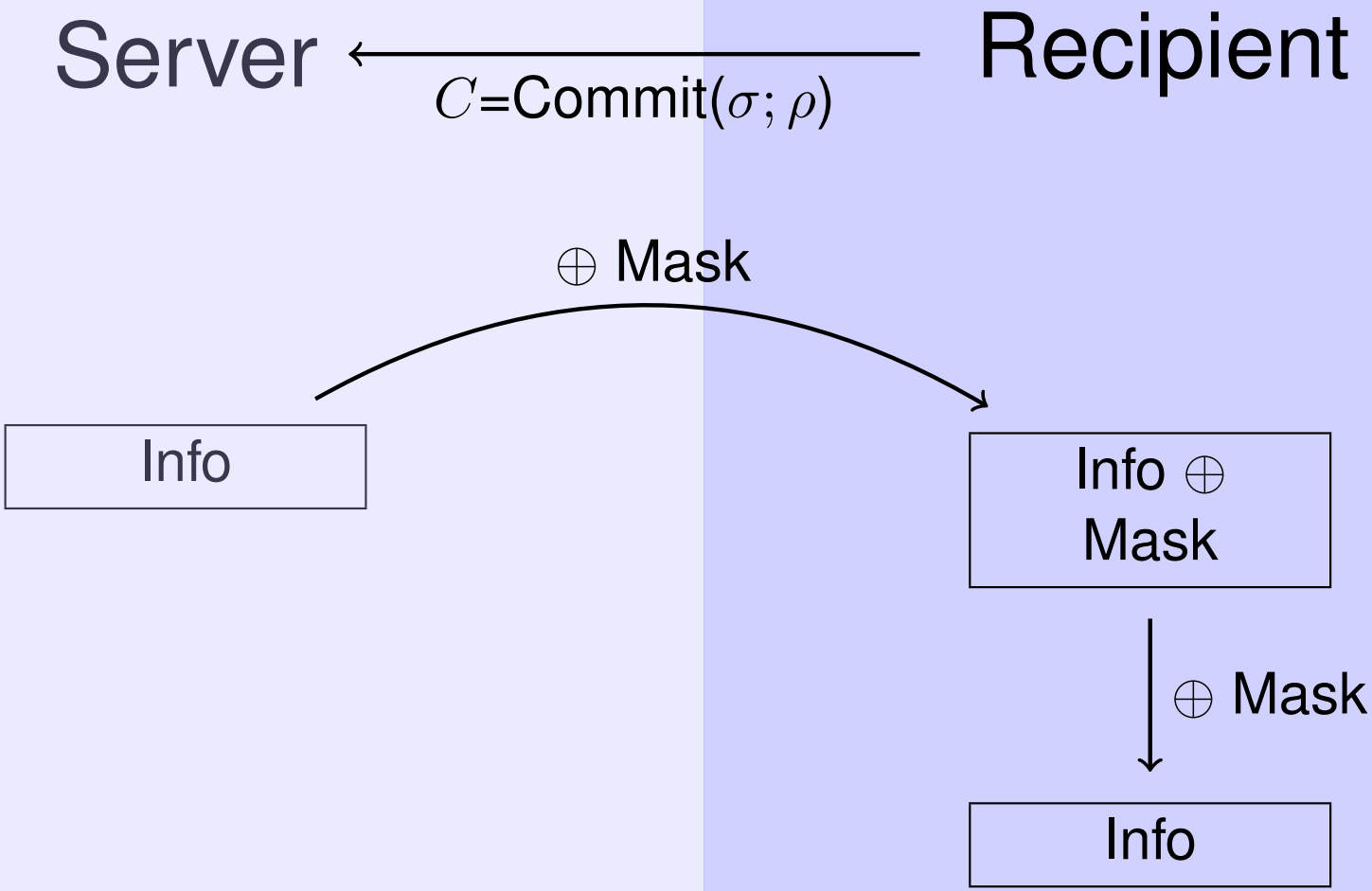

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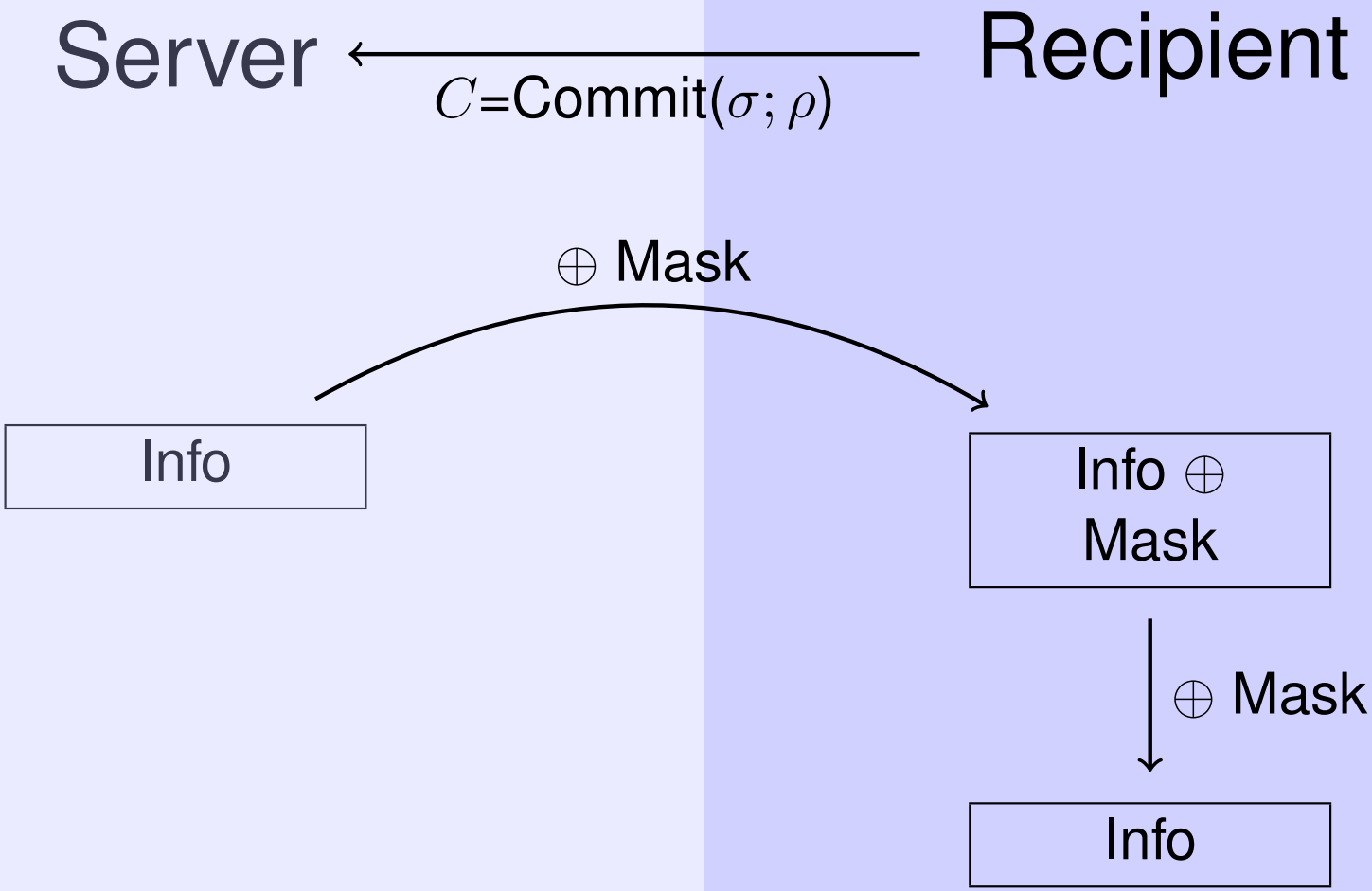

Mask computable for the user if and only if C is a commitment of σ

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$$\text{If } \mathsf{CH}(\mathsf{ck}, m; r) = H \text{ then}$$

$$\mathsf{coll}(\mathsf{ck}, \mathsf{tk}, H; m') = r' \text{ s. t. } \mathsf{CH}(\mathsf{ck}, m'; r') = H$$

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$$\text{Hash}(\text{hk}, (\mathfrak{L}, \text{param}), W) = \text{ProjHash}(\text{hp}, (\mathfrak{L}, \text{param}), W, w).$$

If w is a witness for $W \in \mathfrak{L}$.

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- Pseudo-Randomness: Without w, if $W \in \mathfrak{L}$ it is hard to distinguish a hashed value from a random one.

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- $H' =$ hp$^\alpha$

SPHF And Implicit Decommitment Achieving OSBE

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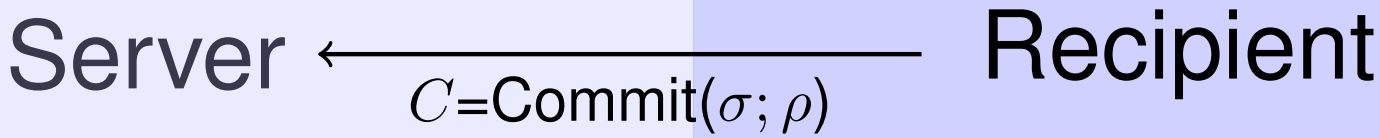

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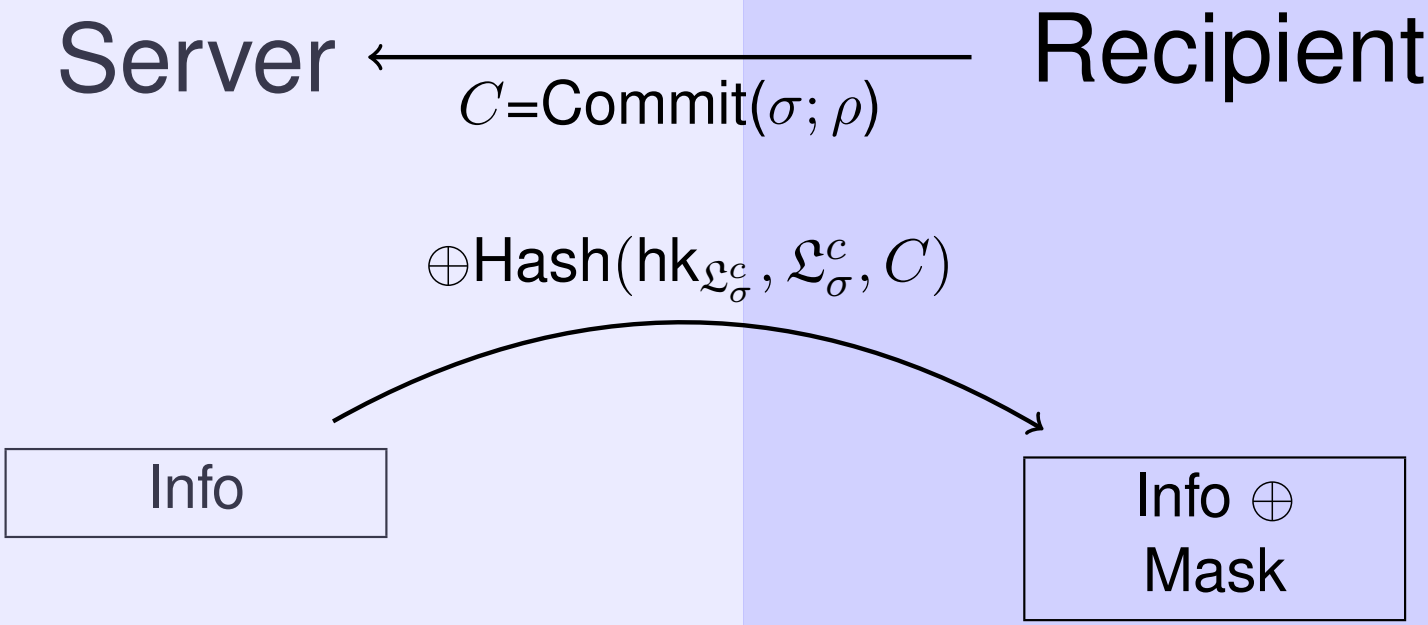

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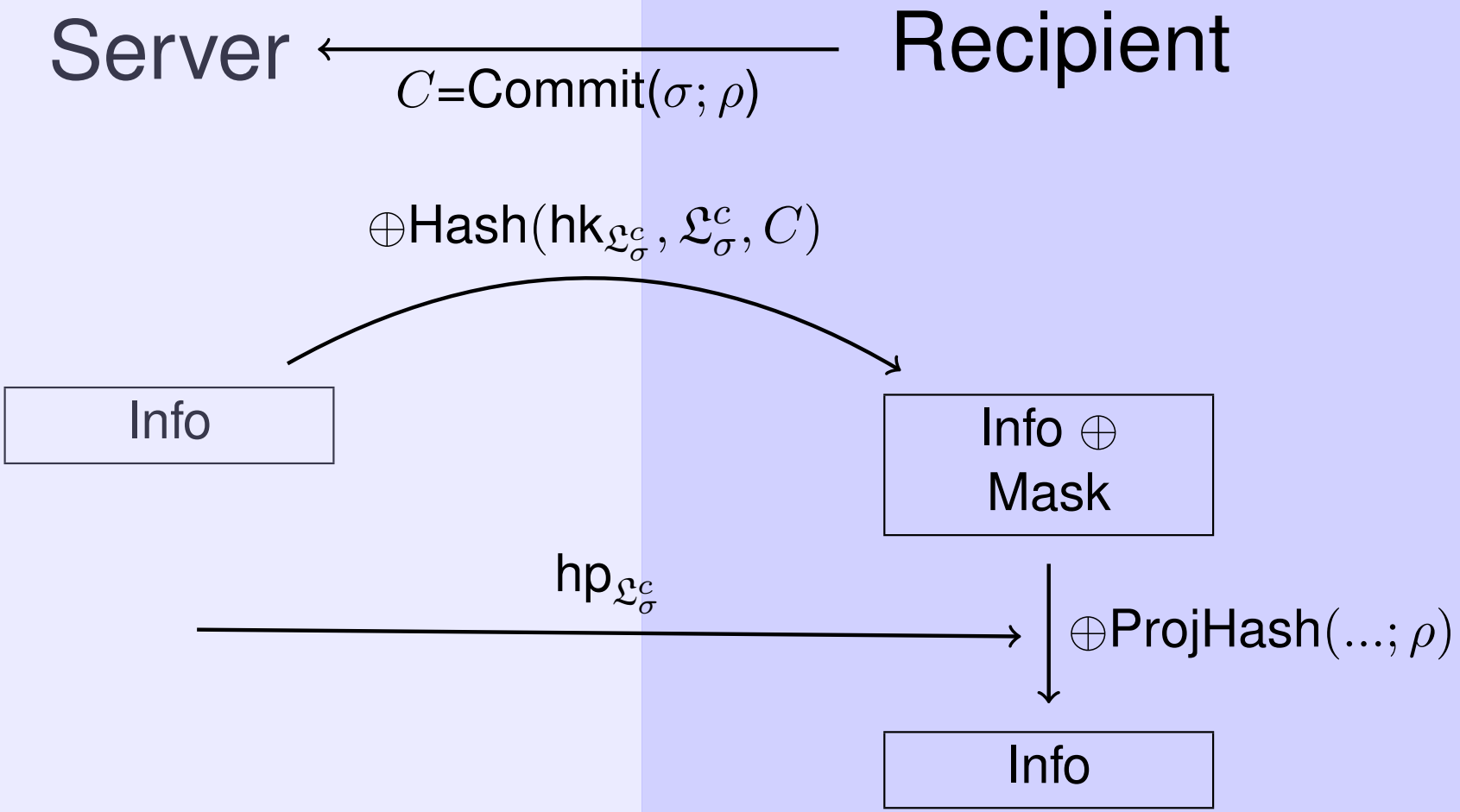
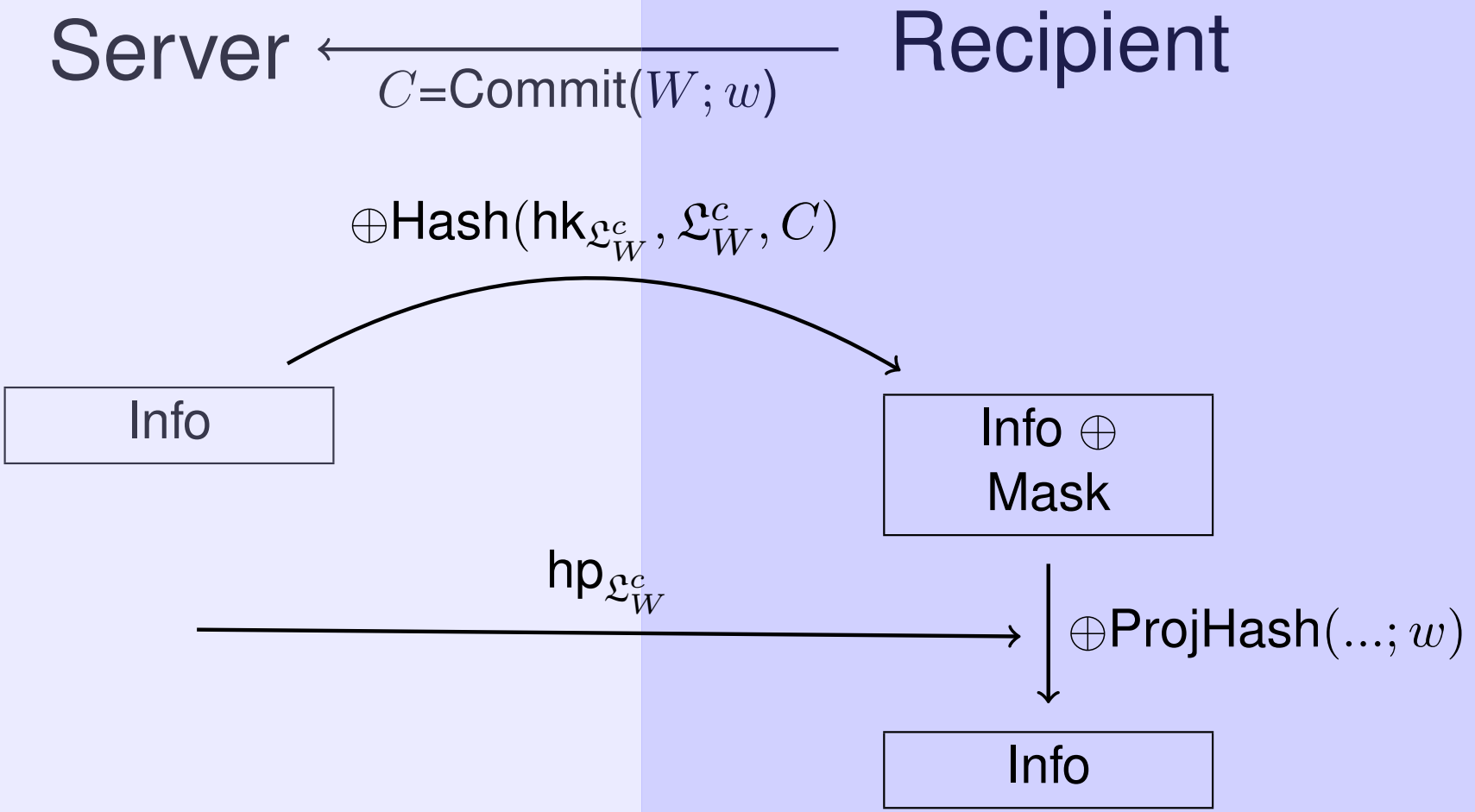


Generalization: Oblivious Language Based Envelope



Examples

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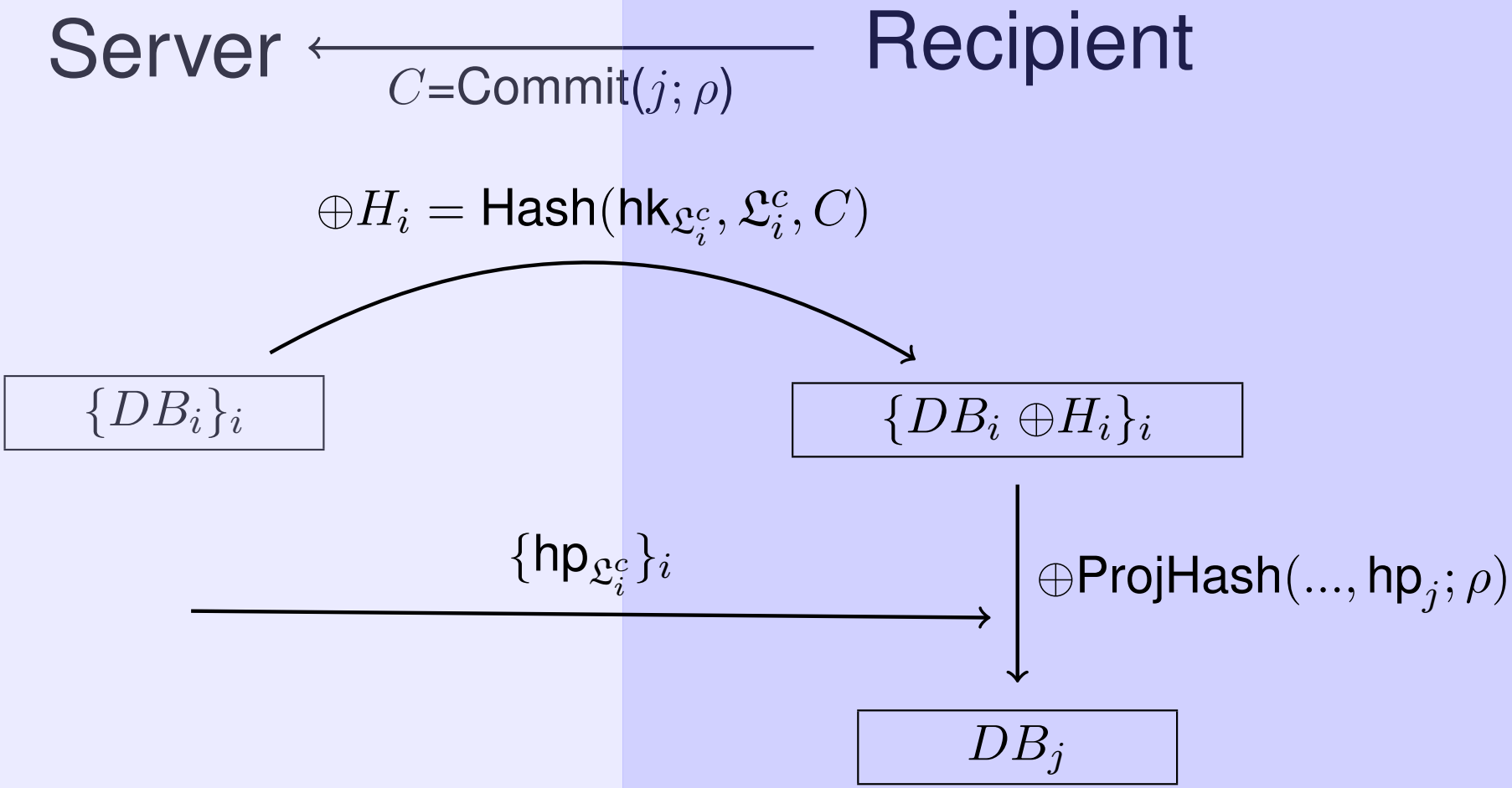
Oblivious Transfer



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- Conditionned Oblivious Transfer

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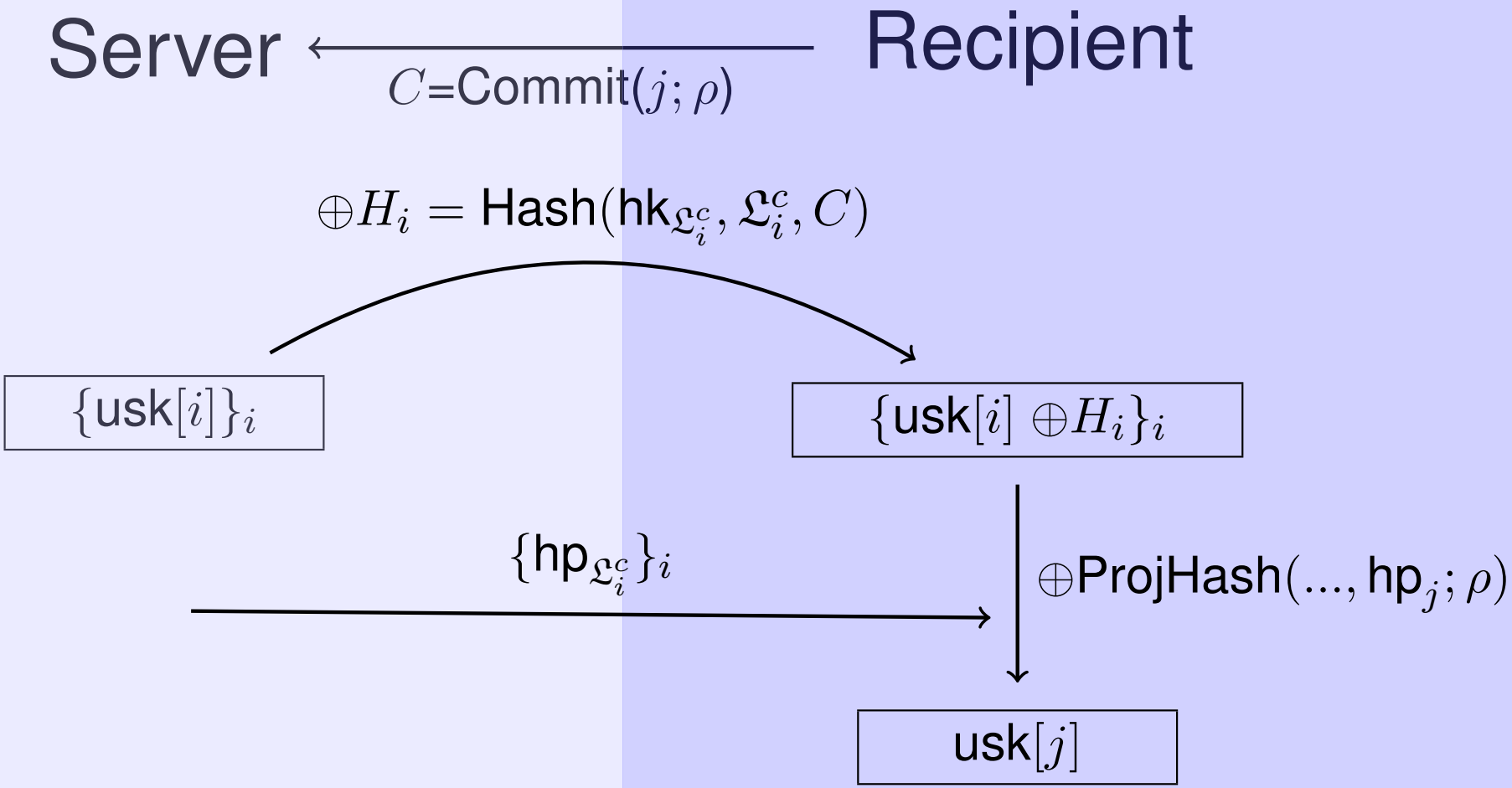
  - Do an OT with the keys used to encrypt.

OT On IBE Keys: A Blind IBE



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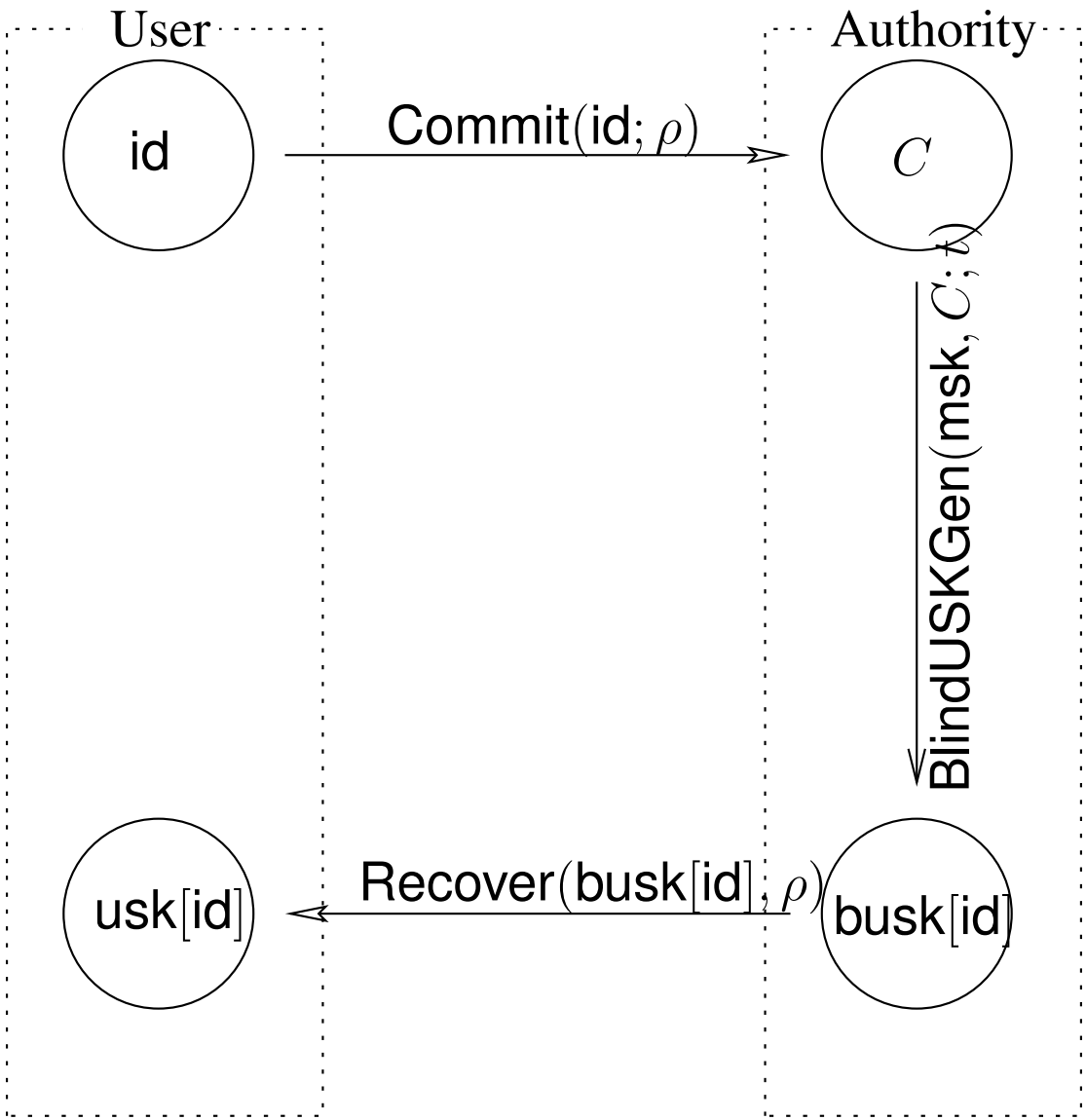
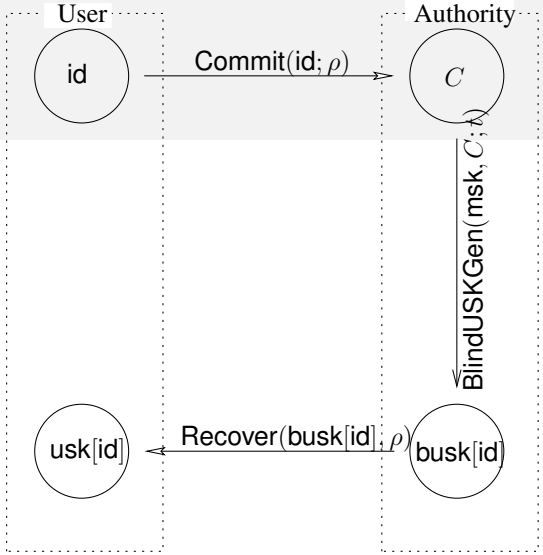


BIBE Generic Construction

1. User: $C = \mathsf{Encrypt}_{\mathrm{cca}}(\mathsf{id}; \rho)$



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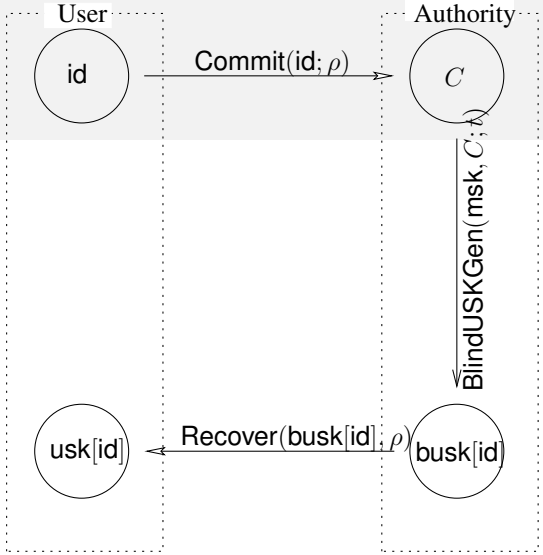

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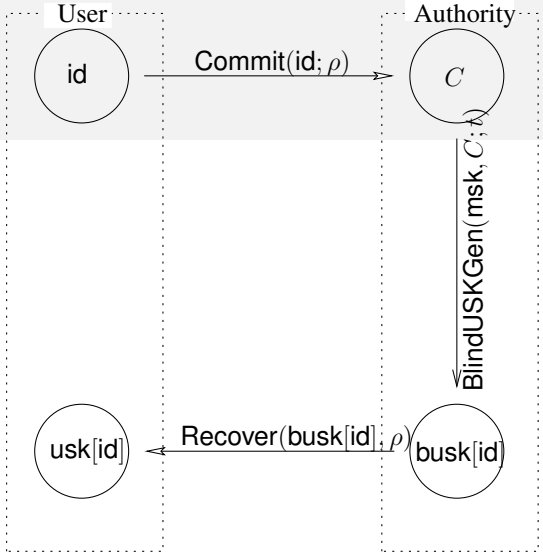

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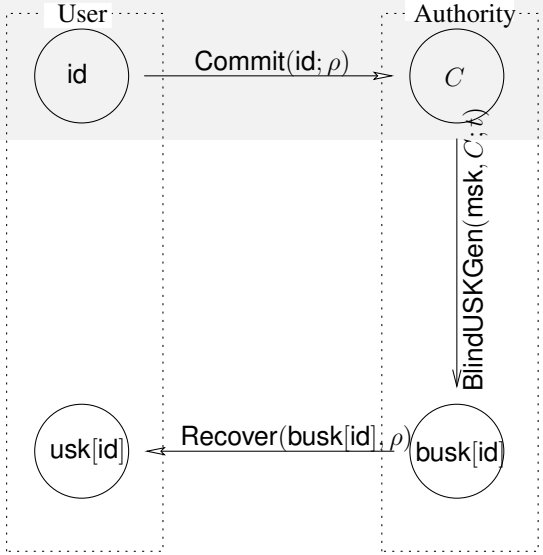

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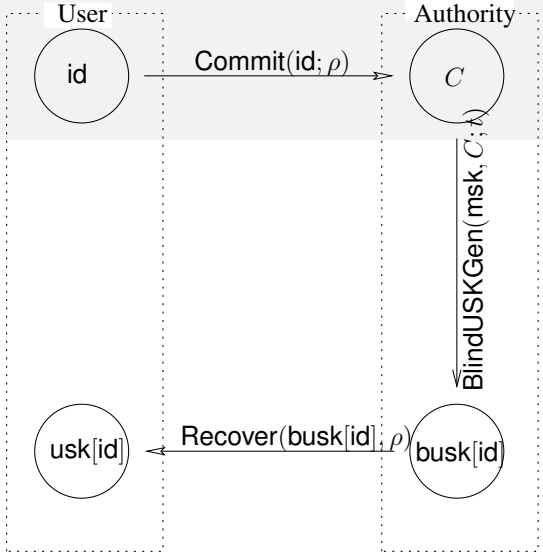

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3. User computes $H'_{\mathsf{id}} = \mathsf{ProjHash}(\mathsf{hp}_{\mathsf{id}}, (\mathfrak{L}^c_{\mathsf{id}}, \mathsf{param}), C, \rho)$
   Recovers $\mathsf{usk}[\mathsf{id}]$

3-flow-Adaptive Oblivious Transfer

Database Preparation:
Data encryption, usk computation, channel key generation.

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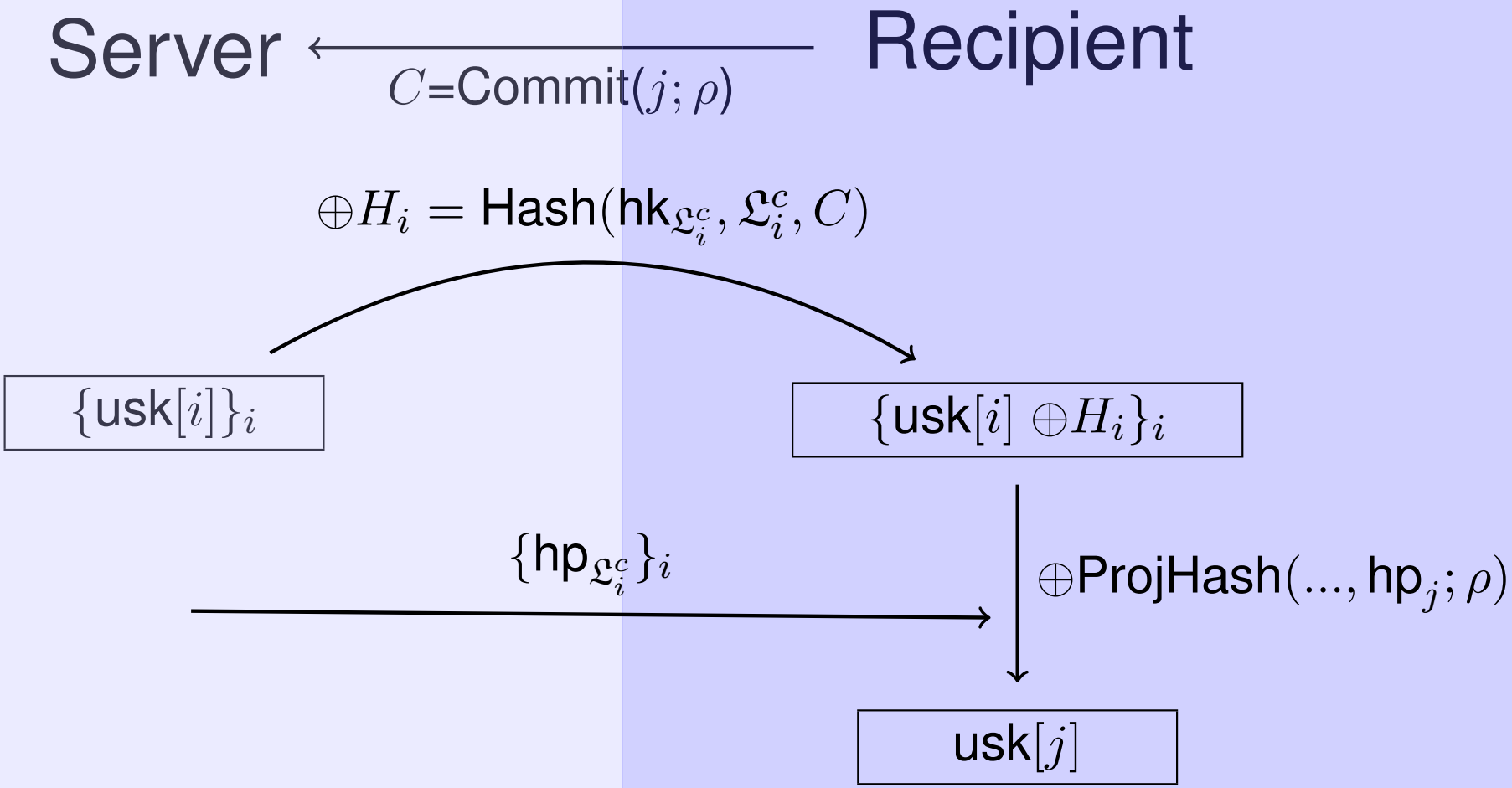
Almost...

What About The Communication Cost?



Issue With The Communication Cost

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Solution: fragment identities into bits.

$\Rightarrow$ communication cost logarithmic in the size of the database.

Affine IBE

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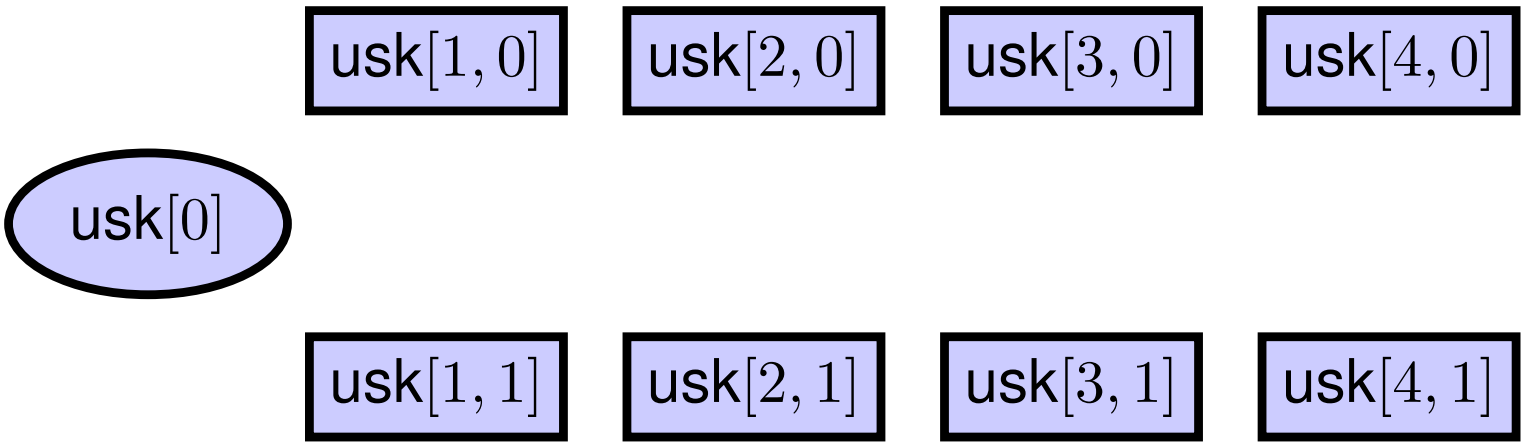
Example with $\mathsf{id} = 0010$



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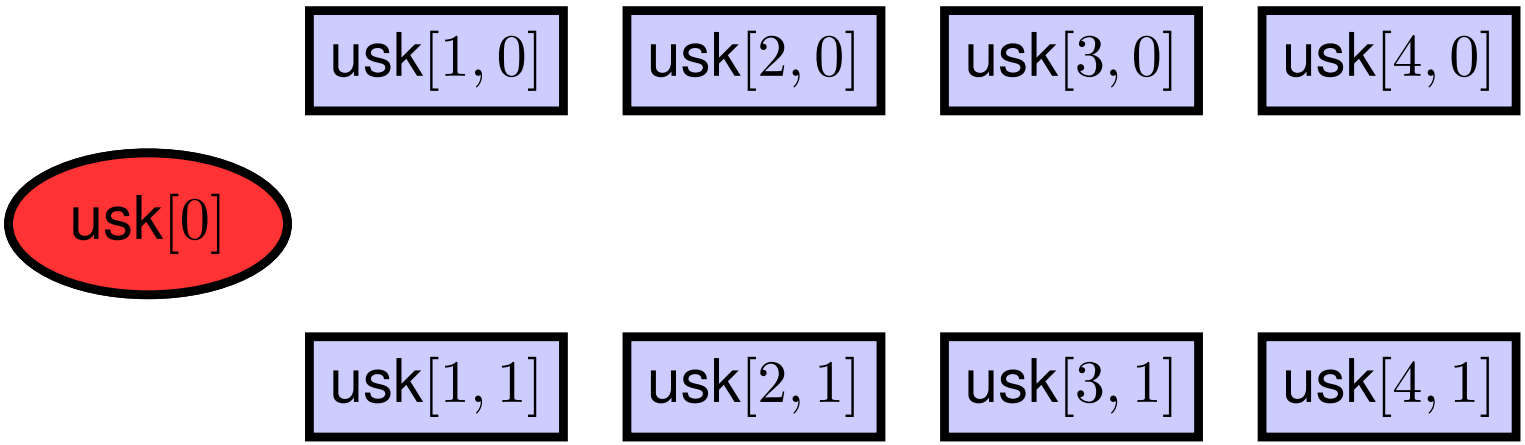

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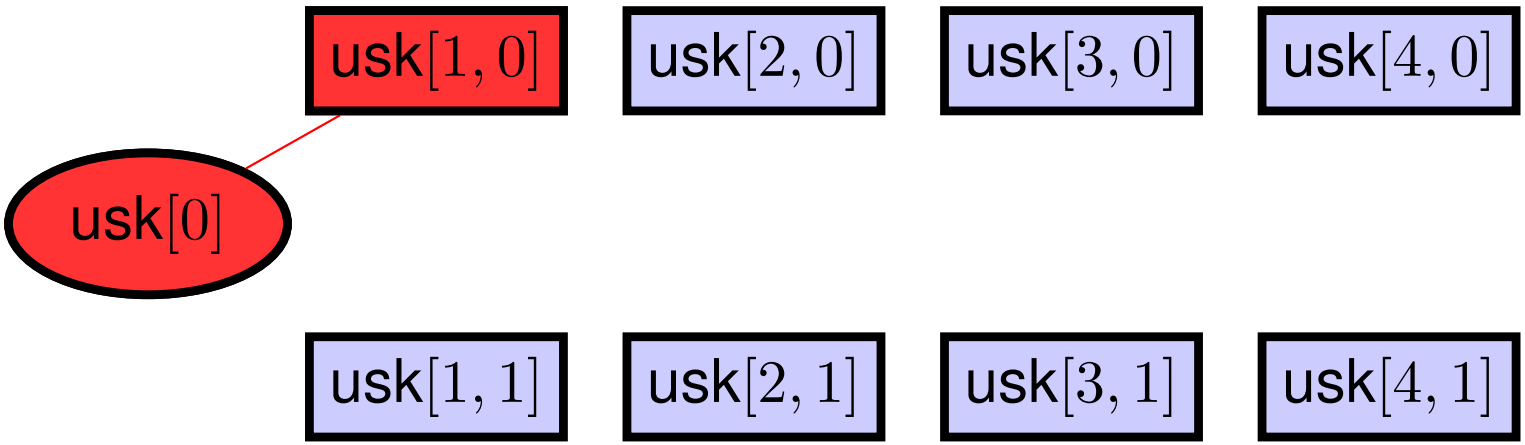

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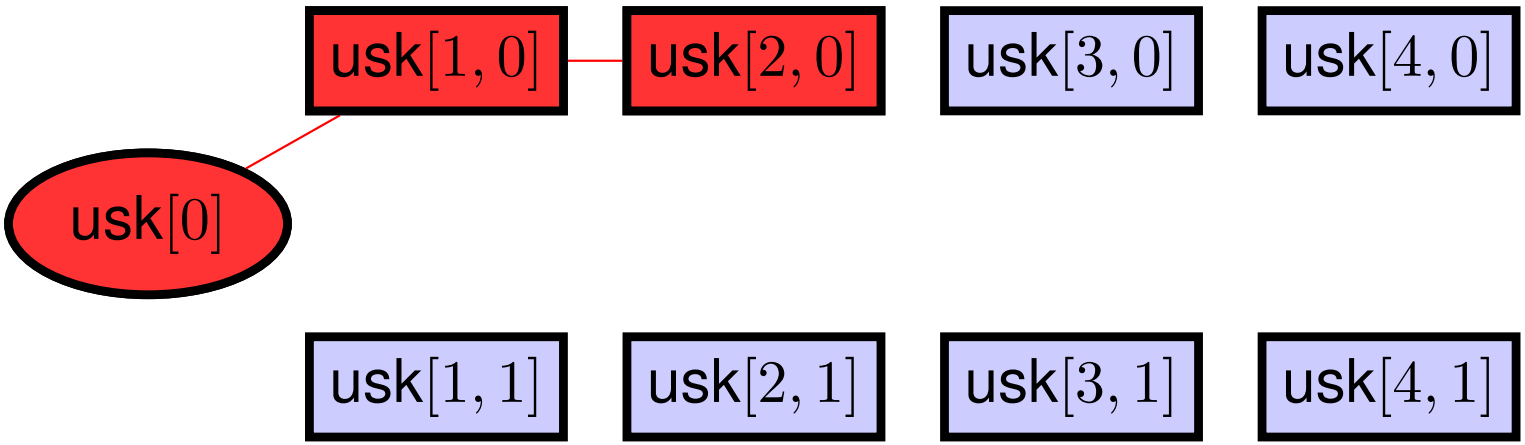

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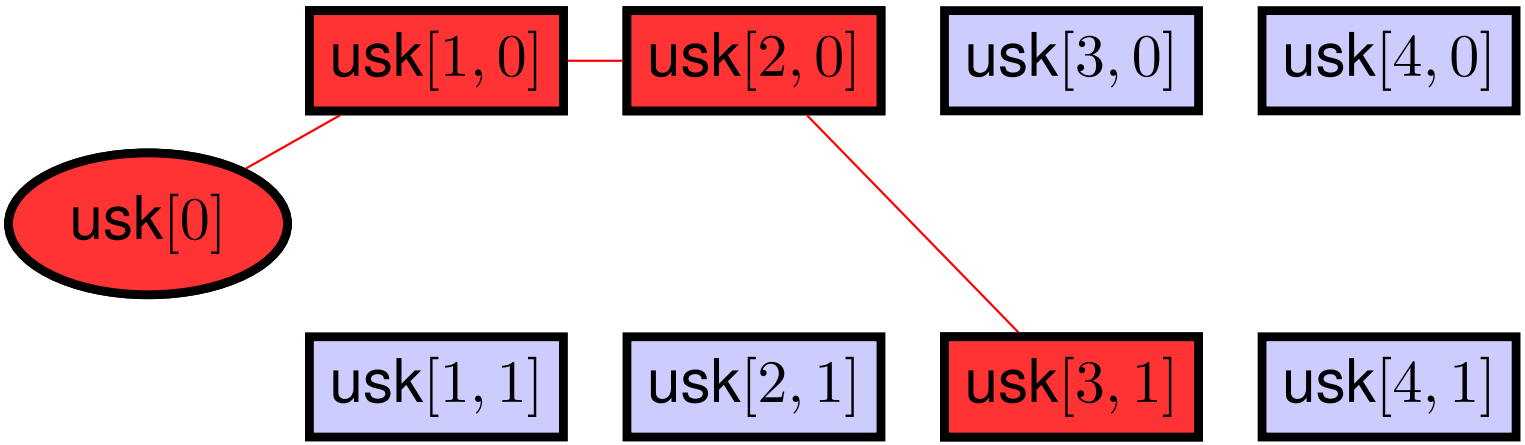

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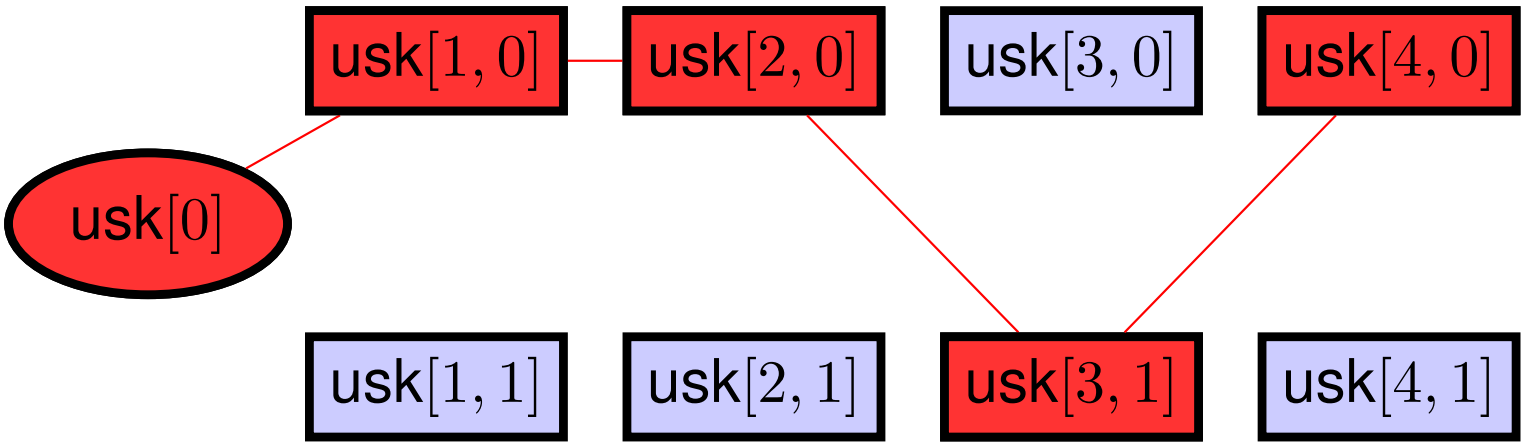

Fragmented BIBE Construction

2. For all i, b S computes:
   - $(\mathsf{usk}[i,b], (\mathsf{hk}_{i,b}, \mathsf{hp}_{i,b}))$ for SPHF on $\mathfrak{L}^c_{i,b}$
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   - $Z = \mathsf{usk}[0] \ominus \left(\bigoplus_i z_i \right)$

   Sends $(Z, \mathsf{hp}_{i,b}, \mathsf{usk}[i,b] \oplus \mathtt{KDF}(H_{i,b}) \oplus z_i)$ for each (i,b)

3. User computes $H'_{i,b} = \mathsf{ProjHash}(\mathsf{hp}_{i,b}, (\mathfrak{L}^c_{i,b}, \mathsf{param}), C, \rho)$

   Recovers $\mathsf{usk}[\mathsf{id}] = \left(\bigoplus_i (\mathsf{usk}[i, \mathsf{id}_i] \oplus z_i) \right) \oplus Z$

An Affine IBKEM Scheme

- **Gen** : $Y_i \xleftarrow{\$} \mathbb{Z}_p^2$, $Z_i = Y_i^\top \cdot A$, $y' \xleftarrow{\$} \mathbb{Z}_p^2$, $z' = y'^\top \cdot A$,
  $\Rightarrow$ mpk $= (g_1^A, g_1^{Z_i}, g_1^{z'})$, msk $= (Y_i, y')$
- **USKGen**: $s \xleftarrow{\$} \mathbb{Z}_p$, $t = Bs$, $w = (Y_0 + \sum h_i(id_i)Y_i)t + y'$,
  $\Rightarrow$ usk[id] $= (g_2^t, g_2^w)$
- **Enc**(**mpk**, **id**): $r \xleftarrow{\$} \mathbb{Z}_p$, $c_0 = Ar$, $c_1 = (Z_0 + \sum h_i(id_i)Z_i) \cdot r$,
  $\Rightarrow K = z' \cdot r$,$C = (g_1^{c_0}, g_1^{c_1})$, sk $= g_T^K$
- **Dec**(**usk**[**id**], **id**, C): sk $= e(g_1^{c_0}, g_2^t) \cdot e(g_1^{c_1}, g_2^w)^{-1}$

The Talk In One Slide

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- Fragmented BIBE + UC folklore $\Rightarrow$ UC secure Adaptive OT