Design Strategies for ARX with Provable Bounds: SPARX and LAX

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ASIACRYPT

UNIVERSITÉ DU LUXEMBOURG

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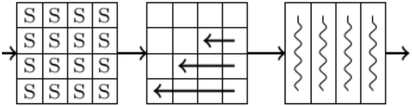

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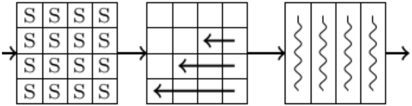
Design of an S-Box based SPN
(wide-trail strategy)



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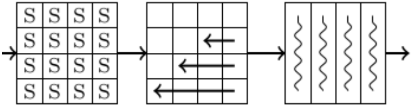

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Can we use ARX *and* have provable bounds?

Outline

Plan

The Wide Trail Strategy (WTS)

Wide Trail Argument

$$\mathrm{MEDCP}(F^r) \leq {p_S}^{\mathbf{a}(r)}$$

- $\mathrm{MEDCP}(F^r) = \max\left(P[\text{any trail covering } r \text{ rounds of } F]\right)$
- $P[S(x \oplus \mathbf{c}) \oplus S(x) = \mathbf{d}] \;\leq\; p_S$
- $\#\{\text{active S-Boxes on } r \text{ rounds}\} \geq \mathbf{a}(r)$

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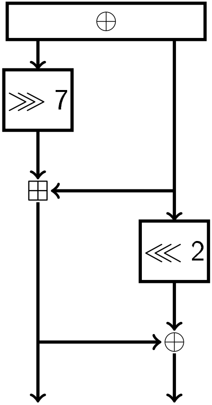
Application to ARX

Can we use this to build an ARX-based cipher?

ARX-Boxes (1/2)

SPECKEY

1. Start from SPECK-32
2. XOR key in full state (Markov assumption)
3. Find best trails



SPECKEY.

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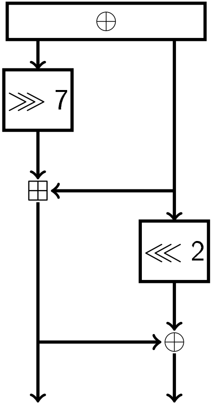
Parameter Search

- Rotations 7, −2
- Second best crypto properties, lightest
- Indeed NSA design strategy (see DAC'15).



SPECKEY.

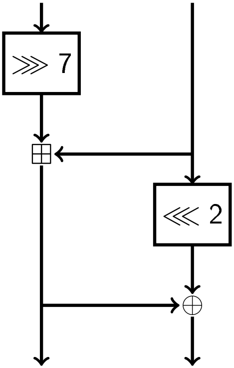
ARX-Boxes (2/2)

Differential/Linear bounds

r	1	2	3	4	5	6	7	8	9	10
MEDCP(A^r)	−0	−1	−3	−5	−9	−13	−18	−24	−30	−34
MELCC(A^r)	−0	−0	−1	−3	−5	−7	−9	−12	−14	−17

Maximum expected differential characteristic probabilities (MEDCP) and maximum expected absolute linear characteristic correlations (MELCC) of SPECKEY ($\log_2$ scale); r is the number of rounds.

Notations



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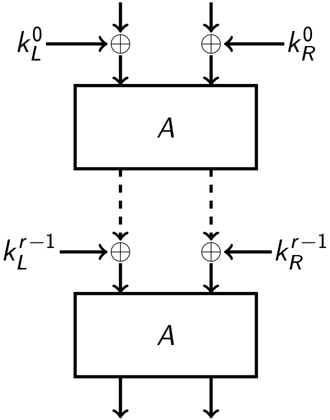
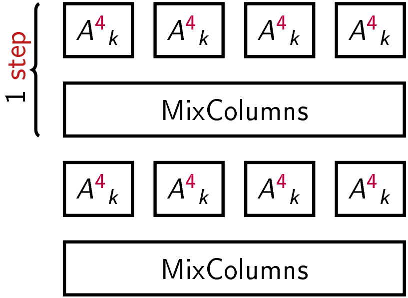


A_k^r.

Naive Approach

S-Box: A^4 ; Linear layer: 128-bit MixColumns.



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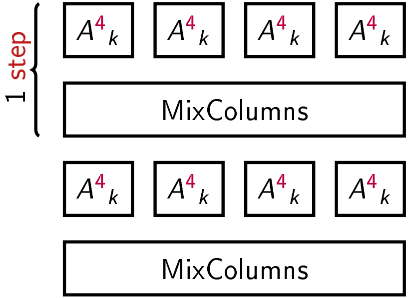

- Active ARX-Boxes: $\mathbf{a}(2s) \geq 5s$,
- $\log_2 \left(\text{MEDCP}(A^4)\right) = -5$

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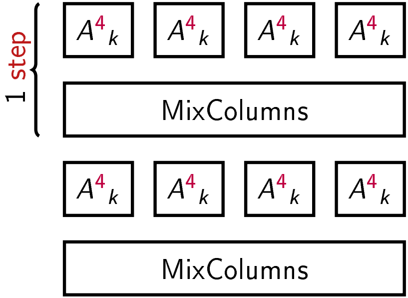

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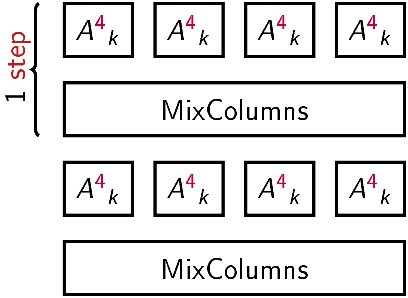

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Need $2\lceil 128/25 \rceil = 12$ steps, i.e. **48 ARX rounds!**

Drawbacks

The Wide Trail Strategy fails here

Two (bad) options:

1. design a very weak cipher, or
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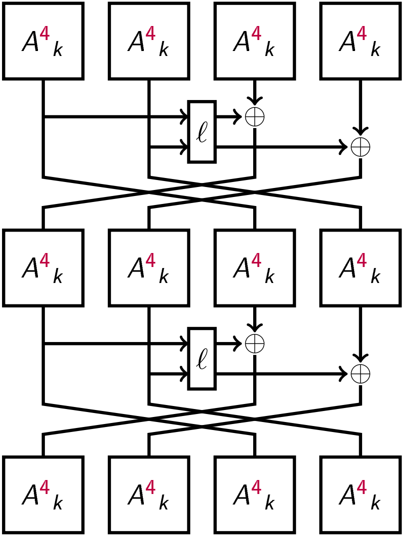
A New Hope

- $\log_2\left(\text{MEDCP}(A^4)\right) = -5$
- $\log_2\left(\text{MEDCP}(A^8)\right) = -24 \ll -5 \times 2$

Better Approach

- New linear layer "chaining" ARX-Boxes.



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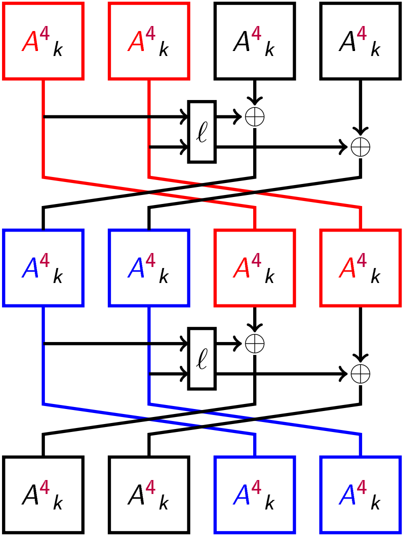

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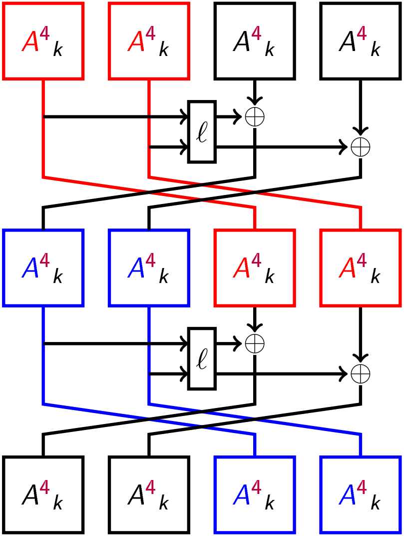

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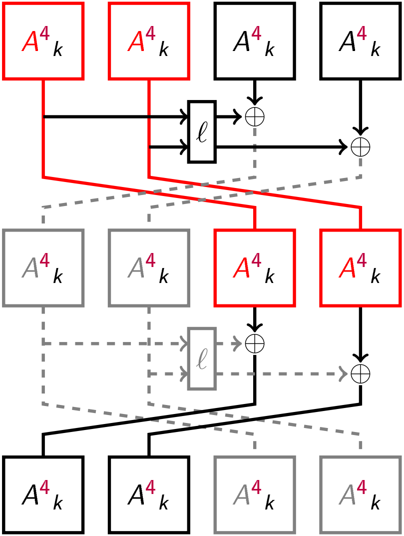

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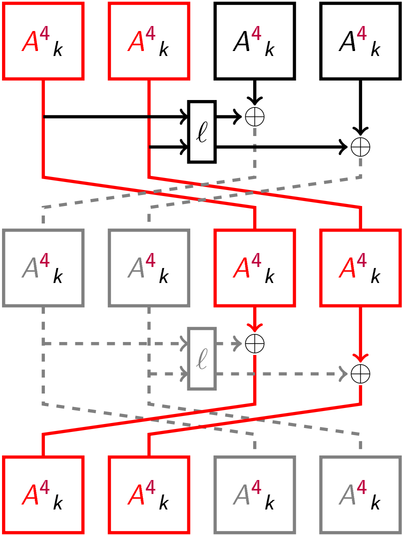
- If left half has zero differences, we can use MEDCP(A^{12}) instead of $(\text{MEDCP}(A^4))^3$.



The Long Trail Argument (1/2)

Definition (Long Trail)

A Long Trail (LT) is a trail covering several ARX-Boxes without receiving any outside difference. Can be *static* (probability = 1) or *dynamic* (depends on the trail).

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Definition (Truncated Trail)

A sequence of values in $\{0, 1\}^4$: 1 if ARX-Box i is active, else 0.

The Long Trail Argument (2/2)

Bounding Differential Probability

For all truncated trails covering r rounds:

1. check if it is coherent with the linear layer,
2. decompose it into long trails (static and dynamic),
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$\implies$ Deduce a bound on the probability of all trails.

Example of a LT bound

After 5 steps, the best trail for four 4-round ARX-Boxes + Feistel linear layer is $< 2^{-128}$.

$5 \ll 12$ steps

The Long Trail Strategy (LTS)

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When using large, weak S-Boxes, it is better to foster Long Trails than diffusion. Thus, the linear layer must be small.

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Wide Trail Strategy

S-Box	Small, cheap.
Lin. Layer	Expensive, complex.

Long Trail Strategy

S-Box	Large, expensive.
Lin. Layer	Cheap, simple.

Plan

1 The Long-Trail Strategy

2 The SPARX Family of LW-BC
- High Level View
- Security Analysis
- Implementation
  - Methodology
  - Results

3 The LAX Approach

4 Conclusion

High Level View

SPARX family of block ciphers

- Designed using a long trail strategy.
- SPARX-n/k: n-bit block, k-bit key ($k \geq 128$).
- Only need 16-bit operations: $\lll i$, $\oplus$, $\boxplus$.

High Level View

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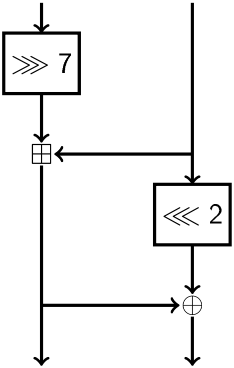
n/k	64/128	128/128	128/256
# Rounds/Step	3	4	4
# Steps	8	8	10
Best Attack (# rounds)	15/24	22/32	24/40

Notations (reminder)



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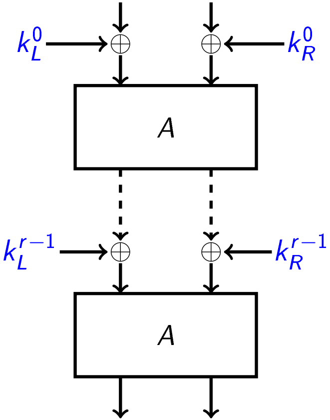

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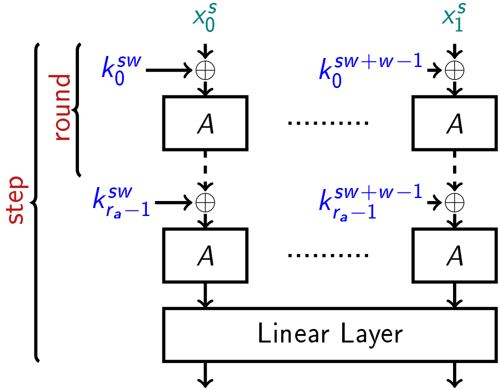
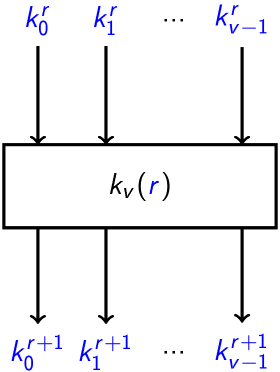
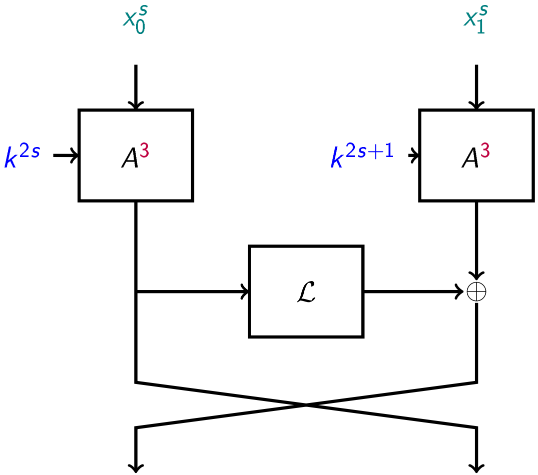


Round function of SPARX.

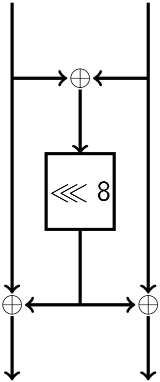


Key schedule.

SPARX-64/128

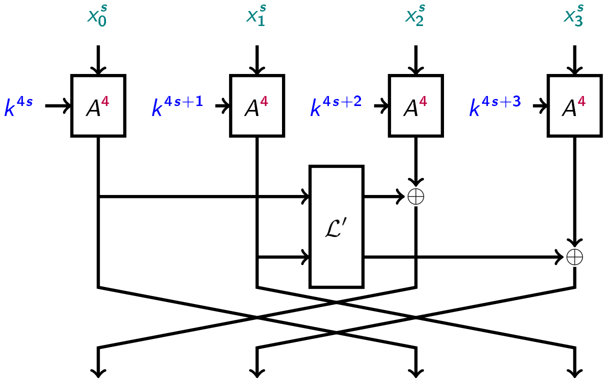


Step Function.



$\mathcal{L}$.

SPARX-128/128 and SPARX-128/256



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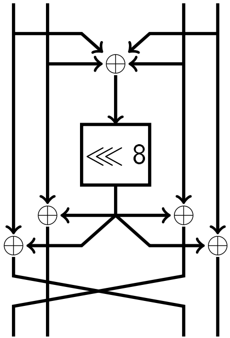

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Security

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Integral Attacks

- Todo's division property: 4-5 steps for n =64-128,
- properties of modular addition: +1 round,
- best distinguishers cover 13-21 rounds for n =64-128.

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n/k	64/128	128/128	128/256
rounds attacked/total	15/24	22/32	24/40
security margin	38 %	31 %	40 %

Benchmarking

https://www.cryptolux.org/index.php/FELICS

- **F**air **E**valuation of **L**ightweight **C**ryptographic **S**ystems
- 8-bit ATMEL AVR ; 16-bit TI MSP ; 32-bit ARM Cortex-M3
- Usage scenarios (e.g. CBC encryption of 128 bytes)
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- Extracts RAM usage, ROM usage, # CPU cycles.
- Figure Of Merit aggregates: all metrics accross all platforms for the best implementations of one algorithm.

Efficiency of the SPARX Ciphers

Rank	Cipher	Block size	Key size	Scenario 1 FOM	Security margin
1	Speck	64	128	5.0	27 %
2	Chaskey-LTS	128	128	5.0	42 %
3	Simon	64	128	6.9	32 %
4	RECTANGLE	64	128	7.8	28 %
5	LEA	128	128	8.0	33 %
6	**Sparx**	**64**	**128**	**8.6**	**38 %**
7	**Sparx**	**128**	**128**	**12.9**	**31 %**
8	HIGHT	64	128	14.1	19 %
9	AES	128	128	15.3	30 %
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Gray: designers did not provide differential/linear bounds.

Plan

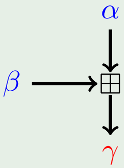
An Alternative Strategy for Provable ARX

The Wallén Challenge

[...] design a simple and efficient cipher that uses only addition modulo 2^n and F_2-affine functions, and that is provably resistant against basic DC and LC.

–Johan Wallén [Master Thesis, 2003]

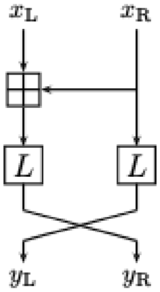
Rationale



- DP and LC drop exponentially with $\mathrm{hw}(\alpha \oplus \beta)$
- Affine part should maximize $\mathrm{hw}(\alpha \oplus \beta)$!

DP = differential probability; LC = linear correlation

The LAX Construction



$(y_L, y_R) =$
$(Lx_R,\ L(x_L \boxplus x_R))$

LAX-2n

- $2n$-bit block, $n \in \{8, 16\}$
- L is $n \times n$ binary matrix that
  1. is invertible,
  2. has branch number $d > 2$,
- $[I\ L]$ is a $[2n, n, d]$ lin. code:
  - LAX-16: [16, 8, 5]
  - LAX-32: [32, 16, 8]

Linear transform, Addition, XOR $\implies$ **LAX**

Differential Bound on 3 Rounds

Theorem

The maximum DP of any trail on 3 rounds of LAX-2n *is* $2^{-(d-2)}$, *where* d *is the branch number of* L.

$2n$	# Rounds	1	2	3	4	5	6	7	8	9	10	11	12
16	p_{best}	+0	−2	−4	−7	−8	−11	−13	−16	−18	−20	−23	−25
	p_{bound}			−3			−6			−9			−12
32	p_{best}	+0	−2	−6	−9	−11	−16	−18	−20	−24	−28	−29	−34
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Open Problem

The bound does not hold for the linear case.

Plan

Conclusion

source: Wiki Commons

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- Dual of the Wide-trail strategy
- Differential *and* linear bounds
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Thank you!