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# Taylor Expansion of Maximum Likelihood Attacks

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#### ASIACRYPT 2016 — Hanoi, Vietnam

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### <span id="page-4-0"></span>Side-Channel Analysis on Embedded Systems





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## <span id="page-5-0"></span> $\Box$  ( $\Omega - 1$ )th-Order Masking: Principle

#### Aim

The sensitive variable Z is randomly split into  $\Omega$  shares:  $\Rightarrow$  need random masks  $\ M_i$  ,  $0 < i < \Omega$ 

Z

### $Z \perp M_1 \perp ... \perp M_{\Omega-1} \qquad M_1 \qquad \cdots \qquad M_{\Omega-1}$



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## $|(\Omega - 1)$ th-Order Masking: Principle

#### Aim

The sensitive variable Z is randomly split into  $\Omega$  shares:  $\Rightarrow$  need random masks  $\ M_i$  ,  $0 < i < \Omega$ 





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## <span id="page-7-0"></span> $|(\Omega - 1)$ th-Order Masking: Principle

#### Aim

The sensitive variable Z is randomly split into  $\Omega$  shares:  $\Rightarrow$  need random masks  $|M_i|$ ,  $0 < i < \Omega$ 



#### **Consequence**

Increases the minimum key-dependent statistical moment.



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### Shuffling: Principle

#### Aim





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### Shuffling: Principle

#### Aim

$$
Z_1 \xrightarrow{\hspace{0.5cm}} Z_2
$$



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### Shuffling: Principle

#### Aim





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### Shuffling: Principle

#### Aim





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### Shuffling: Principle

#### Aim





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### Shuffling: Principle

#### Aim





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### Shuffling: Principle

#### Aim





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### Shuffling: Principle

#### Aim

Randomize the order of execution

 $\Rightarrow$  need a random permutation  $\pi$ 





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## Shuffling: Principle

#### Aim

Randomize the order of execution

 $\Rightarrow$  need a random permutation  $\pi$ 



#### **Consequences**

The attacks are applied on the sum of the variables  $\Rightarrow$  increases the algorithmic noise.



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### Protection Parameters

The security level of the protections depends on these parameters:

#### Masking

- $\triangleright$  Ω: the number of shares (Ω 1 masks);
- $\triangleright$  O: the order (i.e. the minimal key-dependent statistical moment).

Perfect masking scheme  $\Leftrightarrow$   $O = \Omega$ .

### Shuffling

 $\blacktriangleright$   $\sqcap$  the size of the permutation.



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Template attacks are the most powerful in a information-theoretic sense [\[Chari et al., 2002\]](#page-60-0).

#### **Offline Profiling**

The leakage model is learned:

- non-parametric methods (e.g. histogram, kernel methods...);
- parametric methods (e.g. mixture models).

#### Online Attack

Recover the key using the models by applying a maximum likelihood (ML) attack.



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### Parametric or Non-Parametric ?

#### Parametric

The only random part is the noise with known distribution.

- $\blacktriangleright$  easy to estimate;
- $\blacktriangleright$  shuffle and mask are known:
- $\blacktriangleright$  many templates are learned.

#### Non-Parametric

Shuffle and masks are part of the noise.

- $\triangleright$  can be hard to estimate  $\Rightarrow$  curse of dimensionality;
- $\blacktriangleright$  shuffle and mask are unknown.



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### Notations for the Online attack

The attacks are applied on:

- $\triangleright$  Q queries (i.e. the traces).
- $\triangleright$  D dimension (i.e. the number of leakage samples);
- A leakage measurement is  $X = y(t, k^*, R) + N$  where:
	- If  $y(t, k^*, R)$  is the deterministic part of the model;
	- ► the secret key  $k^*$  and the plaintext  $t$  are n-bit words;
	- $\triangleright$  R is the random countermeasure;
	- $\blacktriangleright$  N is a random Gaussian noise of variance  $\sigma^2$ .



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## Maximum Likelihood Attacks

Theorem (Maximum Likelihood [\[Bruneau et al., 2014\]](#page-59-0))

When the model is known the optimal distinguisher (OPT) consists in maximizing the sum over all traces  $q = 1, \ldots, Q$  of the log-likelihood:

$$
LL = \sum_{q=1}^{Q} \log \mathbb{E} \exp \frac{-\|x^{(q)} - y(t^{(q)}, k, R)\|^2}{2\sigma^2} ,
$$

where expectation  $E$  is applied to the random variable  $R \in \mathcal{R}$ and  $\|\cdot\|$  is the Euclidean norm on  $\mathbb{R}^D$ .

For convenience we let  $\gamma=\frac{1}{2\sigma^2}$  be the SNR parameter.



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## Complexity in presence of Masking and Shuffling

$$
\mathcal{O}\left(\left|Q\right|\cdot\left|D\right|\cdot\left|(2^n)^{\Omega-1}\cdot\left|\Pi\right|\right)\right)
$$

- $\blacktriangleright$  number of traces
- $\blacktriangleright$  dimension of the attack
- $\blacktriangleright$  number of possible share values
- $\blacktriangleright$  number of possible permutations



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## Complexity in presence of Masking and Shuffling

$$
\mathcal{O}\left(\left|\frac{Q}{\gamma}\cdot\frac{D}{\gamma}\cdot\frac{(2^n)^{\Omega-1}}{\gamma}\cdot\frac{\Pi!}{\gamma!}\right.\right)
$$

 $\blacktriangleright$  number of traces  $\angle$ 

- $\blacktriangleright$  dimension of the attack -
- $\blacktriangleright$  number of possible share values
- $\blacktriangleright$  number of possible permutations



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## Complexity in presence of Masking and Shuffling

$$
\mathcal{O}\left(\left|Q\right| \cdot \left|D\right| \cdot \left| (2^n)^{\Omega-1} \right| \cdot \left| \Pi \right| \right)
$$

- $\blacktriangleright$  number of traces
- $\blacktriangleright$  dimension of the attack
- $\blacktriangleright$  number of possible share values  $\blacktriangleright$
- $\blacktriangleright$  number of possible permutations



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## Complexity in presence of Masking and Shuffling

- $\mathcal{O}(\begin{array}{|c|c|c|}\n\mathcal{O} & \mathcal{O} & D & (2^n)^{\Omega-1} & \mathcal{O} \end{array})$
- $\blacktriangleright$  number of traces
- $\blacktriangleright$  dimension of the attack
- $\blacktriangleright$  number of possible share values -
- $\blacktriangleright$  number of possible permutations  $\blacktriangleright$



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## Complexity in presence of Masking and Shuffling

- $\mathcal{O}(\begin{array}{|c|c|c|}\n\mathcal{O} & \mathcal{O} & D & (2^n)^{\Omega-1} & \mathcal{O} \end{array})$  $\blacktriangleright$  number of traces  $\blacktriangleright$  dimension of the attack
- $\blacktriangleright$  number of possible share values -
- $\blacktriangleright$  number of possible permutations

Not computable for large Π !



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## <span id="page-29-0"></span>Taylor Expansion of Optimal Attacks in Gaussian Noise

The optimal attack consists in maximizing the sum over all traces  $q = 1, \ldots, Q$  of the log-likelihood:

$$
LL = \sum_{q=1}^{Q} \log \mathbb{E} \exp \frac{-\|x^{(q)} - y(t^{(q)}, k, R)\|^2}{2\sigma^2}.
$$

It can be rewritten using the cumulant generating function:

$$
\mathrm{LL} = \sum_{q=1}^Q \sum_{\ell=1}^{+\infty} \frac{\kappa_\ell^{(q)}}{\ell!} (-\gamma)^\ell \enspace ,
$$

where  $\kappa_{\ell}^{\left( q\right) }$  $\ell_{\ell}^{(q)}$  is the  $\ell$ th-order cumulant of  $\|x^{(q)} - y(t^{(q)}, k, R)\|^2$ .



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### High order Cumulants

The  $\ell$ th-order cumulant of  $\|x-y(t,k,R)\|^2$  is given by:

$$
\kappa_{\ell}=\mu_{\ell}-\sum_{\ell'=1}^{\ell-1}\binom{\ell-1}{\ell'-1}\kappa_{\ell'}\mu_{\ell-\ell'} \qquad (\ell\geq 1),
$$

where  $\mu_\ell$  is the corresponding moment:

$$
\mu_{\ell} = \mathbb{E}_R(||x-y(t,k,R)||^{2\ell}) .
$$



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### Rounded Optimal Attack

#### Rounded Optimal Attack  $(ROPT<sub>1</sub>)$

The rounded optimal Lth-degree attack consists in maximizing the sum over all traces of the Lth-order Taylor expansion  $LL_I$  in the SNR of the log-likelihood :

$$
\mathrm{LL}_{\mathcal{L}} = \sum_{q=1}^Q \sum_{\ell=1}^{\mathcal{L}} (-1)^{\ell} \kappa^{(q)}_\ell \frac{\gamma^\ell}{\ell!} \enspace ,
$$

and we have

$$
\Big|{\rm LL}={\rm LL}_L+o(\gamma^L)\Big|\ .
$$



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## <span id="page-32-0"></span>Complexity Gain

- $\blacktriangleright$  number of possible share values  $\sim$  $\blacktriangleright$  number of traces  $\mathcal{O}\left(\begin{array}{cc} \mathsf{Q} & L \cdot \binom{D+L-1}{L} \end{array} \cdot \ \frac{2^{(\Omega-1)n}}{2^{(\Omega-1)n}} \cdot \ \frac{1}{2^{(\min\left(\left\lceil \frac{n}{2}\right\rceil, L\right)}}\right)$
- $\blacktriangleright$  Factorial terms
	-
	-
	-



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## Complexity Gain

**•** number of possible share values  $\sim$  $\blacktriangleright$  number of traces,

$$
\mathcal{O}\left(\overbrace{Q \cdot L \cdot {D + L - 1 \choose L}}^{L} \cdot \frac{1}{2^{(\Omega - 1)n}} \cdot \left(\underset{\min\left(\left\lceil \frac{n}{2} \right\rceil, L\right)}{\prod_{i=1}^{L} L_i}\right)\right)
$$

 $\blacktriangleright$  Factorial terms

- 
- 
- 



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## Complexity Gain







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## Complexity Gain







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## Complexity Gain



 $\triangleright$  size of the permutation



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## Complexity Gain



 $\blacktriangleright$  size of the permutation



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## <span id="page-38-0"></span>Complexity Gain



Reduces to small constants when  $L \ll D$ 



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## <span id="page-40-0"></span>Implementation of Masking Schemes

In masking schemes, while the implementation of the linear parts is obvious, that of the non linear parts is more difficult.

- $\triangleright$  algebraic methods [Blömer et al., 2004];
- $\triangleright$  global look-up table method [\[Prouff and Rivain, 2007\]](#page-61-0);
- $\triangleright$  table recomputation methods which precompute a masked S-box stored in a table [\[Chari et al., 1999\]](#page-60-1).

In [\[Coron, 2014\]](#page-60-2) a table recomputation scheme secure at order  $\Omega - 1$  was presented.



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### Table Recomputation Algorithm

**input** : t, one byte of plaintext, and  $k$ , one byte of key output: The application of AddRoundKey and SubBytes on t, i.e.,  $S(t \oplus k)$  $1 \;\; m \leftarrow_{\mathcal{R}} \mathbb{F}_2^n, \, m' \leftarrow_{\mathcal{R}} \mathbb{F}_2^n$  // Draw of random input and output masks 2 for  $\omega \in \{0, 1, \ldots, 2^n - 1\}$  do // Sbox masking 3  $\vert$   $z \leftarrow \omega \oplus m$  // Masked input 4  $\mid z^{\prime} \leftarrow S[\omega] \oplus m^{\prime}$  // Masked output  $5 \quad | \quad S'[z] \leftarrow z'$  // Creating the masked Sbox entry 6 end 7  $t \leftarrow t \oplus m$  // Plaintext masking 8  $t \leftarrow t \oplus k$  // Masked AddRoundKey  $9 \t t \leftarrow S'[t] \t //$  Masked SubBytes 10  $t \leftarrow t \oplus m'$  // Demasking 11 return t

- $\blacktriangleright$  usual 2-variate 2nd-order attack:
- $\triangleright$  2-stage CPA attack [\[Pan et al., 2009\]](#page-61-1);
- improved  $(2^n + 1)$ -variate 2nd-order attack on the input [\[Bruneau et al., 2014\]](#page-59-0).



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### Protected Table Recomputation Algorithm

**input** : t, one byte of plaintext, and  $k$ , one byte of key output: The application of AddRoundKey and SubBytes on t

 $m \leftarrow_R \mathbb{F}_2^n$ ,  $m' \leftarrow_R \mathbb{F}_2^n$  // Draw of random input and output masks  $\varphi \leftarrow_{\mathcal{R}} \mathbb{F}_2^{\pi} \rightarrow \mathbb{F}_2^n$  // Draw of random permutation of  $\mathbb{F}_2^n$ 3 for  $\varphi(\omega) \in {\varphi(0), \varphi(1), \ldots, \varphi(2^n-1)}$  do // S-box masking  $\mid\quad z\leftarrow\varphi(\omega)\oplus$   $m$  // <code>Masked</code> input  $\mathsf{5} \quad \quad \quad \mathsf{z'} \leftarrow \mathcal{S}[\varphi(\omega)] \oplus \mathsf{m'}$  // Masked output  $S'[z] = z'$  // Creating the masked S-box entry 7 end  $t \leftarrow t \oplus m$  // Plaintext masking  $t \leftarrow t \oplus k$  // Masked AddRoundKey

- 10  $t \leftarrow S'[t]$  // Masked SubBytes
- 11  $t \leftarrow t \oplus m'$  // Demasking
- 12 return  $t$

Make the index of the loop unknown, use some random permutation  $\varphi$ .



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### <span id="page-43-0"></span>Leakages

**input** : t, one byte of plaintext, and  $k$ , one byte of key output: The application of AddRoundKey and SubBytes on t

```
\n1 
$$
m \leftarrow_{\mathcal{R}} \mathbb{F}_2^n, m' \leftarrow_{\mathcal{R}} \mathbb{F}_2^n / / \text{Draw of random input and output masks}\n2  $\varphi \leftarrow_{\mathcal{R}} \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n / / \text{Draw of random permutation of } \mathbb{F}_2^n$ \n3 for  $\varphi(\omega) \in \{\varphi(0), \varphi(1), \ldots, \varphi(2^n - 1)\}$  do // S-box masking\n4  $z \leftarrow \varphi(\omega) \oplus m' / / \text{Masked input}\n5  $\begin{array}{l} z \leftarrow \varphi(\omega) \oplus m' / / \text{Masked output} \ S'[z] = z' / / \text{Creating the masked S-box entry}\n7 \text{ end}\n\end{array}$ \n7 end\n$
$$

```

8  $t \leftarrow t \oplus |m| / /$  Plaintext masking 9  $t \leftarrow t \oplus k$  // Masked AddRoundKey

10  $t \leftarrow S'[t]$  // Masked SubBytes

- 11  $t \leftarrow t \oplus m'$  // Demasking
- 12 return  $t$



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### <span id="page-44-0"></span>Leakages

**input** : t, one byte of plaintext, and  $k$ , one byte of key output: The application of AddRoundKey and SubBytes on t

1  $m \leftarrow_R \mathbb{F}_2^n$ ,  $m' \leftarrow_R \mathbb{F}_2^n$  // Draw of random input and output masks  $2$   $\varphi \leftarrow_{\mathcal{R}} \mathbb{F}_2^{\pi} \rightarrow \mathbb{F}_2^n$  // Draw of random permutation of  $\mathbb{F}_2^n$ 3 for  $\varphi(\omega) \in {\varphi(0), \varphi(1), \ldots, \varphi(2^n-1)}$  do // S-box masking 4  $\mid\quad z\leftarrow\varphi(\omega)\oplus$   $m$  // <code>Masked</code> input  $\mathsf{5} \quad \quad \quad \mathsf{z'} \leftarrow \mathcal{S}[\varphi(\omega)] \oplus \mathsf{m'}$  // Masked output 6  $S'[z] = z'$  // Creating the masked S-box entry 7 end

8  $t \leftarrow t \oplus m$  // Plaintext masking

- 9  $t \leftarrow t \oplus k$  // Masked AddRoundKey
- 10  $t \leftarrow S'[t]$  // Masked SubBytes
- 11  $t \leftarrow t \oplus m'$  // Demasking
- 12 return  $t$ 
	- ▶ second-order Correlation Power Analysis 2O-CPA;
	- $\triangleright$  OPTimal distinguisher OPT;
		- $\triangleright$  Rounded OPTimal Distinguisher ROPT<sub>2</sub>, ROPT<sub>4</sub>

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### Bi-Variate Attacks





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### Bi-Variate Attacks





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### <span id="page-47-0"></span>Leakages, with Table Recomputation

**input** : t, one byte of plaintext, and  $k$ , one byte of key output: The application of AddRoundKey and SubBytes on t

```
\n1  m <_{\mathcal{R}} \mathbb{F}_2^n, m' <_{\mathcal{R}} \mathbb{F}_2^n // Draw of random input and output masks\n2  \varphi <_{\mathcal{R}} \mathbb{F}_2^n \to \mathbb{F}_2^n // Draw of random permutation of 
$$
\mathbb{F}_2^n
$$
\n3 for  $\varphi(\omega) \in \{\varphi(0), \varphi(1), \ldots, \varphi(2^n - 1)\}$  do // S-box masking\n4  \n5  \n6  \n7  \varphi(\omega) \oplus m // Masked input\n6  \n7  \varphi(\omega) \oplus m // Masked output\n7  \n8  \n9  \n10  \n11  \n12  \n13  \n14  \n15  \n16  \n17  \n18  \n19  \n10  \n11  \n12  \n13  \n14  \n15  \n16  \n17  \n18  \n19  \n10  \n11  \n12  \n13  \n14  \n15  \n16  \n17  \n18  \n19  \n10  \n11  \n12  \n13  \n14  \n15  \n16  \n17  \n18  \n19  \n10  \n11  \n12  \n13  \n14  \n15  \n16  \n17  \n18  \n19  \n10  \n11  \n12  \n13  \n14  \n15  \n16  \n17  \n18  \n19  \n10  \n11  \n12  \n13  \n14  \n15  \n16  \n17  \n18  \n19  \n10  \n11  \n12  \n13  \n14  \n15  \n16  \n17  \n18  \n19  \n10  \n10  \n11  \n12  \n13  \n14  \n15  \n16  \n17  \n18  \n19  \n10  \n10  \n11  \n12  \n13  \n14  \n15  \n16  \n17  \n18  \n19  \n10  \n11  \n12  \n13  \n14  \n15  \n16  \n17
```

8  $t \leftarrow t \oplus m$  // Plaintext masking

- 9 t ← t ⊕ k // Masked AddRoundKey
- 10  $t \leftarrow S'[t]$  // Masked SubBytes
- 11  $t \leftarrow t \oplus m'$  // Demasking
- 12 return  $t$



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### Leakages, with Table Recomputation

**input** : t, one byte of plaintext, and  $k$ , one byte of key output: The application of AddRoundKey and SubBytes on t

```
\n1  m <_{\mathcal{R}} \mathbb{F}_2^n, m' <_{\mathcal{R}} \mathbb{F}_2^n // Draw of random input and output masks\n2  \varphi <_{\mathcal{R}} \mathbb{F}_2^n \to \mathbb{F}_2^n // Draw of random permutation of 
$$
\mathbb{F}_2^n
$$
\n3 for  $\varphi(\omega) \in \{\varphi(0), \varphi(1), \ldots, \varphi(2^n - 1)\}$  do // S-box masking\n4  z <_{\varphi(\omega) \oplus m} // Masked input\n5  z' <_{\varphi(\omega) \oplus m} // Masked output\n6  S'[z] = z' // Creating the masked S-box entry\n7 end\n
```

8 
$$
t \leftarrow t \oplus m
$$
 // Plaintext masking

- 9  $t \leftarrow t \oplus k$  // Masked AddRoundKey
- 10  $t \leftarrow S'[t]$  // Masked SubBytes
- 11  $t \leftarrow t \oplus m'$  // Demasking
- 12 return t

 $\triangleright$  optimal distinguisher NOT computable due to the term 2<sup>n</sup>!



[Introduction](#page-3-0) [Rounded Optimal Attack](#page-28-0) [Case Study](#page-39-0) [Protected Table Recomputation Implementation](#page-40-0) [Bi-Variate Attacks](#page-43-0) [Multi-Variate Attacks](#page-47-0)

### <span id="page-49-0"></span>Leakages, with Table Recomputation

**input** : t, one byte of plaintext, and  $k$ , one byte of key output: The application of AddRoundKey and SubBytes on t

```
\n1  m <_{\mathcal{R}} \mathbb{F}_2^n, m' <_{\mathcal{R}} \mathbb{F}_2^n // Draw of random input and output masks\n2  \varphi <_{\mathcal{R}} \mathbb{F}_2^n \to \mathbb{F}_2^n // Draw of random permutation of 
$$
\mathbb{F}_2^n
$$
\n3 for  $\varphi(\omega) \in \{\varphi(0), \varphi(1), \ldots, \varphi(2^n - 1)\}$  do // S-box masking\n4  z <_{\varphi(\omega) \oplus m} // Masked input\n5  z' <_{\varphi(\omega) \oplus m} // Masked output\n6  S'[z] = z' // Creating the masked S-box entry\n7 end\n
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t \leftarrow t \oplus m
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- 10  $t \leftarrow S'[t]$  // Masked SubBytes
- 11  $t \leftarrow t \oplus m'$  // Demasking
- 12 return  $t$ 
	- ighthird order attack MVA<sub>TR</sub> [\[Bruneau et al., 2015\]](#page-59-2);
	- $\triangleright$  Rounded Optimal Distinguisher ROPT<sub>3</sub>.



[Protected Table Recomputation Implementation](#page-40-0) [Bi-Variate Attacks](#page-43-0) [Multi-Variate Attacks](#page-47-0)

### Complexity of the Case Study



The time of execution have been computed on a Intel Xeon X5660.



[Protected Table Recomputation Implementation](#page-40-0) [Bi-Variate Attacks](#page-43-0) [Multi-Variate Attacks](#page-47-0)





[Protected Table Recomputation Implementation](#page-40-0) [Bi-Variate Attacks](#page-43-0) [Multi-Variate Attacks](#page-47-0)

## $(2^{n+1}+2)$ -Variate Attacks on Shuffled Table Recomputation



Figure: Number of traces to reach 80% of success



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### Conclusion

#### **Results**

We have presented a practical, truncated version of the theoretical, optimal distinguisher:

- $\blacktriangleright$  becomes efficient:
- $\blacktriangleright$  remains effective

#### **Perspective**

How to quantify the accuracy of the approximation?



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### Conclusion

#### **Results**

We have presented a practical, truncated version of the theoretical, optimal distinguisher:

- $\blacktriangleright$  becomes efficient:
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#### **Perspective**

How to choose the degree of the Taylor Expansion?



## Thank you for your attention.



30/30 December 2016 Taylor Expansion of Maximum Likelihood Attacks

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